THE STRUCTURED HEDGING OF FINANCIAL VALUE:
WITH APPLICATIONS TO FOREIGN EXCHANGE RISK MANAGEMENT

by

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Statement of Sources

The work presented in this thesis is, to the best of my knowledge and belief, original to the thesis. Parts of the thesis are based upon papers written jointly with one or more of the thesis supervisors and are published at a number of professional journals. Material based on such papers is explicitly footnoted in the text. Full acknowledgement is made and references given to all other sources used. The material has not been submitted either in whole or in part for a degree at this or any other university.
Abstract

The objective of the thesis is to develop a structured financial hedging framework that is empirically implementable and consistent with a corporate finance perspective. Value at risk provides a suitable framework for this purpose. The aversion implied in the value at risk and its generalised theory arises from a firm’s concerns about contingent financial distress costs, which can be considered as the payoff of a put option written by stockholders of firms in favour of third parties. This enables the development of a hedging framework to explore how a firm’s welfare might be enhanced by replacing natural exposures with hedged outcomes. An ideal hedging decision is to maximise the financial value in good times at minimal cost in terms of the generalised value at risk penalty function. In an efficient market, a fully hedged policy using forwards is generally the optimal decision, while alternatives should be taken into account where markets are not efficient. In such cases, the underlying empirical methodology should be able to detect inefficiencies and feed into the objective functions for maximising firm value.

The empirical implementation is explored with a variety of econometric methodologies. These include the development of new semi-parametric or non-parametric techniques based upon wavelet analysis, as well as an incomplete forecasting algorithm. Such methods have been preferred to classical linear and stationary models, because they have broader application in an inefficient market where information is technically fuzzy and financial data may exhibit non-linearity or non-stationarity. Further decision dimensions concern exposure duration or path risk, in which individuals’ perspectives of risk is time-dependent and linked to the evolution of value at risk through time. The proposed approaches find their main application in foreign exchange risk management, a topic of considerable importance and sensitivity in New Zealand. A statistically well-adapted hedge object for an exporter such as the dairy industry is the corporate terms of trade, which balances up output and expense prices as a single index related to the net profit margin. Further applications are to strategic fund management where the objective is to derive optimal foreign exchange forwards based hedges.
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List of Symbols

\( a \) – Log of currency reference rate
\( A \) – Wavelet decomposition approximations; or currency reference rate
\( b \) – Risk aversion parameter
\( B \) – Standard Brownian motion; Bankruptcy point in the welfare function
\( d \) – Fractional integration indicator
\( D \) – Wavelet decomposition details
\( e \) – Exchange rates; or identity matrix
\( E_k \) – Wavelet energy at scale \( k \)
\( F \) – Forward exchange rate
\( f \) – Log of forward exchange rate
\( h \) – Hedging ratio
\( i \) – interest rate; or commodity currency in the exchange rate
\( i^* \) – The interest rate at time \( t \) in term currency country
\( i^r \) – The interest rate at time \( t \) in commodity currency country
\( I \) – Indicator signals
\( J \) – Highest scale of the wavelet decomposition
\( j \) – Scale indicator in the wavelet decomposition; or terms currency in the exchange rate
\( \kappa \) – Frequency scale indicator
\( L \) – Lag indicator; or sum of log likelihood function
\( l \) – Likelihood function
\( N \) – Total number of observations
\( p \) – Probability
\( P \) – Value at Risk indicator, or the corporate’s financial distress point
\( r \) – return indicator; or index function in the directional forecasting model
\( R \) – Welfare or exposure variable; or regime in the exchange rate forecasting model
\( S \) – Bilateral exchange rate
\( s \) – Log of bilateral exchange rate
\( t \) – Time
\( T \) – Maturity
\( U \) – Utility
\( V \) – Accumulating portfolio value
\( x \) – Portfolio choice at time \( t \)
\( X \) – The history of portfolio settings; or predictor variables in GARCH model
\( Y \) – Dependent variable
\( Z \) – Exogenous indicator variables
\( \alpha \) – Confidence level; or parameter in the exchange rate forecasting model
\( \beta \) – Parameters
\( \gamma \) – Order of tail moment
\( \Gamma \) – Probability measure
\( \omega \) – Path history
\( \tau \) – Interval time
\( \mu \) – Mean
\( \mu_d \) – The effects of hedging on the mean part
\( \nu_d \) – The value added by hedging through decreasing the GVaR
$\sigma^2$ – Variance

$\varphi$ – Path risk metric

$w$ – Weight

$\Phi$ – Standard normal distribution

$\lambda$ – Loading weight

$\Lambda$ – Collective loading weights

$\nu$ – Risk constraint

$\rho$ – Time weight

$\delta$ – Boundary marker parameter

$\xi$ – Random variable

$\pi$ – Risk premium

$\Re$ – Positive definite function

$\eta$ – Lagrange multiplier

$\varepsilon$ – Error term

$\theta$ – Generalised Rubinstein risk premium; or parameters in the exchange rate forecasting model

$\Delta$ – Difference

$P(\cdot)$ – Probability function

$F_R(\cdot)$ – Distribution function of the variable $R$

$SF(\cdot)$ – Step function

$E[\cdot]$ – Expectation operator
List of Abbreviations

ADF – Augmented Dickey-Fuller
ARMA – Autoregressive moving average
AWE – Average wavelet energies
CD – Certificate deposit
CIP – Covered interest rate parity
CRR – Currency reference rate
CVaR – Conditional value at risk
DWT – Discrete wavelet transform
ECR – Effective conversion rate
EM – Expectation maximisation
FTT – Farmer terms of trade
GDP – Gross Domestic Product
fit – log of farmer terms of trade
GARCH – Generalised autoregressive conditional heteroskedasticity
GVaR – Generalised value at risk
IAS – International Accounting Standard
ML – Maximum likelihood
MODWT – Maximal-overlap discrete wavelet transform
MV – Mean variance
NPM – Net profit margin
OCR – Official cash rate
OMD – Ordered mean difference
PI – Profitability index
PP – Phillips-Perron
PPP – Purchasing Power Parity
QML – Quasi maximum likelihood
QMLE – Quasi maximum likelihood estimator
RBNZ – Reserve Bank of New Zealand
REF – Reward energy frontier
SFAS – Statement of Financial Account Standard
TWI – Trade weighted index
UIP – Uncovered interest rate parity
List of Abbreviations

VaR – Value at risk
Chapter 1 Introduction and Overview

The risks arising from fluctuating prices, interest rates, or exchange rates have not been diminished by the recent liberalisation and globalisation of financial and commodity markets. These developments have expanded the range of instruments available to manage the risks, whilst at the same time introducing new risks associated with such usage. In using them effectively, managers must pay considerable attention to the underlying welfare objective. This thesis is mainly concerned with the interactions that arise between objectives and instruments. Firstly, an adequate objective function is specified on the basis of value at risk and its generalised theory, which is consistent with firm value or the investment value of a managed fund. Then a structured risk management framework is established to improve that value. This will include an analysis of when hedging instruments should be used and how these can be utilised most effectively. These two issues will be largely explored through the empirical implementation, with the aid of a variety of developed econometric models and methods.

The motivation for the thesis originated in a series of adverse risk events which will be described in the case studies of Chapter 2. It has become evident that hedging decisions have to be empirically based upon some structured decision rules or algorithms. A number of tasks or considerations arise in the development of such a framework, the more important of which are summarised in the following decision dimension section.

1.1 Decision dimensions

(a) The first task in developing a structured risk management framework is to define the financial criteria. From the corporate finance viewpoint, risk management is appropriate only to the extent that it can maximise the market value of the firm. The firm’s value increases with real or expected value creation and decreases with the costs associated with any form of financial distress. Such financial distress costs can arise from the implicit put options written by the firm’s stakeholders in favour of third parties. When a corporate experiences financial difficulty, liquidators, statutory managers, management consultants, and competitors all benefit from the event.

Out of all risk management criteria, such as variance, semi-variance, extreme value and capital adequacy, it is the value at risk (VaR) and its generalised theory that
correspond most closely with firm value. The generalised value at risk (GVaR) can be utilised as a tool in representing an adverse exposure to put options that diminish the value of a firm. This allows corporate managers to have the freedom to pursue gains. Hence, the financial criterion developed in this thesis for optimal currency risk management is based on the VaR and its generalised theory. Since its inception in the 1990s, VaR, as a well known quantile risk management tool, has gradually become an industry standard for implementing risk management. This thesis rationalises the G VaR in the context of corporate finance, by explaining and emphasising the linkage between the measurement of GVaR and firm’s financial distress cost.

(b) The thesis further extends the usage of the GVaR in long term risk management. A static financial criterion, which is applicable for short or intermediate term hedging against the risk that diminishes the financial value of a corporate or a portfolio, may be inadequate for developing a long term dynamic risk management framework. Over a long period, financial prices may evolve with longer run cyclical influences which could arise from business cycles, interest rates, exchange rates, and economic policy changes. An overall GVaR indicator usually based on single-period returns, perhaps with simple extensions, might fail to identify this macro-scale variation pattern, because the cyclical influences could be swamped by short run noise.

The thesis therefore develops dynamic value at risk metrics in a multi-scale approach to assess the relative risks between various paths. The proposed model enables a better reconciliation between measurement and design methodology, on the one hand, and underlying welfare objectives on the other. The methods that result are adapted to long term risk management in an environment where both longer run - macro-scale and shorter run market disturbances impact on asset values. The developed long term risk management techniques are applied to strategic fund investment. Investment outcomes are compared in terms of the value paths which are distinct from a more narrow focus on either short run or longer run returns in isolation. Path risk becomes a secondary welfare objective, arising from path exposures and measurable in the form of a penalty functional of spectral power.

(c) The implementation of a risk management policy that depends on a GVaR approach can be either passive or active, depending on whether a firm uses its own information in making decisions. If a firm’s managers believe markets are informationally efficient, they are more likely to adopt a passive approach. On the
other hand, an active stance, would indicate that managers are confident that their own information might be superior to that of the market as a whole. Managers that actively use their own information can improve their risk management outcome provided that their information or expertise remains confidential from the market.

In developing an active risk management approach, this thesis attempts to contribute to the existing literature by constructing a directional exchange rate forecasting model in a market where information is incomplete and fuzzy. Information is considered incomplete in a currency market when macroeconomic indicators present conflicting signals about exchange rate changes or when the role of indicators in determining the direction of currency movements varies from time to time. In these circumstances, a precise level estimation over long term or even intermediate intervals is difficult and thus directional calls alone are usually more successful. This thesis develops a directional forecasting model to assign numerical probabilities to possible outcomes (for example, to move up, stay the same, or move down) based on available information. Unlike previously published models for directional exchange rate forecasting, the proposed model allows for both incompleteness and asymmetry of available information. This is achieved by incorporating into the model artificial techniques, such as fuzzy logic and neural nets.

To integrate directional calls and probabilities into risk management actions, a suitable welfare or loss function needs to be associated with possible outcomes. Loadings can then be devised that weigh the directional probabilities according to the welfare consequences of state transitions, so that the resulting risk management decision reflects not only directional probabilities, but also welfare outcomes. The results can also provide the statistical inference associated with the variable selection in the estimation phase.

In order to verify the value added by such an active risk management approach, both in-sample and out-of-sample diagnostic tests are developed. The outcomes from both tests may also indicate whether the financial market is efficient.

(d) The present thesis examines the usefulness of the proposed GVaR model, along with the developed hedging algorithms and associated theory and practice, in the context of corporate risk management. Addressing these issues requires the selection of a specific hedging data framework. From the corporate finance perspective, a complete solution would by necessity entail hedging the total free cash flows to investors. This is impractical for a number of reasons, including the inadequacy of
available data, and the difficulties associated with normalising variables. All variables should be consistently expressed, in order to avoid the problem of hedging stationary with non-stationary time series data. The thesis resolves the problem by isolating the most critical exposures in the form of ‘corporate terms of trade’, and in effect creates a new single indicator that accounts for corporate net profit margin.

Utilising the ‘corporate terms of trade’ as the underlying exposure indicator distinguishes the current hedging application from most previously published hedging literature which focuses on the single risk factor, of either the financial rate (for example, exchange rate or interest rate) or commodity price. Employing this form of compound exposure measure also alerts users to the dependence structure in the critical corporate exposure and its impact on the potential risk management strategy. Although hedging is widely considered a value increasing activity, it may have negative effects on firm value when there are natural buffering effects among various exposure components.

(e) The thesis shows how the implementation of the proposed risk management strategy for both corporate and fund management draws on wavelet analysis. Static distributional theory does not adapt well to the inter-temporal context and requires restrictive hypotheses as to the underlying data generation mechanism. Wavelet analysis on the other hand provides an elegant way of overcoming these restraints. As an analytical tool, wavelet analysis has been widely used in disciplines such as acoustics, astronomy and engineering. Its application in economics and finance, however, has been much more limited (Crowley 2005). This thesis contributes to wavelet and financial literature by extending the application of wavelet analysis to hedging and portfolio management.

In the applications described, wavelet analysis serves as an instrument for examining the exchange rate fluctuations as well as a tool of decomposing corporate natural exposures. The energy decomposition under wavelet analysis reveals variation patterns in terms of both time and frequency. Decomposition results can lead to a better understanding of the underlying variation patterns. When applied to corporate natural exposures, the results can also shed light on the fundamental origins and causes of adverse exposures, which in turn are valuable for deriving the optimal hedging decision. The multi-scale decomposition of time series into wavelets can, furthermore, be employed to design portfolios tailored to preferences between long
and short run variation. Consequently, portfolios can be constructed based on band pass filters, which take into account designated long or short-term value fluctuations.

(f) In this thesis, the proposed risk management structure is principally applied to foreign exchange-based hedging from the perspectives of New Zealand trading companies and strategic fund managers. Perhaps owing to the small size of New Zealand’s economy, existing research has paid little attention to the currency hedging problem in New Zealand due to the size of New Zealand’s economy. On the other hand, currency risk management must play an important role in New Zealand as its economy relies heavily on international trade and is vulnerable to fluctuations in currency values. In addition, New Zealand has a particularly unstable currency that exposes the economy to significant currency risks. The current monetary policy in New Zealand, which relies on official interest rate adjustments, is considered by many to exacerbate the exchange rate volatility (e.g. Bowden, 2006c; Zettelmeyer, 2004). A narrow commodity-based economy, along with soaring offshore funding requirements due to housing booms, also contributes to the instability of the NZ dollar. Whatever the macroeconomic causes, the micro-economic consequences of currency variations have on occasion been severely adverse. New Zealand therefore provides an ideal market to test the developed risk management model and theory, although the model is also useful in a more general context.

As a case study, the thesis offers a multi-dimensional analysis of currency exposure encountered by New Zealand dairy farmers who make a significant contribution to the New Zealand economy but are particularly exposed to exchange rate variations because of the large proportion of their revenues denominated in foreign currency. Although the cyclical behaviour in the New Zealand dairy industry is well-known to academics and other practitioners, this thesis adds to existing research by articulating through wavelet analysis how the cycles are generated and sustained. An understanding of the origin and the regularity of these cycles should assist dairy farmers to develop production and risk management strategies.

This thesis also explores how New Zealand corporates derive an optimal hedging ratio with the proposed risk management framework. Capital importing countries like Australia or New Zealand have interest rates that are persistently higher than their trading partners. This implies a forward discount on their exchange rates and a natural motivation for exporters to sell forward their foreign exchange receipts. However, exclusive use of forwards by exporters is only attentive to the expected
positive return on the forward conversion rate relative to the spot rate at that time and does not take potential risks into account. The thesis argues that a more appropriate hedging portfolio for New Zealand corporates would contain both the unhedged spots and either forwards, or a range of forward maturities, in proportions that depend upon managerial preferences.

Finally, locational and productional decisions are frequently based on perceived currency strength and weakness. A relevant contribution of this thesis is the development of a ‘reference exchange rate’ or ‘absolute exchange rate’. This aims to isolate the currency value of a particular country from the quoted two-way exchange rate and enable the comparison of currency stability in different countries. The currency comparison based on the reference exchange rate assists the company in assessing exposures with respect to a particular currency. This assessment will further support the corporate managers in developing a global exchange rate risk management strategy.

The quoted exchange rate is bilateral and its variation could arise from a change in either home or foreign currency. If a fluctuation in the exchange rate is caused by events in a foreign country, a multinational company that has business in several countries might experience low currency exposure due to the diversification effects. In other words, an exposed company might reduce its level of exchange rate risk by geographically diversifying its business. On the other hand, if exchange rate variation relates to circumstances in the home country, the currency risk is systematic and not geographically diversifiable. In these circumstances, financial hedging generally plays an important role in a multinational company’s risk management strategy.

To construct a reference exchange rate, the currency value is measured in relation to a common reference basket so that a set of no-arbitrage prices arise as a result. The no-arbitrage feature distinguishes the proposed method from the trade-weighted index, which is the classical methodology for measuring currency fluctuation in one country. Further, the proposed reference basket for constructing an absolute exchange rate is analogous to a portfolio, and its choices can be dependent on specific economic interpretations or uses. Historical variability in different currencies over specific cyclical bands is compared on the basis of the developed no-arbitrage reference rate.
1.2 Thesis organisation

This thesis is structured as follows.

Part I discusses the motivation of the thesis. Chapter 2 reviews a series of case studies on foreign exchange risk management and illustrates a variety of problems that hedging can cause. With the benefit of hind-sight, these case studies are used to distil some key questions or issues that are addressed in the latter part of the chapter.

Part II considers the basic conceptual issues that are important in a risk management framework, including how to decide whether or not to hedge in the first place. Chapter 3 starts by establishing an appropriate guiding principle, and hence the objective function, through an analogy between contingent distress costs and GVaR. A discussion about the perspectives of corporate hedging follows. Chapter 4 extends the GVaR management framework to a long term and dynamic context to include the aversion to the path risk, from a strategic fund manager’s perspective.

Part III, spanning Chapters 5 and 6, explores the econometric background in order to provide an empirical input into subsequent hedge algorithms. These chapters explore the issues of how to measure the variation of currency value and whether exchange rate markets are efficient. Alternative empirical models, such as wavelet frameworks and directional models, are developed to detect the market inefficiency and derive a directional forecasting.

Part IV establishes empirical hedging algorithms in the framework of corporate currency risk management. Chapter 7 focuses on operational considerations of corporate hedging. Chapter 8 firstly considers hedges based on unconditional distributions and, as an example, uses a specially constructed terms of trade index to account for farmer exposures. Hedges derived from conditional distributions are established in the second part of the chapter.

Part V discusses the context of portfolio investment and funds management. The main issue explored here is the duration of exposures, and the differential preferences between short and longer term variations in portfolio value. Chapter 9 formalises these issues in terms of a concept of path risk and shows how to implement it with an extension of spectral utility functions through wavelet analysis.

The thesis concludes with a review of the principal insights from the study.
Part I The Background

Chapter 2 The Importance of Currency Risk Management

The global trend of currency regimes changing from fixed to floating has led to different currency risk exposures, with growing short and intermediate term volatility and unpredictability. This has increased the importance of currency risk management in corporations that face significant exchange rate exposures. In the past few decades, growth of the financial derivative market has enabled corporations and individuals to hedge the currency risk in the financial market. However, financial instruments may have their own risk and ineffective uses of them can result in serious problems for hedgers, as shown by a variety of case studies (to follow). In these cases, corporates employed financial derivatives as hedging instruments in order to reduce the currency risk. Nevertheless, the consequent outcome was contradictory to the hedging objective. Some companies were falling into financial distress because they misapplied the hedging instruments, such as over-hedging the exposures and leading firms to face net exposures in the financial market. Companies that matched the hedging instruments with their spot exposures, on the other hand, were sometimes also threatened with bankruptcy because of significant losses on their hedging position. Several research questions are thus derived from these case studies, mainly about whether and how financial value can be enhanced by replacing natural exposures with hedged outcomes. They form the motivation for this thesis.

This chapter is organized as follows. In section 2.1, the effects of increasing uncertainty on currency risk management are explored. This is followed by a short section discussing currency risk management tools. Section 2.3 introduces some case studies that reveal how corporations can suffer from their hedging position. Section 2.4 discusses the lessons that can be learned from these cases. Section 2.5 describes the proposed research questions. Section 2.6 discusses the New Zealand context. Section 2.7 concludes the chapter.

2.1 Effects of more volatile exchange rates

Since the breakdown of the Bretton Woods System, currency markets have been confronted by increasing uncertainty. The Bretton Woods System, which prevailed from the end of World War II until the early 1970s, is commonly understood as a
pegged rate currency regime. In such a system, the United States dollar served as the reserve currency that could be converted into gold while currencies for other nations were pegged against the US dollar. Governments were required to intervene in currency markets in order to maintain exchange rates within the target band. When a significant fundamental dis-equilibrium occurred, however, the rate needed to be adjusted.

Despite the success at the beginning of its establishment, this system experienced a series of crises from 1967 to 1973. Among the many economic and political reasons for the breakdown of the Bretton Woods System, the growth of international capital and trading flows made it difficult to maintain a pegged rate. The resulting instability in currency markets and national economies led economists to advocate a floating rate system. The key currencies, as a result, started to float, for example, the British sterling floated in 1972 and the US adopted a fully floating currency system in 1973. Many other major currencies, such as the Canadian dollar, were floating against the US dollar by 1973. The Australian and New Zealand dollar were set afloat in 1983 and 1985 respectively. In contrast to the expectation that floating exchange rates might stabilize the real exchange rate, Figure 2.1 reveals that the NZD/USD\(^1\) exchange rate time series from 1948 to 2007 exhibited as high or higher volatility after the 1970s, in terms of both nominal and real exchange rates.

\(^1\) Exchange rates are expressed in the way of A/B, with A as the commodity currency and B as the terms currency, e.g. NZD/USD=0.75 implies 1NZD=0.75USD, where USD is the US dollar and NZD is the New Zealand dollar. All the exchange rate data employed here and in what follows are mid rate. Real exchange rate is derived from adjusting the nominal exchange rate with the CPI indexes for NZ and US. The monthly exchange rate and CPI indexes are all from Global Financial Data.
Besides the growing instability in the exchange rate market, the transformation of currency regime also affects our notion of currency risk. Under a fixed or pegged rate regime, the exchange rate changes suddenly and by a large amount. The consequent currency exposures are infrequent, with fewer problems on a day to day basis. Even with the sudden changes, corporations and individuals usually have better signals making it easier to anticipate the adjustment. For instance, when the central bank appears to have lost control of the currency value that is under the pressure of depreciation, the currency might very likely move down. Simultaneously, because corresponding financial currency derivatives are hard to write under a fixed exchange rate regime, corporations can do little in this circumstance to manage currency risk in a financial market. In addition, the rigidity of regulations for cross-nation currency movement prohibits corporations from hedging currency exposure with other financial tools.

When floating exchange rates prevail world-wide, the nature of currency exposures and the way of managing them have changed. The currency exposures become more frequent and have increasing short term variation. Given the complex causations of these short intervals as well as long term fluctuations, the duration and magnitude of exposures become more difficult to predict.

Foreign exchange risk management literature usually identifies three sorts of currency exposures:
(1) Translation or accounting exposure: refers to the impacts of currency fluctuations on the financial statements of the firm.

(2) Transaction exposure: refers to the effects of currency changes upon identifiable transactions committed or contracted at some point of time in the future.

(3) Economic exposure: refers to the longer term or ultimate effects of currency changes upon the value of the firm. They encompass both strategic and competitive exposures, which arise from cost or pricing pressures with respect to the currency variations.

The overall concern in the present study will be to analyse the effects of currency changes upon corporate value, which relates mainly to economic exposure, but also to transaction exposure to some extent. The translation exposure, also known as accounting exposure, is ignored in the current context. This is because existing empirical research shows that translation exposure management does not add value to firm value (e.g. Hagelin, 2003; Hagelin & Pramborg, 2004).

Foreign exchange exposures can be experienced by any institutions with cross-border transaction. Trading companies are one of the major groups which encounter significant currency exposures. The mis-match caused by revenue and expenses being denominated or determined in different currencies results in uncertainty in net cash flows as well as the corporate values. In the face of a volatile exchange rate market, companies may not know exactly what they will receive or pay in the future, despite knowing in advance the prices in foreign currencies. An unfavourable currency fluctuation may turn a trade that is profitable at the time of signing the contract into a significant loss when it is settled. The corporate currency exposure can be further exacerbated by a rival’s actions in a competitive environment. In this circumstance, a well run firm may face the threat of bankruptcy just because it fails to manage currency risk effectively, even though the company is still competitive in terms of production technology and marketing strategy.

With the globalization of the capital market, financial institutions – especially those which possess a portfolio comprising a large proportion of overseas investments or liabilities – are also exposed to currency risk. How exchange rates evolve during the entire investment period can have a major impact on the overall return, irrespective of whether the asset class is equity, fixed interest security or real estate. Given the high volatility in currency markets, to ignore the exchange rate variations in
deciding asset or liability allocations might result in a considerably risky portfolio, as in the examples of Malaysia, Indonesia, and some other Asian countries in late 1997 (Corsettia, Pesenti & Roubinic, 1999). In these instances, the soaring overseas debt obligations resulting from depreciated Asian currencies caused the financial situation to deteriorate and eventually, along with other factors, led to a financial crisis in the region.

The adverse outcomes arising from currency fluctuation extend beyond corporate exposure to the economic fortune of nations. Following a considerable boom in the 1980s, the Japanese economy slumped quickly, mainly because exporters were hurt by the strong Japanese Yen. Recently, Chinese products have been highly price-competitive in the world market, which has generated a boom of the Chinese economy. One of the reasons is that the low Chinese Yuan encourages exporters who can afford to sell the goods at a lower foreign currency-denominated price. On the other hand, exporters in the US have been discouraged by the high US dollar since the late 1990s. In the face of a strong domestic currency, a large number of Japanese manufacturers and US corporations have been forced to exit the market or move production abroad.

2.2 Currency risk management tools

In the last several decades, a number of operational tools have been developed to manage currency risk. One way to eliminate the adverse impacts arising from currency variations is to adjust the local product price. The currency exposures can then be absorbed by raising the sale price or decreasing the payout price. Given a corporation’s fear of losing its share of the market, such a pricing strategy may, however, not be applicable in a price sensitive market. In fact, product pricing is usually employed as the tactic to raise a market share when the currency moves in a favourable direction, rather than as a tool to eliminate currency exposures. An efficient currency exposure management device that has been widely used by multinational companies is the geographic dispersion of subsidiaries over countries and regions. With multiple possible sources of production, multinational companies have bought themselves the option of reweighing sources towards whichever currency bloc is temporarily most advantageous. Another operational tool for corporations to manage currency risk is to enhance productivity. Currency fluctuations can encourage a firm to become more efficient in order to survive in the market.
Bradley and Moles (2002) show it is common for multinational corporations to manage their long term currency exposure with operational tools. However, the authors also find that operational risk managing tools are not effective substitutes for financial market-based hedging instruments. This thesis will concentrate on discussing the financial hedging with derivatives. Since the 1970s, a range of financial instruments, including forwards, options, and swaps have been developed to meet the needs of companies’ risk management. Recently, the financial market is characterised by the further development of more complex forms of instruments, such as exotic options and hybrid securities. Nevertheless, the focus of this thesis is not to investigate the evolution of financial derivatives. Instead, the task is to explore whether companies can manage the currency risk with financial instruments by following an established structured financial hedging framework. Therefore, the discussion of currency risk managing tools will be limited to standard financial derivative instruments, such as forward and option contracts. This, however, does not rule out the use of more complex derivative products.

2.3 Case studies

Although some empirical studies have shown how firms benefit from currency hedging (Geczy et al, 1997; Allayannis & Weston, 2001), the issue of hedging with financial instruments, among strategies for foreign exchange risk management, can become controversial in some circumstances. The case studies that follow mainly describe what happened when export companies attempted to protect themselves against a strong or stronger domestic currency. Whether or not the adopted strategies were optimal, given the information available at the time, is reviewed in the section following (section 2.4).

2.3.1 New Zealand Dairy Board

The following discussions are largely based on Bowden (2005b). The New Zealand Dairy Board, a monopoly producer which was the forerunner of the largest New Zealand dairy company, Fonterra, encountered significant losses on its hedging position in 1997.

The currency exposures arose for the Dairy Board when the price of dairy products sold overseas was mainly denominated in terms of foreign currencies, principally US dollars, while the expenses for farmers were largely in New Zealand
dollars. In the face of a floating exchange rate, the Dairy Board’s cash flows as well as profits became unstable. The instability caused by the currency variation would be eventually passed on to farmers, who were suppliers and also shareholders of the cooperative company, the Dairy Board. To protect farmers from currency fluctuation, financial instruments were usually employed to manage the currency risk and the hedging strategy had worked well until 1997.

Figure 2.2 shows the NZD/USD exchange rate history from 1986 to 2007. In 1995, the New Zealand dollar started to appreciate and reached to a historical high of US 71 cents in 1997. The Dairy Board decided to increase the hedging ratio under the consideration that business for some farmers might not survive if there was any further appreciation in the New Zealand dollar. As a consequence, the following mixed option-forward based strategy was employed:

1. Buy at the money call options on the NZ dollar, at a strike price of US69c.
2. Sell a series of put options on the NZ dollar, at a strike price of US64c.
3. Purchase some straight forwards of the NZ dollar against the USD.

Maturities with regards to the above forwards and options were mixed, with some of them as long as three years. Put options were written for funding the purchasing of call options. The combination of short a put and long a call have similar effects as holding forwards: locking the rate at a pre-determined level, which can be a particular point or a band according to whether short and put option strike prices differ or not. The reason for heavy reliance on forwards and option-based forwards was due to the fact that extremely high exchange rate volatility at that time had resulted in very expensive options.
What happened subsequently was that the NZ dollar dived rapidly. This was compounded by contagion effects of the Asian financial crisis. As a consequence of the depreciated home currency, the Dairy Board booked a loss of half a billion NZ dollars in the derivative market. The hedging policy of this dairy company was broadly criticised by farmers who compared the hedging exposures with outcomes when no hedging occurred. Under the existing hedging, farmers were locked at an extremely low price for up to three years, though the subsequent recovery of dairy prices eased the burden to some extent.

2.3.2 New Zealand Apple and Pear Board

The fluctuation of the NZ dollar in 1997 also affected the New Zealand Apple and Pear Board. The following discussion of this case is based on Bowden (2005b). The New Zealand Apple and Pear Board was another statutory monopoly producer cooperative. In a similar strategy to that of the Dairy Board, the Apple and Pear Board took a heavy position in the forward market in the late 1990s, in order to mitigate the future currency exposure arising from a further rising New Zealand dollar. When the domestic currency depreciated subsequently, the hedging losses occurred in the derivative market, especially from some long-dated forwards. These losses significantly impaired the corporate’s profits. In order to reduce these opportunity

\[ \text{Figure 2.2 Historical exchange rates of NZD/USD}^2 \]

\[ \text{The data of monthly exchange rates and CPI index comes from Global Financial Data and spans from Feb 1986 to Mar 2006.} \]
costs, the Board entered into a deal with Citibank to adopt a straddle put call options strategy. It sold NZD puts at a lower strike price but bought NZD calls at a higher strike price than the current spot rate. When the exchange rate depreciated again, the Apple and Pear Board suffered a large loss related to the short put options.

After the huge losses in the derivative market, an official enquiry found the losses derived from the Board taking a net exposed position in foreign exchange derivative market. This was not allowed within the regulations governing the Board. In this case, the total amount of put options and residual forwards held by the Apple and Pear Board exceeded the natural exposures. For such a portfolio, some of the opportunity losses occurring to financial instruments could not be off-set by gains at spot market. The situation was worsened by the world fruit prices simultaneously experiencing a downward trend. As the dairy farmers had experienced in the previous case, the apple and pear growers, who are suppliers and shareholders of the Apple and Pear Board, suffered from the unfavourable currency hedging outcome and additionally low product prices.

### 2.3.3 Solid Energy New Zealand Limited

Another company that failed to hedge optimally was Solid Energy New Zealand Limited, a state owned entity that sells a large amount of coal to the foreign market. The following study is based on the New Zealand Office of the Auditor-General Report (1999), which describes how state-owned enterprises manage foreign exchange risk.

In 1998, the total revenue of Solid Energy New Zealand Limited was $187 million, of which $94 million were foreign currency income. Given the high proportion of overseas revenue, Solid Energy practised an explicit currency risk management policy that guided the firm’s hedging decision, in terms of the hedging ratio and maturity period. The firm’s hedging strategy was relatively passive as a regular revision on the items of policy was absent. Since the company was established in 1987, Solid Energy had reviewed the hedging policy only twice, in 1995 and 1997. The following table 2.1 illustrates the policy in 1997.
Table 2.1 Solid Energy Limited policy for foreign exchange cover: 1997

<table>
<thead>
<tr>
<th>Export Sales Exposure</th>
<th>Forward Exchange Contracts</th>
<th>Option Cover</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>min %</td>
<td>max %</td>
</tr>
<tr>
<td>Forecast receipts up to 3 months</td>
<td>35</td>
<td>90</td>
</tr>
<tr>
<td>Forecast receipts 4 to 12 months</td>
<td>35</td>
<td>90</td>
</tr>
<tr>
<td>Forecast receipts 13 to 24 months</td>
<td>35</td>
<td>80</td>
</tr>
<tr>
<td>Forecast receipts 25 to 36 months</td>
<td>30</td>
<td>65</td>
</tr>
<tr>
<td>Forecast receipts 37 to 60 months</td>
<td>30</td>
<td>65</td>
</tr>
</tbody>
</table>

The policy reviewed in 1997 extended the maximum long term exposure cover period from three years to five years. Long term hedging was favoured because the firm felt comfortable with the available exchange rate. The currency value, the product price and the sale volume at that time, were expected to result in a satisfactory return for the company. Therefore, corporate managers were inclined to forgo the potential gains arising from the future depreciation of currency value, for the purpose of ensuring a long term return. However, they did not take into account the possible changes of price and volume of the exported coal, which determined the expected return along with the currency value.

Solid Energy Limited decided the amount of hedging instruments on the basis of the forecasted export sales as well as contracted sales. From 1997 to 1998, Solid Energy raised its foreign exchange cover substantially. The total cover held at 30 June 1998 was recorded as $468 million while there was only $165.2 million at the same period in 1997. Of the new contracts, 87% were bought between September and December 1997, when the New Zealand dollar was historically high. The rise in foreign exchange cover was mainly because of the higher forecasted sales, which were described in a five-year business plan. In 1997, the plan included two projects, namely the Mount Davy Mine and the West Coast Jetty. Given the expectation that the completion of the two projects would boost the export sales, Solid Energy decided to go long more forward contracts to cover the consequent higher foreign exchange risk. However, the projects had not even been approved when the hedging contracts were bought. At that time period, it was still uncertain when the projects would be completed and how much additional revenue could be generated.
In addition, to be consistent with the policy that encouraged the long term currency hedging, more than half of the new currency forward contracts purchased in 1997 and 1998 were for 37 to 60 months. Options covering periods 1 year to 5 years ranged from $10 million to $40 million, although the policy in table 2.1 prescribed the maturity of options could not exceed 12 months. Because there were no contracted sales beyond 3 years at 30 June 1998, long term hedging instruments were entered into mainly on the basis of the forecast sales. Considering the high uncertainty in sale volume and sale price over a long time period, this hedging policy implied a large amount of quantity risk.

Such a hedging profile exposed Solid Energy to significant risk with respect to the depreciation of the domestic currency. Unfortunately, the New Zealand dollar moved against the company’s hedging position after 1998. In the subsequent five years, the New Zealand dollar had depreciated by more than 40%, down to a historical low. As a result, Solid Energy had $3.6 million realised losses on the derivative contracts in 1998, relative to nil in 1997. In addition, the unrealised losses rose from $8.2 million to $138.1 million from 1997 to 1998. These amounts of losses are significant relative to the $187 million total sales and $94 million international revenues in 1998. In the face of huge losses, the Board of Solid Energy considered closing out some forward contracts in February 1998 but such steps were not taken. When the exchange rate dropped further in 1999 and 2000, the losses arising from the derivatives were exaggerated further.

To aggravate the situation, the price and the quantity of the export sales shrank dramatically due to the downturn of the Asian economies in 1997 and 1998. The reduced demand for coal meant that customers not only could not meet the company’s predicted sales, on which a large proportion of foreign exchange cover was determined, but also had difficulty to fulfil their existing contracts. In fact, the international sales decreased from $115.6 million in the year ending 30 June 1997 to $94.8 million in the year ending 30 June 1998. The decline in sales together with the rise in forward covers implied a serious issue of overhedging. The derivative net losses regarding the overhedging position deteriorated further the company’s financial situation, which had been poor as a consequence of the diminished sales volume and price. A new Board was appointed by the government to mitigate the foreign exchange risk. The new Board reviewed the policies with the assistance of a newly
appointed adviser and instituted a new strategy to manage only the currency risk over the next 12 months, which excluded hedging for longer than one year.

2.3.4 Pasminco Limited

The following discussion is largely based on Brown and Ma (2006) and Pasminco Annual Report (2000). The Australian company, Pasminco Limited, was formed in 1988 and soon became the world’s largest integrated zinc and lead producer. On 30 June 2001, however, the company was placed under voluntary administration, which meant an administrator was appointed to arrange an agreement between creditors and the firm. According to the regulations, the company would move into liquidation if the arrangement was unsuccessful. The issues faced by Pasminco included lower commodity prices and high debt burden. The debt ratio had increased as a consequence of fast expansion, such as Pasminco’s acquisition of Century Mine in 1997 and takeover of Savage Resources Ltd in 1999. In addition, the company’s currency hedging policies exacerbated the losses and eventually led the company to collapse.

As an Australian metal exporting company, Pasminco Limited was exposed to the fluctuations in zinc and lead prices, and exchange rates. The latter mainly involved the AUD/USD, as the exporting prices were largely denominated and determined in USD. To cover its currency exposure, the company bought a large amount of AUD/USD forward contracts in the late 1990s (A$684.6 million in 1998 and A$679.6 million in 1999). The subsequent depreciation of the Australian dollar resulted in significant losses on the firm’s hedging position. As shown in the Pasminco Annual Report (2000), the forward hedging portfolio was restructured into options from 1999 to 2000. The losses on the forward contracts were then covered by income from writing call options on USD. However, the option portfolio of long USD put options and short USD call options exposed Pasminco to the risk of strong USD. The following table reflects the company’s foreign currency position as stated in 2000.
Table 2.2 Currency financial instrument positions

<table>
<thead>
<tr>
<th>Maturity date</th>
<th>30/6/2001 A$m</th>
<th>30/6/2002 A$m</th>
<th>30/6/2003 A$m</th>
<th>30/6/2004 and beyond A$m</th>
<th>Total A$m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long USD put option (AUD/USD)</td>
<td>698.2</td>
<td>646.6</td>
<td>769.9</td>
<td>1155.7</td>
<td>3270.4</td>
</tr>
<tr>
<td>Average rate</td>
<td>0.6593</td>
<td>0.6551</td>
<td>0.6867</td>
<td>0.7051</td>
<td></td>
</tr>
<tr>
<td>Short USD call option (AUD/USD)</td>
<td>773.8</td>
<td>697.5</td>
<td>799.7</td>
<td>1200.1</td>
<td>3471.1</td>
</tr>
<tr>
<td>Average rate</td>
<td>0.5949</td>
<td>0.6073</td>
<td>0.6611</td>
<td>0.6858</td>
<td></td>
</tr>
</tbody>
</table>

In addition, as Brown and Ma (2006) argued, Pasminco’s currency option position was extremely high relative to other firms. The ratio of nominal value of the short call options to the total asset value was nearly 90% for Pasminco, while the survey done by Hentschel and Kothari (2001) showed the average ratio of derivative to asset value was only 12% for a sample of 394 non-financial firms. This outstanding option position exposed the company to significant potential net loss when exchange rates moved against it, as did happen later. On 19 September 2001, the spot exchange rate dropped down to 0.4939. The put options purchased by Pasminco were out-of-the-money but the short call options were significantly deep in-the-money. Realized and unrealised losses during 2001 were of $A609 million, which damaged the company’s financial situation.

Along with low metal prices and high debt costs, the adverse impact of a weakening Australian dollar on the firm’s currency hedging position led Pasminco to be placed under voluntary administration in 2001. In 2004, Zinifex Limited was established by acquiring certain key assets from Pasminco. According to the financial report (ABN 2005, 2006), Zinifex remained unhedged to foreign exchange risk and hedged only a small proportion of commodity risk, around 5% at the end of June 2006. The subsequent rise in zinc prices and the US dollar benefited the company, generating profits of A$238 million and A$955 million in the financial years 2005 and 2006 respectively.

2.3.5 Bonlac Foods

In a competitive environment, more problems may arise from a hedging decision. When the consequent competitive disadvantage is exploited by rivals, the company
might be forced out of business. A typical example is Bonlac Foods, one of Australia’s major dairy exporters which, in 2003, fell under the control of Fonterra. Fonterra is the successor of the New Zealand Dairy Board, a company which had once experienced the disastrous effects of its hedging position.

Bonlac was established in 1986 and had a long history of hedging export revenues. Before 1996, its hedging policy worked well. In 1996, Phil Scanlan, a former Coca-Cola Amatil managing director was employed to expand the business aggressively. It acquired Spring Valley Fruit Juices, launched Spring Valley Bottled Water and obtained the rights to Gatorade (The Sydney Morning Herald, September 1, 2003). The fund requirement for the rapidly growing business led Bonlac to face a high debt ratio in the company’s capital structure. The bank covenants required the company to adopt a high hedging ratio, one which might be incompatible with the currency market situation at the time of decision.

Table 2.3 depicts the position and fair value of forwards and options existing for Bonlac in 2001.

<table>
<thead>
<tr>
<th>Table 2.3 Forward and options</th>
</tr>
</thead>
<tbody>
<tr>
<td>Face Value ($A Million)</td>
</tr>
<tr>
<td>Forward USD sales</td>
</tr>
<tr>
<td>Delivery within 12 Months</td>
</tr>
<tr>
<td>Delivery 12 to 24 Months</td>
</tr>
<tr>
<td>Delivery 24 to 36 Months</td>
</tr>
<tr>
<td>Bought AUD call options</td>
</tr>
<tr>
<td>Expiry within 12 Months</td>
</tr>
<tr>
<td>Sold AUD put options</td>
</tr>
<tr>
<td>Delivery within 12 Months</td>
</tr>
</tbody>
</table>

(Source: Bonlac Foods Annual Report 2001)

In addition to utilising standard forward and option contracts as hedging instruments, Bonlac also employed a large amount of contingent collars which trigger only when the specified event occurs. The collars are composed of long AUD call options and short AUD put options. Table 2.4 shows the firm’s contingent collars in 2001.
Table 2.4 Contingent collars

<table>
<thead>
<tr>
<th>Contingent collar</th>
<th>Face Value ($A Million)</th>
<th>Net Fair Value ($A Million)</th>
<th>Relevant contingency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expiry within 12 months</td>
<td>96</td>
<td>-18</td>
<td>AUD below 0.6000 on single date in December 2001</td>
</tr>
<tr>
<td>Expiry 12 to 24 months</td>
<td>193</td>
<td>-37</td>
<td>AUD below 0.6000 on single date in December 2002</td>
</tr>
<tr>
<td>Expiry 24 to 36 months</td>
<td>193</td>
<td>-36</td>
<td>AUD below 0.6000 on single date in December 2003</td>
</tr>
<tr>
<td>Expiry 36 to 48 months</td>
<td>193</td>
<td>-36</td>
<td>AUD below 0.6000 on single date in December 2004</td>
</tr>
<tr>
<td>Expiry 48 to 54 months</td>
<td>96</td>
<td>-18</td>
<td>AUD below 0.6000 on single date in December 2005</td>
</tr>
</tbody>
</table>

(Source: Bonlac Foods Annual Report 2001)

The Australian dollar reached a historical low in the early 2000s, as shown in figure 2.3. Hence, Bonlac experienced negative net fair value of those financial instruments and had to pay the opportunity costs on their hedging position. However, the hedging loss did not lead directly to the collapse of the company. Bonlac failed to survive in the market mainly because opportunity costs regarding the hedging were significantly exaggerated by the rival’s actions. Bonlac’s main competitor, Murray Goulburn co-operative, exploited the relative advantage arising from a non-hedging position. In the face of a depreciated Australian dollar in the late 1990s and early 2000s, Murray Goulburn raised the milk price it paid to the suppliers, who are usually also the shareholders of dairy companies in Australia and New Zealand (*The Sydney Morning Herald*, September 1, 2003). Murray Goulburn could afford to do this as it did not lock its income at a previously high exchange rate, as would happen with hedging.

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3 Contingent collars will come into existence only if the spot AUD/USD exchange rate is below the specified exchange rate (the contingency). If the contingency never arises, the relevant options comprising the collar will never exist. (Bonlac 2001 Annual Report)
Subsequently, the Tasmanian dairy farmers who supplied Bonlac became irritated because they were receiving less than their Victorian counterparts, who were suppliers for Murray Goulburn. According to the report from [www.infarmation.com.au](http://www.infarmation.com.au) (*Tasman dairy anger at Bonlac* 2003), the difference between the milk price paid by the two companies led Tasmanian farmers to averagely earn $10,000 fewer a year than Victorian farmers. As a consequence, farmers began to leave Bonlac. As reported on September 1, 2003 by *The Age*, Bonlac suppliers had fallen from 3800 farmers in 1992 to 1825 in 2002. As a result, its share of Victorian milk production had declined from about 40 per cent in 1992 to 16 per cent in 2002. Bonlac’s total sales decreased from $A1237 million in 2000 to $A700 million in 2003.

The diminishing market share worsened the company’s financial situation, as a number of fixed costs were still incurred despite the dramatic drop in revenue. Standard & Poor’s Rating Services downgraded Bonlac’s long-term corporate credit to reflect the company’s weak financial performance, high debt levels, weak liquidity, and narrow focus. In order to survive in its current form, Bonlac had to sell off part of its business. With an agreed takeover, Fonterra bought a 25% stake in Bonlac. In addition to the 25% stake it already owned, Fonterra then held 50% of Bonlac’s shares till 2003 (*The Sydney Morning Herald*, September 1, 2003). Bonlac Foods Ltd

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eventually became a wholly-owned subsidiary of Fonterra through the votes on September 2005.

2.3.6 Other cases

The depreciation of the Australian dollar after the Asian financial crisis also hit Savage River Mines, which was owned by Ivanhoe Mines Ltd. According to *The Sydney Morning Herald* (Robins, April 8, 2002), the market value of the company fell from $US 125.3 million to US$36 million within two years. In addition to the poor operational performance due to the global slowdown in the steel industry, the deep out-of-the-money currency hedging position contributed significantly to the collapse of the company. Although the currency dropped to around US50c in 2002, the company fixed its exchange rate at US68c for each Australian dollar through a delivery agreement with UBS Australia.

Cases regarding currency hedging are not exclusive to New Zealand and Australia. A similar example is offered by Japan Airlines, which lost ¥45 billion (US$450 million) on foreign currency hedging of future revenues in 1992. It is ironic that the company suffered at a later date from not hedging. From October to December 2005, Japan Airlines was caught by a net loss of up to ¥11.1 billion (US$136.8 million) in three months. The main cause was that Japan Airlines had not hedged its costs against the soaring jet fuel prices. To relieve the impact of high fuel costs, Japan Airlines was forced to raise fares, cut labour costs and eliminate unprofitable flights. At the same time, its competitor, All Nippon, Japan’s second-largest airline, protected itself from increasing fuel prices by taking a hedging position in the derivative market. As a result, All Nippon enjoyed a comfortable period as it won over customers from Japan Airlines (*The New Zealand Herald* Feb 7, 2006).

Another Japanese giant, Sony Corporation of Japan, also experienced significant losses on its hedging position in the early 2000s. Since 75% of its products were sold overseas, the company was exposed to a great extent of currency risk. To protect the firm from depreciation in the USD or Euro, the two main foreign currencies the firm received for exports, Sony hedged roughly 80% of its USD receipts and 90% of its Euro receipts through a combination of forwards and options in the late 1990s. The subsequent strength of the USD and Euro resulted in a hedging loss of USD140 million, which accounted for 14% of the earnings at the same period. In response to
the appreciation of domestic currency, the company reduced its hedging ratio to 15% for USD exposures and 30% for Euro (Bowden, 2005b).

Although exchange-traded derivative contracts imply less credit risk, they may exaggerate disastrous effects through additional obligations of cash outflow before maturity, due to the exchange’s ‘mark to market’ requirement. An example is South African Airways (SAA), which was once considered one of the most enduring symbols of South Africa. AFP (Johannesburg, 2004) reported that SAA encountered a hedge loss of US$1 billion. The company’s hedging policy was based on the assumption that the domestic currency, the Rand, would depreciate significantly over the long term. To protect the company from the weak local currency, some derivative contracts were entered into for as long as 10 years, with a strike price of 10.85 Rand per US dollar, despite the current spot exchange rate of 6.5. When the subsequent exchange rates actually moved upwards, the consequent financial obligations created such difficulties that the Government had to provide a credit guarantee (Clarson, 2005).

There are many other examples of this type of hedging loss, such as the well-known case of Metallgesellschaft that suffered from the scarcity of cash flows arising from margin calls on future contracts for hedging. However, discussion in the later chapters will mainly focus on the use of over-the-counter (OTC) derivatives as the cash flow problem regarding the exchange traded derivatives is beyond the focus of the current discussion.

2.4 Lessons from cases

Most corporations discussed above bought derivative contracts in order to reduce foreign currency risks. However, the consequence of such a strategy was the financial distress. Some companies failed because of the misplaced hedging policy, such as New Zealand Apple and Pear Board, Solid Energy and Pasminco. These companies had over-hedged the spot risk and thus had net exposures in the financial derivative market. Other corporations, such as the New Zealand Dairy Board, Bonlac Foods and Japan Airlines, which did not have the problem of misplacing derivatives, also encountered financial difficulties regarding hedging. These companies were criticised for having employed a passive hedging strategy in which the current exchange rate risk profile and the corporate exposures had been ignored.
Take the New Zealand Dairy Board case as an example. As discussed by Bowden (2005b), the risk and market assessments in 1997 indicated significant downside potential in the New Zealand dollar. For instance, the current account deficit reached a historical high level and had been in that region for some time. In addition, the housing market started to decline from 1997. This eased inflation and relieved pressure for the Reserve Bank to continue tight money policy. The quiet New Zealand housing market also diminished demand for overseas capital, formerly an important source for funding mortgages. Furthermore, it can be observed from figure 2.2 that both the nominal and real NZ dollar was at a historical peak in the mid-1990s. If a mean reverting process exists in the real exchange rate, as suggested by Sollis et al (2002), Kapetanios et al (2003), Liew et al (2004), Cerrato and Sarantis (2006), one might expect the NZ dollar to come down in the longer run. A forward hedging policy with some contracts long up to three years locking the exchange rate at an unfavourable level for a long time could result in significant opportunity costs, especially for shareholders considering the spot rate as the benchmark. In this circumstance, some short term forwards or deep out-of-the money options may perform better than long term derivative contracts, as suggested by Bowden (2005b).

The New Zealand Dairy Board designed such a hedging policy mainly for minimising farmers’ bankruptcy risk and costs arising from further appreciation in the New Zealand dollar. The Board, however, overlooked the fact that farmers had started to experience distress in the face of the currency value at that time. Locking the exchange rate at such a value could not free the farmers from distress costs, although hedging could still be helpful to decrease the probability of bankruptcy. Furthermore, the Board utilised relatively long term forwards instead of options or short term forwards for hedging purposes. The use of long term forwards by the Board had the effect of extending the length of distress period, for which farmers experienced significant financial distress costs.

The experience of Bonlac Foods and Japan Airlines has additionally raised the issue of how to incorporate the effects of competition on corporate exposures and hedging costs into the hedging decision. Bonlac failed at the time when its core operation was still working well. Its failure was due to its hedging position that created huge disadvantages for the firm’s product market. Japan Airlines had experienced significant losses when it remained unhedged but its competitors hedged
against high petrol costs. Japan Airlines lost the market share to its hedged competitor which could afford to keep prices unchanged in the face of the significantly inflated spot petrol market. The Bonlac Foods and Japan Airlines examples show that, in a competitive market, advantage or disadvantage caused by a financial hedging policy can be quickly transferred to the product market. As a result, any losses resulting from a hedging or unhedged position may be exaggerated to the extent that a firm’s survival is under threat. To ensure a corporate’s continued existence in the face of competitors, superior understanding of corporate exposures and relevant financial markets play an important role in deriving its risk management policy.

2.5 Research questions motivated

These examples reveal the difficulties as well as the complexity of establishing an optimal hedging strategy. Such difficulties prompt a number of questions on hedging in general, and currency hedging in particular.

(a) Generally, what is the primary purpose of risk management? Associated with this question are other issues, such as should the firm hedge; and if so, how should the hedging function be defined? The traditional view, which is still very popular in practice and literature, is that hedging should be designed to reduce volatility. In this respect, forwards and options could be efficient hedging instruments as they lock the price at a certain rate or within a band. However, as shown within the New Zealand Dairy Board and Bonlac examples, hedging could still bring the firm to the brink of bankruptcy even though it does contribute to stabilizing the cash flows.

Volatility reduction is not necessarily consistent with a firm’s primary objective. A more basic concern of a corporate manager is whether a firm’s operation can continue or not. From this perspective, firms should design hedging policies with a focus on bankruptcy risk, or more generally, the risk of financial distress, rather than risks from both up and down side. On the other hand, exclusive concern with the downside risk should not ignore benefits to be gained from good outcome which in many cases account for the core business of firms. A more embracing formulation would allow for both, namely the expected payoff in the good zone and the expected damage in the bad zone.

(b) How can the natural exposure faced by corporates be defined? A corporate’s exposure is not always restricted to one single financial risk. For some companies, a relatively complete description of the natural exposure could include a series of
components, some which are hedgable while others are not. For instance, an exporting or importing company’s profits depend not only on the currency variation but also on product prices. These natural exposure components could interact with each other and eventually affect hedging outcomes. The Solid Energy limited example showed the impact of hedging losses due to exchange rates changes was amplified by unfavourable commodity price fluctuations. On the other hand, in the case of New Zealand Dairy Board, high dairy product price actually relieved dairy farmers’ losses on expensive forward rates.

Both examples suggest that currency exposures encountered by a corporate can be affected by fluctuations in other financial variables. A further question thus arises as to how natural exposures can be defined to capture the main risk facing the company and how the variation in all the exposure components will interact with each other. These understandings would facilitate corporate managers in developing a structured risk management framework to minimise the natural exposures of firms.

(c) The role of information is also important. Should a company take a view as to future values of exchange rates and other environmental variables? If a decision to hedge is made, should the hedging be active or passive; and if active, how could it be implemented? Hedging with a view of future asset prices cannot perform better than a passive hedging in an efficient market where spot prices reflect all the available information and no individuals can successfully second guess the market. However, there remains a controversy regarding market efficiency. The mean-reversion element is a popular topic in the real exchange rate literature. If a correction for significant appreciation in real home currency value is expected, exporting companies’ value could be harmed by a heavy hedging policy. If the market is inefficient, corporate managers with superior information and knowledge are expected to increase the firm value through more effective hedging decisions. Therefore, the first task for determining the active risk management strategy is to examine the market efficiency. If there is indeed inefficiency in the market, a model of forecasting exchange rates could be designed with an aim to improve the hedging outcomes.

Active hedging is particularly important in a market where opportunity costs of hedging are high. One example is a market where the benchmark for assessing the hedging outcome is the natural (unprotected) exposure, such as that experienced by the New Zealand Dairy Board. The Board was criticised for implementing a passive
hedging policy and thus undergoing significant hedging losses relative to spot position. Another example is the competitive market, in which hedging losses would be exaggerated by the rival’s actions and passive hedging could not always protect the firm from financial distress, as shown in the Bonlac example.

(d) The duration of potential exposures is another issue that requires some investigation. Is the corporate more vulnerable to short term or long term exposure? What is the difference between short run and long run risk management? Currency risk management literature has a focus on a relatively short term exposures hedging. It is rational from two perspectives. One is that it has been widely accepted that parity conditions tend to hold over a long term period (e.g. Taylor, 1995; Froot & Rogoff, 1995). If this is true, the risk tends to be more prominent over a short period. Quantity risk is also relevant. Uncertainty in the quantity of exposures in a far future gives hedgers incentives to avoid long term hedging. As discussed earlier, Solid Energy Limited took a large position in the long term forward hedging, resulting in disastrous results due to the rising quantity risk.

However, for some organisations, such as superannuation fund managers who focus on long term fund investment, short term currency exposures are not particularly relevant and their main concerns should be on the fluctuation of currency value over a long time period. The long term exposure may exhibit its unique characteristics. The ways to control it may differ from those used to manage short period exposures. The longer run variation in the value of financial variables is usually affected by the macroeconomic or structural influences. For these strategic funds, different association with these macroeconomic causes may lead to various dependence structures of fund values over time. Final distribution of final value may be related to a variety of conditional dependence structures over time. A series of measurement techniques have been developed to account for such conditional distribution over a long period. The problem requiring more exploration is how to develop measurement techniques that are also useful for portfolio construction. Such measurements should be well adapted to the longer run movements associated with business cycles or economic growth and they should also be useful in constructing portfolios that can exploit longer run sources of investment value.
2.6 New Zealand context

Although the theoretical and methodological development in this thesis is intended to be general, the empirical application in the current context is selected for New Zealand exporting companies and fund managers. Exporting companies play an important role in the New Zealand economy. According to Statistics NZ (2003), New Zealand Gross Domestic Product (GDP) relies heavily on the exporting sectors of the economy, with the exports of goods and services accounting for 33% of GDP in 2003. In addition, New Zealand, as a small country, is a ‘price taker’ rather than ‘price maker’ in the world market. New Zealand’s exports are usually priced in foreign currencies, such as the US dollar. The volatility in the exchange rate between NZD and USD or other foreign currencies can therefore directly influence overseas revenue and indirectly influence the market share of exported products.

Currency exposure is also extremely high in New Zealand investment portfolios. A survey undertaken by Statistics NZ (2003) shows that New Zealand’s total investment abroad is NZ$77.53 billion at the end of March 2003, while overall GDP for the country is NZ$126.26 billion at the same period. As a small country, New Zealand does not have a massive capital market. To diversify the portfolio risk and pursue superior return, NZ financial institutions as well as individual investors are inclined to allocate a large proportion of their portfolios to foreign securities investments. Since the fluctuations in currency values have big impacts on the total return of an international investment portfolio, how to manage the consequent currency risk also becomes a crucial issue for New Zealand financial investors.

In addition, the extremely high volatility in the New Zealand dollar implies that New Zealand trading companies and international investors are exposed to highly fluctuating cash flows. The NZ dollar has been found to be one of the most unstable currencies in the world on the basis of a study of the value history of thirteen currencies (for details see Chapter 5). Take the exchange rate between New Zealand and US as an example. In the last ten years, the lowest NZ dollar was worth only 0.39 USD in November 2000 while the highest NZD was worth 0.82 USD in August 2007. The total appreciation was 110% in five years and resulted in a yearly return of 16%. The high volatility exhibited in the New Zealand currency ensures the importance of the currently proposed issue – exchange rate risk management in a New Zealand context.
2.7 Conclusions

The reforms of currency regimes and the globalisation of financial markets have been accompanied by a changed notion of currency risk, along with the corresponding risk management for exporters, importers, financial institutions and even nations. Exchange rate exposures have become frequent and unpredictable under a floating rate regime. At the same time, the development of financial derivative products is enabling corporations to manage foreign exchange risk with financial market-based instruments. Arguments in favour of financial hedging have gained wide acceptance among academics and practitioners. The descriptions of a series of cases in this chapter, however, have shown how companies’ financial situations can be damaged by using financial hedging instruments.

In some cases, hedging leads to bad outcomes because it has been incorrectly applied. In other circumstances, a hedging strategy that has been initiated on a seemingly correct basis also brings the firm into financial distress, partly because of the ignorance or misinterpretation of market information. This research has been motivated to develop a structural and effective hedging framework, which is particularly important in these days when the growing complexity of exposure and contemporary financial environment increases the difficulty of currency hedging. In the face of a comprehensive financial market, those corporates that have a thorough understanding of their exposure profiles and have expertise in establishing superior hedging strategy, have greater prospects of survival.

The aim of this thesis is to explore solutions to specified research questions as to whether a firm’s wealth can be enhanced by the risk management strategy and if so how. Issues to be addressed include the choice of an appropriate objective function, the impact of managerial private information, and other basic features of a structured hedging framework. The next chapter will concentrate on building the welfare criteria function of a risk management system. The specification of a hedging objective function will be built upon the analysis of a GVaR criterion. The proposed approach forms a connection between the currency hedging and maximising the market value of a firm.
Part II Risk Management Decision Theory

Chapter 3 Generalised Value at Risk (GVaR) – An Objective Welfare Criterion for Corporate Risk Management

There is an extensive literature describing decision theories under risk; for example mean variance analysis, expected utility theory, prospect theory, regret theory, value at risk theory and many others. The choice of an appropriate risk management criterion for a corporate, which will be discussed in this chapter, depends on the extent to which the criterion can be related back to the company’s primary objective for corporate hedging, namely maximising firm value. Among the decision theories mentioned above, the closest correspondence is with GVaR, which includes the value at risk and related theory. The general idea is that disutility caused by imposing GVaR constraints is consistent with the negative payoff in the form of financial distress costs. Such financial distress costs diminish the firm value that is otherwise available to stockholders and debtholders. A widely accepted view is that hedging enhances firm value by reducing the financial distress risk (e.g. Smith & Stulz, 1985; Rawls & Smithson, 1990). In the present chapter, questions regarding whether and how hedging increases firm value are raised and this will be examined under the proposed GVaR decision criterion.

There might be conflicts between firm value optimisation and equity value maximisation as shareholders are not necessarily averse to bankruptcy costs given the fact that they can walk away when the firm goes bankruptcy. The present chapter will discuss some specific circumstances where shareholders’ interests tend to differ from those of the firm’s other stakeholders.

Some theoretical results are explored regarding the use of this proposed GVaR approach in the context of corporate hedging. Corporate managers might derive different hedging strategies according to the nature or circumstances regarding the financial market, product market and exposure composition. Some qualifications about the proposed welfare function will be explored, in particular relating to concerns of the potential opportunity costs of hedging.

The chapter is organized as follows: Section 3.1 describes the GVaR theory

Section 3.2 develops the primary objective welfare function for a structured risk

5 Section 3.1 is based on Bowden and Zhu (2006b).
management decision. The function is inherent to the GVaR theory. Section 3.3 rationalises the generalised objective function from the perspective of corporate finance. Section 3.4 explores some theoretical results of the corporate hedging in different markets. Section 3.5 provides some qualifications related to the hedging opportunity costs. Section 3.6 discusses the corporate hedging in a competitive market. Section 3.7 concludes.

3.1 GVaR theory

Value at risk (VaR) measures the minimum loss for a given probability that a firm or individual may expect over a specific time period under normal market conditions. It originated as a portfolio diagnostic tool when the Chairman of JPMorgan, Sir Dennis Weatherstone required a daily report regarding the total risk of his firm. A VaR of $1 million at a confidence level of 95% implies that the overall loss at a probability of 5% will be at least $1 million for a given time period (Jorion 1996, 1997; JPMorgan Risk Metrics, 1995, 1996, 1997; Linsmeier & Pearson, 1996, 1997). Since its initiation, this risk measurement tool has been widely employed in risk disclosure and reporting. The Basle Committee requires banks to perform regulatory capital calculations with VaR. Another advocate of using VaR is US Securities and Exchange Commission which authorized VaR as one of three quantitative risk disclosing methods for the listed firms to report the risks associated with market sensitive assets, such as derivative products (Linsmeier & Pearson, 1996, 1997). In addition to the use in risk reporting or disclosure, the VaR has also been widely used in firm-level risk management, including deriving hedging decisions, constructing portfolios and managing overall cash flows. An advantage of VaR is that it emphasizes downside risk only. Due to this feature, most managers and investors prefer it to some traditional risk indicators such as variance that penalises both upside and downside movements.

Despite the popularity of VaR measurement, it has been widely criticised for ignoring the losses beyond the critical point. As a result, minimising VaR may increase rather than decrease the tail risk (Larsen et al, 2002; Gaivoronski & Pflug, 2000; Artzner et al, 1997, 1999, 2002). To overcome the deficiency of VaR, Urysaev (2000), Palmquist et al (2001), Rockafellar and Uryasev (2002), Alexander and Baptista (2004) develop and stimulate the use of another percentile risk measure, conditional value at risk (CVaR), also described as ‘expected shortfall’, a term used
widely in the insurance industry. The CVaR is defined as the weighted average of VaR and losses exceeding VaR (Rockafellar & Uryasev, 2000). The CVaR approach is concerned with the expected tail length in the form of the truncated or censored expectation. The motivation comes from the long left hand tail possibility, one that may cause considerable damage to managerial welfare but would not be captured by the VaR limit (Basak & Shapiro, 2001). The VaR essentially refers to lump sum type penalties if the outcome return or value falls below an acceptable benchmark; while CVaR, refers to the disutility to be attached to the left hand tail as a whole. Bowden (2006a) defines either or both VaR and CVaR as ‘generalised value at risk’.

Some equivalent option structures have been noted in the GVaR theory (e.g. Artzner et al, 1999; Basak & Shapiro, 2001; Bowden, 2006b). Firstly, VaR incorporates, at its most basic level, a simple binary option as a loss measure. Let $R$ represent an exposure outcome, e.g. portfolio value or return, and $P$ a VaR point, so that $F_R(P) \leq \alpha$, where $F_R(P)$ denotes the distribution function of the variable $R$; and $\alpha$ stands for the pre-set confidence level. (Commonly in the range of 5-20%.) Then the cumulative distribution function involved in VaR can alternatively be written as

$$F_R(P) = E_R[SF(P-R)],$$  

(3.1)

where $SF(x)$ is the elementary step function such that $SF(x)=1$ if $x >0$, = 0 otherwise. Thus the given VaR condition $F_R(P) \leq \alpha$ is equivalent to placing an allowable bound on the expected value of the payoff to a binary put option that has value 1 whenever $R \leq P$ and zero otherwise.

The option interpretation is also apparent with CVaR. CVaR-based decision rules seek to place a limit on the expected loss given that a designated VaR critical point $P$ is exceeded. In other words,

$$E[R|R \leq P] \geq c,$$  

(3.2)

for some user-assigned tolerance constant $c$. Now

$$E[R|R \leq P] = \frac{E[(R-P)_-]}{F_R(P)} + P,$$  

(3.3)

where $(R-P)_- = \begin{cases} 0 & R \geq P \\ R-P & R < P \end{cases}$. Hence if $F_R(P) = \alpha$ in the VaR restriction, then the CVaR constraint can be written:

$$E_R[(R-P)_-] \geq \nu, \nu = \alpha(c-P).$$  

(3.4)
The left hand side of the equation is the expected value of the payoff to a put option with strike price at \( P \). Imposing CVaR is equivalent to requiring that the put option value with strike at \( R = P \) is not too large. In this particular context the option is valued just as the expected value of the future payoff.

### 3.2 Primary objective welfare function – Generalised value at risk

Decision problems involving GVaR can be formulated in various ways (for a review see Bowden, 2006a). One version seeks to minimise the VaR for a given significance level (critical probability), subject to a specified minimal performance on other metrics such as the expected value. Other versions set at the outset a given value \( P \) to be the VaR critical point and seek to maximise the expected portfolio value outcome, subject to the requirement that the probability of falling short of \( P \) is less than or equal to the chosen significance value (such as 5% or 10%). In effect, VaR provides the risk aversion. The latter versions amount to asserting that the firm’s core business is to maximise the expected portfolio value, but that the VaR critical point \( P \) now provides the tolerance level for adverse outcomes. In addition, CVaR imposes a further requirement that the expected shortfall is to be less than some given constraint \( \nu \) (Urysaev, 2000; Palmquist et al, 2001).

A convenient way to capture the essence of the VaR and CVaR approach is to specify at the outset a utility function of the form

\[
U(R; P) = R - P + b(R - P)_-, \tag{3.5}
\]

where \( R \) stands for an exposure outcome: value, cash flow, or some other dimension of corporate or managerial exposures. In the empirical application discussion in Chapter 8, this is taken specifically as the net profit margin, but in the current context can be quite general. When taking the expected utility,

\[
E[U(R; P)] = E[R] - P + bE[(R - P)_-]
\]

\[
= E[R] - P + bF_R(P) \times E[R - P | R \leq P], \tag{3.6}
\]

where \( F_R(P) \) is the distribution function of \( R \) evaluated at \( R = P \) and it represents the concerns on the VaR. The term \( E[R - P | R \leq P] \) incorporates the CVaR. The parameter \( b \) is set as a risk penalty to the GVaR component relative to the overall or unconditional expectation \( E[R] \).
Figure 3.1 above illustrates the objective function from which it can be seen that the underlying utility function has a segmented or a smoothed segmented form. The managers’ risk averse attitude is asymmetric around the reference point $P$. Above the critical point $P$, utility increases linearly with the return or value outcome. Below the critical point $P$, the slope increases from unity to $1+b$, indicating a higher disutility as the value sinks below the critical point. The effect of figure 3.1 is as though an otherwise risk-neutral manager with a linear utility function has been compelled to write $b$ put options with strike price at $P$. The contingent cost of doing so increases as $P$ increases and also as the number $b$ increases.

This asymmetric utility can also have the bad zone ‘powered up’ if necessary as:

$$ U(R;P) = R - P - b \frac{1}{\gamma} |R - P|^{\gamma} SF(P - R) $$

(3.7)

where the constant $\gamma > 1$. This can cover cases such as those pointed out by Yamai and Yoshiba (2002), where the expected shortfall fails to eliminate the tail risk beyond a specific threshold (such as very long tail effects). Of course, it could also be the case that marginal disutility is increasing with larger losses. In terms of option equivalence, the effect would be as though the manager had written power options on the downside. Note that the special case $\gamma=2$ and $P=E[R]$ would correspond to using the semivariance as a downside risk metric. Ferreira and Goncalves (2004) also employ a powering up metric, though they use the conditional $\gamma$th order tail moment. The conditionality implied in their methods is similar to omitting the factor $F_{\gamma}(P)$ in equation (3.6).
3.3 Rationalising GVaR from a corporate finance perspective

As discussed earlier, there is a common rationale between the theory and practice of GVaR and the corporate finance literature, namely the avoidance of bankruptcy costs.

3.3.1 Aversion to bankruptcy costs

Firms are facing the threat of bankruptcy when they are unable to make required payments to their debt holders or other creditors. Because the debtors hold fixed claims on the firm which are regardless of the firm’s performance, the rise in the leverage increases the possibility of bankruptcy. An all equity firm, however, may still go bankrupt as the firm’s non-financial stakeholders, such as employees and suppliers, have the similar claims on the firm’s cash flows as debt holders. In the face of bankruptcy, the firm’s assets would be transferred from equity holders to other stakeholders. The transformation would be costless if there were no additional costs arising from the event of bankruptcy. As traditionally debated in the corporate finance literature, the possibility of bankruptcy is irrelevant to a firm’s capital structure or other financial decision, as long as there are no extra cash outflows due to the event of bankruptcy. The aversion to bankruptcy arises from the cost of bankruptcy as these costs are paid to third parties rather than the firm’s stakeholders.

White (1983) and Altman (1984) classify bankruptcy costs into two categories – 
*ex post* direct or *ex ante* indirect bankruptcy costs. Direct costs include the cash outflow relating to administrating the bankruptcy, such as accountants’ and lawyers’ fees. Indirect costs on the other hand, can include those associated with the threat of bankruptcy prior to liquidation. Implicit indirect bankruptcy costs can occur from the impaired ability to conduct business, including

(a) lost sales from declining demand as customers have concerns about ongoing support services for the product they buy;

(b) increasing input expenses from suppliers due to the difficulty of obtaining credit in distress;

(c) inputs of management time and effort;

(d) growing debt costs as rational bondholders will require a higher rate of return when the firm is close to bankruptcy.

Both direct bankruptcy costs and indirect financial distress costs should be taken into account in reaching the risk management decision. Significant distress expenses could worsen a firm’s financial position and eventually increase the probability of a firm’s
collapse. This was illustrated in the New Zealand Dairy Board example where policies designed for hedging against the company’s bankruptcy risk had been criticised for exaggerating its financial distress costs.

The objective welfare function shown above incorporates concerns regarding bankruptcy costs in the form of VaR and CVaR. The VaR accounts for the probability of financial distress, as shown in the earlier equation (3.1). Therefore, an aversion to the VaR results in the reduction of possibility for any losses beyond the critical level. Since direct bankruptcy costs that occur at insolvency are generally fixed, no matter what causes the event, imposing a VaR constraint amounts to decreasing direct bankruptcy costs. However, in some circumstances, the costs may vary in accordance with the firm’s financial position, for example, a firm involving a larger number of stakeholders and facing serious insolvency problems might have more complicated and thus more costly liquidation processes.

The indirect bankruptcy costs, along with some variable direct bankruptcy costs, have been represented in the form of CVaR. The indirect bankruptcy costs are negatively correlated with the firm’s expected cash flow. The worse the firm’s financial position, the higher the indirect bankruptcy costs could be. For instance, given the same default probability, debt holders can receive less in the event of default if a more considerable loss occurs. In this case, debtholders tend to request a higher return for compensating the risk involved in their investments in a firm more likely to have financial problems. More expensive debt could be taken to signal higher indirect bankruptcy costs for equity holders. The VaR penalises the bankruptcy possibility but takes no account of the magnitude of losses caused by the threat of bankruptcy. As indicated earlier, the CVaR overcomes the limitation regarding the VaR by penalising all the exposures exceeding the critical point. In this respect, the expected welfare function expressed by equation (3.6) incorporate firm’s aversion to the indirect bankruptcy costs by imposing the CVaR constraints.

The criteria function can be further connected with firm value. Suppose that the first factor of equation (3.5) is the payoff to a firm in the world without bankruptcy costs. The second factor of the equation would then be interpreted as the costs of bankruptcy, as discussed earlier. The hedging decision derived from maximising the expected objective welfare, as expressed by equation (3.6), amounts to optimising the firm’s expected payoff in a world with bankruptcy costs. Firm value can be derived by discounting the firm’s expected payoff with the appropriate discount rate. The
discount rate should reflect the risk involved in those cash flows. Another approach for valuing a firm or a project is to convert the risky cash flows to certainty equivalents and then discount the certain cash flows by the risk-free rate (Ross et al., 2002). In the equation (3.6), the risk regarding the payoffs can be absorbed by the choice of $b$ (detailed discussions see Chapter 8). In this case, the consequent decisions derived from optimising the expected GVaR criterion, as expressed in equation (3.6), is in effect consistent with the purpose of maximising firm value.

Such a connection between the GVaR and the corporate finance assists in specifying the GVaR constraint. According to the definition of the VaR and the CVaR, the choice of a confidence level is crucial to determine the value of GVaR. When used for the portfolio diagnostic, the confidence level is usually arbitrarily set in the range of 5-20%. In the current context, however, the confidence level can be derived from the point where companies start to experience the financial stress.

### 3.3.2 Impacts of capital structure

A firm’s managerial concerns on the GVaR can be affected by its capital structure. In a leveraged firm, the maximisation of firm value is not necessarily consistent with shareholder value. When the firm objective is specified as optimising firm value, the corporate managers are assumed to maximise returns to all stakeholders, including both stockholders and bondholders. However, since Jensen and Meckling (1976) have formalised the agency problems between different stakeholders, it is well known that the interests of shareholders may differ from bondholders’. In the face of bankruptcy, shareholders have the option to exit from the liabilities that they could not afford. Shareholders that can exercise the liability exit option are not necessarily averse to a potential volatile position. In fact, they may have the tendency to stay unhedged in a difficult time as the uncertainty encompassed in the financial variable brings shareholders the opportunity of relief from stress. In this respect, shareholders increase their own value at the cost of debt holders. Within the corporate finance, this is the ‘asset substitution’ effect.

However, the shareholders may only exercise their exit option in some limited circumstances. According to corporate finance theory (Ross et al., 2002), the shareholder exit option will not be in the money unless all four of the following conditions are satisfied.
(1) The firm has been close to bankruptcy point at the time financial decisions are determined;
(2) The firm has a high financial leverage;
(3) Information is asymmetric between shareholders and debitholders and thus the former can take advantage of the latter;
(4) There are no conflicts between corporate managers and shareholders regarding the view of bankruptcy. It implies that managers will act from the perspective of shareholders by choosing the risky projects regardless of the consequent rising bankruptcy risk.

It is easy to understand why the first condition has to hold. In good states, the value of the shareholder liability put option, or namely direct bankruptcy cost, is almost zero as the firm rarely goes to collapse in these situations. The potential financial distress costs, which are expected to be very low in such circumstances, are mainly composed of indirect bankruptcy costs. As discussed earlier, shareholders are not free of indirect bankruptcy costs. Therefore, maximising firm value is consistent with equity value optimisation when the firm is far from bankruptcy.

In terms of the condition (2) above, the value of the liability exit put option owned by a shareholder is positively correlated with the leverage ratio. Shareholders in a firm of greater financial leverage have more incentives to exercise the option. However, such incentives could also be conditional upon the time of the firm’s leverage decisions being made. As Leland (1998) argued, managers could posses various incentives for deriving hedging decisions depending on whether these are determined prior to or after the debt being in place. When the hedging choice is made \textit{ex ante}, any costs arising from the agency issues could be transferred from rational debt holders to shareholders in a form of higher cost of debt. In this circumstance, shareholders can hardly take any advantage of debt holders and maximising firm value is generally consistent with equity value optimisation. Leland (1998) concludes that hedging usually leads to a higher debt ratio. On the other hand, when hedging is determined after debt holders have delegated the control of money to stockholders, the equity holders have the opportunity to exploit the debt holders.

In addition, as described by the above condition (3), it is hard for shareholders to take advantage of debt holders when a market is informationally efficient. Since the United States introduced the Statement of Financial Account Standard (SFAS) 133 in 1998, which was followed by the release of International Accounting Standards (IAS)
32 and 39, presenting and disclosing financial instruments and hedging activity became mandated for firms. These standards are intended to increase the information transparency and consequently reduce the agency costs between a firm’s debtholders and shareholders, although empirical evidence regarding the impacts of the above accounting standards on information transparency is still mixed.6

Furthermore, to reduce the expected indirect bankruptcy costs related with the agency cost of debt, firms may be inclined to accept bond covenants restricting their risk management choices. In a survey of hedging activities, Geczy, Minton and Schrand (1997) find that some of their sample firms were constrained by the debt covenants or credit arrangements to hedge some portion of their interest rate exposures. Evidence can also be found with Bonlac Food Limited. Given the financial situation of Bonlac following the exchange rate and commodity price crises, debt holders specified in the debt covenants that the firm must hedge their currency exposures (The Sydney Morning Herald, September 1, 2003).

Another constraint for shareholders to exercise their liability exit option lies in the potential managerial distortion, as between corporate managers and shareholders. Apart from the agency issues among shareholders and debtholders, Jensen and Meckling (1976) also articulate the agency problem between shareholders and corporate managers. Though managers are supposed to act on behalf of firm’s owners, they have their own interests which may deviate from those of stockholders. For instance, in the theory of corporate finance, shareholders are inclined to hold a diversified portfolio and be indifferent to the unique risk regarding the specific firm, such as the bankruptcy risk. However, corporate managers, who are the primary individuals to approve the risk management policy, usually lock their human capital with the company and which is not diversifiable.

The managerial personal costs arising from the bankruptcy may be extremely high for those managers operating in large multinational companies, who are particularly keen to preserve their reputation in the market. Gilson (1989) investigates the exchange listed firms and finds that senior managers who lose the job due to the financial distress of companies have no chance to being re-employed by another exchange listed firm for at least three years. Cannella et al (1995) also find evidence for significant managerial personal costs relating to the bankruptcy of banks. Other

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literature focusing on the impacts of managers’ personal bankruptcy costs and risk aversion upon the optimal financial decisions include Zwielbel (1996), Novaes & Zingales (1995), Subramanian (2002), Morellec (2004) and Berk, Stanton & Zechner (2007), among many others. Asset substitution arising from conflicts between shareholders and debt holders could be alleviated by the particular interests of managers.

Given the discussion on the difficulty of exercising shareholders’ liability exit options, the empirical application in later chapters will employ the equation (3.6) in deriving the corporate hedging decision. In equation (3.6), the risk aversion is restricted within the GVaR framework so that there are no concerns about the liability exit option or concerns about adverse put options outweigh liability exit options.

However, it does not exclude the possibility that corporate managers might take into account the corresponding impacts of the liability exit option on corporate hedging when the exit option value is extremely high. The effects of such exit options on corporate decisions, such as the optimal leverage ratio and cash holdings have been largely explored in the literature (see Jensen, 1986; Leland, 1998; Dittmar & Mahrt-Smith, 2007; Pinkowitz et al, 2006; Bowden, 2006b). Leland (1998) and Bowden (2006b) extend these discussions to the risk management decision. Bowden (2006b) incorporates the concerns of this agency problem by specifying an objective function in the form of payoffs from two styles of options, including the liability exit option as well as the adverse financial distress put option. Based on this specification, the utility function in the form of equation (3.5) could be modified to:

\[ U(R; P) = (R - B)_+ + b(R - P)_- , \quad (3.8) \]

where \( B \) is the bankruptcy point and \( P \) is the financial distress level and \( B < P \). The first part of equation (3.8) represents the payoff of a liability exit call option, while the second part stands for the payoff of a adversity put option. The hedging decision is therefore the consequence of balancing values of the adversity and exit options. Bowden (2006b) goes further to suggest that although hedging adds value to shareholders in most cases, there is a ‘no hedge zone’ where firm value gets very low and the shareholders’ liability exit option value is high.

3.4 Corporate hedging and market efficiency

The issue of market efficiency plays an important role in deciding whether to hedge or not. In the following illustrative example, hedging with a market-based forward is
used to describe some of the essential choices of the corporate hedging in an efficient or an inefficient financial market. The forward fulfills the hedging purpose by simply eliminating uncertainty. This usually entails a trade-off between risk protection and opportunity cost. The fixing will take the form of buying or selling a forward with the same maturity as the natural exposure and holding to maturity. Futures, on the other hand, do not automatically fix the exposure due to the requirement of marking to market. Using options is a third choice which complements the value framework. This will be discussed in later chapters.

Let $S$ denote the value (as a price or rate) of an environmental exposure that can be hedged with a forward of current price $F$. If $h$ is the chosen hedge ratio, then the hedged exposure is

$$S^h = hF + (1-h)S$$

(3.9)

This will be referred to as the ‘conversion rate’. It contains one component known at the start of the period ($F$) and one component that is not known at that time ($S$). The key issue is then to choose the optimal hedge ratio $h$ that maximises the expected utility, as expressed in equation (3.6). Issues regarding the derivation of hedge ratio $h$ will be discussed in Chapter 8, in an environment of empirical application. In the current context, the two extreme scenarios – fully hedged or nothing hedged will be compared to explore the value added by hedging. By assuming there is no interaction among the market participants’ hedging strategies, the exposure indicator $R$ is simply represented as $S^h$. When the corporate chooses to hedge nothing, the equation (3.6) can then be written as

$$E[U(S; P)] = E[S - P] + bE[(S - P)_+] .$$

(3.10)

while the expected utility for a firm fully covered by the forward is:

$$E[U(F; P)] = F - P + b(F - P)_+ .$$

(3.11)

The value added by hedging can be expressed as the welfare difference between hedged and unhedged corporates:

$$\Delta EU = F - E[S] + b(F - P)_+ - bE[(S - P)_+] .$$

(3.12)

A positive $\Delta EU$ implies that the hedging does add value to the firm and thus the forward hedging is preferred to the unhedged position. The exact reverse situation applies in the case of a negative $\Delta EU$. 

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Let $\mu_d = F - E[S]$ represents the effects of hedging on the mean part and $\nu_d = b(F - P)_+ - bE[(S - P)_+]$ stands for the value added by hedging through decreasing the GVaR.

The hedging decision is then determined by balancing the value of $\mu_d$ and $\nu_d$. Given the forward rate, the value of $\mu_d$ relies on the expectation of spot exposure. The second part in the expression of $\nu_d$ corresponds to the payoff of the adversity put option, which is positively correlated with the variance of spot exposure. As a result, the value of $\nu_d$ is in addition dependent on the volatility of $S$. In this respect, given the expected value of $S$, the hedging adds more value to the firm if the variable is very volatile. Empirical hedging problems are often solved in a framework of conditional mean variance analysis. In the current context, the optimal hedging strategy can also be derived from the trade off between the mean and the variance of $S$ but the aversion to the volatile $S$ arises instead from the adversity option effect.

The value added by hedging differs for the following three scenarios:

1. If $F = E[S], \mu_d = 0, \nu_d > 0$;
2. If $F > E[S], \mu_d > 0, \nu_d > 0$;
3. If $F < E[S], \mu_d < 0, \nu_d$ can be either positive or negative, depending on whether the value of $F$ is below $P$:

$$
\nu_d = b(F - P)_+ - bE[(S - P)_+] = \begin{cases} 
    b(F - E[S]) + bE[(S - P)_+] & \text{if } F < P, \\
    bE[(P - S)_+] & \text{if } F > P.
\end{cases}
$$

(3.13)

In the third case, $\nu_d$ is equal to the expected payoff of a put option when $F > P$ and thus always positive. Under adverse market conditions, the forward rate, commonly determined by the covered interest parity, might be lower than the critical point $P$. In these circumstances, the value of $\nu_d$ becomes negative when the expected spot rate exceeds the forward rate by more than implied expected payoff of the call options. Consequently, the firm might encounter less financial distress costs by remaining unhedged.

Some simple conclusions regarding whether the corporate should be protected fully or be exposed to the financial risk, at least partially, can be drawn from the

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7 If $F < P$, $\nu_d = b(F - P) - bE[(S - P)_+] = b(F - E[S]) + bE[(S - P)_+]$;
If $F > P$, $\nu_d = -bE[(S - P)_+] = bE[(P - S)_+]$. 
above specifications. Firstly, as the above discussion for the first scenario indicates, a complete hedging is the best option in a world where the forward is the unbiased predictor for the future spot rate. However, as will be explored largely in Chapter 5, such a lack of bias on the forward rate has been commonly rejected by the empirical research.

Secondly, as shown in the above third scenario where $F < E[S]$, the value of hedging diminishes when corporate managers have superior information about the probability of a significant rise in the future spot rate. By remaining exposed, the appreciated spot price not only improves the firm’s wealth through increasing normal operational cash flows but could also reduce the financial distress costs, especially when the forward rate $F$ has been very low. Based on what the New Zealand Dairy Board experienced, the farmers started to suffer from the unfavorable exchange rates in 1997. It implies that the value of $F$ at that period was close or even lower than the distress point $P$. In such circumstances, hedging does not always reduce the corporate’s financial distress costs and thus is not necessarily a value improving approach.

It can be seen that firm value enhanced by the hedging decision largely depends on the expectation of the future spot rate. Therefore, a better forecasting of the exchange rate plays an important role in the currency risk management, in particular when the exchange rate market is not efficient. In this circumstance, firms owning private information or having better access to the true distribution of the future currency value are more likely to derive the optimal hedging policy which adds value to the firm.

Corporate managers who decide the hedging policy on the basis of their view of future markets are regarded as employing an active hedging strategy. Managers tend to choose an active stance at certain times when they feel that they know more than the market (Hakkarainen et al, 1997; Jilling, 1977). This confidence could arise because the firm has a dominant position in the market and thereby has private information about the impact on the financial market from its own operation decisions. Such confidence could also arise when corporate managers have access to superior forecasting techniques or advanced consultancy services.

The above discussions regarding hedging are concentrated on hedging against only one risk exposure, e.g. exchange rate risk, denoted by $S$. In reality, however, the corporate is very likely to face more than one risk. For instance, commodity exporters
may be exposed to the currency risk, commodity price volatility as well as input price fluctuation. In addition, since the current market is far from complete, some market risk can not be directly hedged with the financial instruments. In these circumstances, the risk management decision can deviate from those derived in the single exposure case, especially when the exposure components arising from various market risks are interacted with each other. In this case, the interaction between the two risk factors has impacts on the value added by hedging. The dependence among multi-risks facing the corporate can play an important role in the hedging decision. For instance, the negative interaction of output and input prices brings a natural hedge to the firm. In this case, the corporate may be harmed by hedging since using the forward cover can destroy the buffering effects and increase the downside risk. Strategy for managing the multiple exposures should start from analyzing the origin and construction of exposures. A detailed discussion about hedging against composite exposures is provided in Chapter 7.

3.5 Some qualifications: aversion to the opportunity losses

The segmented welfare function penalises the downside extreme losses through setting a higher marginal disutility for the wealth below the critical point $P$, as depicted in the figure 3.1. Under this form of managerial utility function, a hedging that reduces the downside adversity exposure by foregoing the upside gains is regarded as favourable. However, managers may on occasion have different risk attitudes toward the opportunity costs of hedging, for example, they may have higher risk aversion on the upside gain than appeared in the figure 3.1. Such concerns could increase the corporate’s marginal utility for wealth above the point $P$ and thus reduce the managerial incentives for a heavy hedging decision.

Aversion to hedging opportunity costs may arise from regret theory. According to Bell (1982, 1983) and Loomes and Sugden (1982, 1983), ‘regret’ is a feeling derived from the fact that wrong choices have been made and better decisions could have been taken in the past, given the realised actual outcomes. Individuals, who make decisions according to regret theory, compare each possible outcome arising from the choice they made with what they would have experienced under the same state of the world but assuming they made different decisions.

Solnik and Michenaud (2005) present a regret-theoretic approach for managing currency risk. The approach accounts for both traditional risk and regret. Traditional
risk is represented by a monotonically increasing and concave utility function. A concave regret function is based on the difference of outcomes derived from the adopted action and results caused by the best possible currency hedging decision given the \textit{ex post} outcome. Suppose a decision is made between hedging and non-hedging. Foreign investors may experience regret if they have hedged but their home currency depreciates later. On the other hand, investors who have not hedged may also be regretful if their home currencies appreciate. Solnik and Michenaud (2005) conclude that a fully hedged decision is the best hedging policy from the perspective of managing the traditional risk but a regret averse investor always hedges less than 100%. The optimal hedging ratio will be 50% if investors have infinite risk aversion.

Furthermore, experimental evidence of regret-influencing decision making under uncertainty (Gilovich & Medvec, 1995; Zeelenberg \textit{et al}, 2000; Bell, 1983; Bechara \textit{et al}, 2000; Camille \textit{et al}, 2004) shows that regret is usually more intense when people take actions and later find that no action could have led to a better result than when people fail to take some beneficial actions. If this statement is true, the regret on the losses arising from the hedging activity can be higher than the regret on the losses caused by a non-hedging decision and thus discouraging corporate managers from employing a high hedging ratio.

Aversion to the opportunity losses can also arise when the future spot exposure is regarded as a benchmark to examine whether a hedging policy is optimal or not in corporate currency risk management. Shareholders expect better outcomes from a hedging policy than those that can be achieved from unhedged exposures. The example of the New Zealand Dairy Board in 1997, as discussed in Chapter 2, showed how a hedging policy can be criticised when evaluated against the unhedged exposures. One reason, among others, was due to the farmers’ assessment of the hedged outcome against what they might have had without hedging contracts in place. Bearing the potential criticism from shareholders in mind, company managers could be more concerned about the opportunity losses arising from hedging than those appearing in the above utility function.

Similar considerations can be made with fund managers, especially when they allocate funds to international equity investments. The traditional benchmark for global investment is absolute asset return in the corresponding currency, such as S&P500 or MSCI US index for US stock investments. Van Eyk (2001) and Dunstan (2001) find that fund managers view currency hedging for international equities as
creating an additional risk because of the potential deviations from the unhedged benchmark. Thorp (2005) also found that Australian international equity managers allocating assets to overseas markets was generally assessed against the unhedged return. As a consequence, they may be penalised for hedging global investment against depreciation in home currency, but need not be criticised for failing to hedge an appreciated domestic currency. This form of assessment could intimidate the fund managers towards utilising financial instruments for hedging purposes.

3.6 Corporate hedging and market competition

Corporate hedging may have a more significant role in a competitive market. Some researchers show that financial decisions can interact with the firm’s production decisions when the market is competitive (Brander & Lewis, 1986; Bolton & Scharfstein, 1990; Brown, 2001; Nain, 2005; Mello & Ruckes, 2005). One firm’s advantages arising from a superior financial decision could very possibly be transformed to the product market to defeat its rivals. In such an environment, the opportunity costs of hedging include not only the potential upside gain in the financial market but also the possible losses in the product market, such as the decline in the market share. Especially in an oligopoly competitive market, where there are only a few players that have the power to influence market price. The competition is highly intense and rivals are always prepared to take full advantage of any profitable opportunities. A hedging policy that is efficient in reducing fluctuations in the financial price or rate changes might expose the firm to a significant disadvantage over competitors in the product market.

As discussed in Chapter 2, in the case of Bonlac Foods, the heavy hedging policy locked the corporate’s profitability at a fixed level. The competitor, Murray Goulburn, initiated predatory pricing to expand its market share when the Australian dollar reached a historically low level. Bonlac could not sustain the equal payment without making losses and thus lost the market share to Murray Goulburn. The predatory pricing strategy could also be employed by hedgers when the financial market moved in the direction favourable to them, as occurred in the case of Japan Airlines. However, in an intensely competitive market, the marginal profit is low and the predating would more likely occur when the spot market is unexpectedly good. In this respect, the value arising from the predatory pricing strategy might be asymmetric by exaggerating the opportunity cost of hedging and thus favouring a decision of less
hedging. This might in part explain why Briggs (2004) find in a New Zealand currency hedging survey that firms in a competitive industry, such as meat producers tend to hedge less and shorter than those companies in a less competitive environment.

Some research papers discuss and explore how corporates manage financial risk in a competitive market. Nain (2005) demonstrates that a firm’s hedging decision relative to the hedging pattern of the industry affects its competitive position in the product market. A firm may encounter higher exposure if it employs a hedging strategy different from others in the same industry. Brown (2001) provided evidence of firms benefiting from investigating and responding to competitors’ hedging choices. With the introduction of restricted financial instrument disclosure accounting standards (e.g. IAS 32 and SFAS 107), information regarding a listed company’s hedging with financial instruments becomes public. It enables other companies, such as competitors to have access to the firms’ existing financial hedging positions by investigating the financial statements, although currency risk management with operational strategies still remain as inside information to firms’ managers.

In addition, Adam, Dasgupta and Titman (2007) explore the structured solution for a corporate to derive an optimal hedging decision in a competitive environment. They derive the hedging ratio along with the optimal production decision under Nash equilibrium (e.g. Nash, 1950; Fudenberg & Tirole, 1991). Adam, Dasgupta and Titman (2007) derive the optimal hedging decisions from a Cournot-Nash equilibrium, in which firms maximise their profits given the output decisions. In this research, hedging is valuable because hedging brings certainty to investment in machines and other fixed assets, which consequently result in certain raw material inputs. Given a convex marginal production cost function, this certainty implies lower expected costs. However, unhedged firm, as argued by Adam, Dasgupta and Titman (2007), have production flexibility. In a two stage Cournot game, the firm determines whether to hedge at stage one and how much to produce at stage two. Unhedged firms could therefore adjust the production decision at a later stage to respond to the realised costs, such as the decision to increase the production in a good state. The authors conclude that firms’ hedging choices could vary according to the number of market participants, the degree of rivalry, demand elasticity and convexity of production costs in the industry.
The impacts of interactions between market players on corporate hedging will not be considered in the later empirical investigation. This is because the empirical application mainly relates to the New Zealand dairy industry in which there is no intense competition currently. Fonterra, the biggest dairy company in New Zealand, dominates the dairy market.

However, even if there were intense competition facing the New Zealand dairy industry, such as that seen in the Australian dairy industry, the pattern of interaction between the production market and the currency risk management decision would be different from the case described by Adam, Dasgupta and Titman (2007). In the NZ dairy market, a large proportion of production will be exported to foreign countries. Since the current global demand for dairy production is high while the total amount of supply is limited by the land and weather conditions, competition in the New Zealand dairy industry is mainly on domestic supply side. The market share of one dairy firm largely depends on its payout price to dairy farmers, relative to what the firm’s domestic competitors can afford.

In addition, the New Zealand dairy industry heavily relies on a co-operative ownership structure so that the firm’s suppliers are its shareholders as well. Given the determined cost composition, the amount that the dairy company will pay its suppliers, who are also shareholders, is largely dependent on how much it can sell its products for in the global dairy market. The international dairy prices and exchange rates are generally exogenous and, therefore the same to all New Zealand dairy companies. The distinctions among the selling prices for various firms may arise from the exchange conversion rates for the foreign receivables, which vary due to firms’ different foreign exchange hedging policies. The firm with the higher ex post exchange conversion rate can afford to pay a more attractive payout price to win suppliers over its competitors. In this respect, corporates with superior information or a better understanding of the future exchange rate movements have more survival chances in the market. Even in a market which is efficient in a long term context but not in a short time period, companies that know more about the financial market than their rivals can take advantage of their temporary stronger position against others. As a result, a more active hedging is advocated for companies in a competitive environment.
3.7 Conclusions

This chapter extended the VaR and related theories to establish the primary welfare function for corporate hedging. The consequent hedging objective is directly related to the corporate value function as both of them exhibit a trade-off between upside gains and downside extreme losses. However, there could be conflicts between the maximisation of firm value and that of shareholder’s value, as the shareholder has the option of exiting from the liability when bankruptcy is triggered. These agency conflicts have impacts on the choice for optimal hedging policy, mainly leading to a preference for staying unhedged. Nevertheless, the exiting options owned by shareholders can only be realised if some conditions are satisfied. Given the common ground of maximising firm value and the shareholder’s value, the rest of the thesis concentrates mainly on the perspective of firm value maximisation.

By applying the proposed GVaR approach to corporate hedging, the chapter has examined some theoretical results under a variety of circumstances. A complete hedging against one single exposure is suggested as the best policy in a market where the forward is an unbiased predictor of the future spot rate. In other circumstances, corporate managers may choose a partial hedging or unhedged decision according to their risk averse attitudes and their informational advantage. The empirical application for corporate hedging will be given in Chapter 8, with a focus on hedging against composite exposures.

The next chapter will extend the discussions on risk management objective welfare to other aspects. The GVaR criterion that has been proposed in this chapter is related to the expected value of upside gain and downside loss at a future point in time. The consequent utility function is time separable and has no consideration on what is happening along the path. It may not matter whether a time separable objective function is assumed when the path dependence is not apparent over a short time interval. However, when the risk management focus shifts to a long period, the inter-temporal nature of exposures could have big impacts on managerial preferences for optimal risk management structure. The two paths with identical expected gain or loss could well have various serial correlation patterns and thus exhibit different path exposures. To account for the considerations on such path risk, a modified objective function based on the dynamic VaR will therefore be explored in the next chapter.
Chapter 4 Long Term GVaR: Path Risk and Fund Management

When the hedging period is exceptionally long, a structured hedging framework further needs to be considered in the light of other managerial concerns. The objective welfare function established in previous discussions that derive the hedging decisions by balancing the expected upside gains and downside losses at one time point usually has a time independent utility function. Based on these discussions, the expected utility for one period is assumed not to be relevant to anything that happened prior to that period. One drawback of decision methods of this type is that it does not take into account the conditional dependence structure of exposures that occur during the time period. This issue is minor when the hedging period is short as the market transients arising from random market noises generally dominate the short interval returns. However, the problem can be significant if the focus of risk management is shifted to a long time period, in particular for strategic funds that are exposed to macro-scale variations corresponding with economic environmental factors. In this respect, investment outcomes should be compared in terms of the value paths rather than a more narrow focus on short run or longer run returns in isolation. The notion of path risk is motivated by the application of GVaR to entire time paths, rather than a particular point in time. Given the potential connection with the VaR indicator, the path risk can also be described as a dynamic version of VaR, or namely VaR duration. The portfolio dominated by long scale variation usually exhibits higher VaR duration, as the value of such a portfolio would remain at a risky zone for a long period when a recession is experienced.

To account for such long term dynamics of exposures, value rather than return is chosen as the indicator for measuring portfolio performance. Although return and value should convey the same information, current statistical tools are sometimes not sensitive to long scale variations displaying in return measurements. Short period return could be typically dominated by market transients which might obscure more fundamentally based signals that are strong in the long run but weak over the short term. Therefore, path exposures associated with macroeconomic influences are statistically more apparent with value indicators. Given the non-stationarity exhibited in the value measurement, a wavelet based framework will be chosen in the current context. This is because wavelet analysis can decompose portfolio variations along
both time and scale dimensions and requires no strict assumption for data generation processes. The path pattern of a portfolio value can then be expressed in terms of their wavelet multi-scale variation distribution.

The chapter is organized as follows: Section 4.1 briefly discusses the rationalities for the choice of dependent variables. Section 4.2 defines the path risk and specifies the corresponding objective welfare function. Section 4.3 further illustrates the path risk as the dynamic VaR. Section 4.4 concludes.

### 4.1 Choice of dependent variables: Values versus returns

Before defining the risk and designing the portfolio risk management rules in a long term horizon, some discussions on the choice of dependent variables are useful. The exploration will be mainly about whether value or return should be used to measure portfolio performance. To facilitate subsequent discussion, assume the discrete time interval is small so that returns can be treated as synonymous with changes in the log of value. Portfolio analysis has traditionally been formulated and conducted in terms of security returns. It is understandable why this should be the case: one period returns are stationary or nearly so, and hence are well adapted for standard statistical measures of reward and variation, which in turn can be easily incorporated into mean-variance portfolios. Extensions such as GARCH (generalised autoregressive conditional heteroskedasticity) or similar volatility modelling can be used to derive hedges that vary over time, along with conditional properties of asset returns.

Recent developments in stochastic modelling, however, have raised an issue as to whether security modelling and portfolio design should utilise alternative constructs. As an example, suppose security values are generated in terms of a fractionally integrated time series of order $1+d$, where $d$ is the fractional component $0<d<1$. Such representations have been used as a way of capturing long memory time series (Candelon & Gil-Alana, 2004). A more suitable return concept in such a case might be based on $r_t = (1-L)^{1+d} \log V_t$ rather than the conventional $r_t = (1-L)\log V_t$, where $L$ is the backward lag operator. Alternatively, one might choose some specific return definitions and work at the outset in terms of $\log V_t$ itself.

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8 This chapter is largely based on Bowden and Zhu (2009).
or some scaling transformation such as $\sqrt{1/t} \log V_t$, which can be useful where geometric Ito processes are supposed to generate data.

Logically, there is no difference in using returns or log values; they contain exactly the same information. However, inference based exclusively on returns can suffer from a swamping with shorter run market noise. This fact makes it harder for the return indicator to capture those statistically significant macroeconomic effects that work slowly but are important over years, rather than weeks or months. Short-interval rates of return are typically dominated by market transients, including not only one-period white noise, but also transients such as local bubbles or market overshooting in response to news or sentiment. Transient effects of this kind have been described by authors such as Barnett et al (1989), Lux (1995), Kojima (2000), Scheinkman and Xiong (2003). Market transients can generate episodic short term serial correlation that will magnify the variance of short term returns, but become smoothed out over longer horizon. This fact allows underlying fundamentally based signals to become manifest over a long interval. A similar effect has been noted in the literature on efficiency losses or gains from the use of overlapping observations, where the implicit smoothing enables better detection of longer run movements (e.g. Fama & French, 1988; Campbell & Shiller, 1988; Boudoukh & Richardson, 1993).

The situation is analogous to business cycle indicators, which are often presented in two forms, the value version and the growth or ‘growth cycle’ version, the latter referring to percentage rates of change. Locating cycles and their turning points is easy with the first, but quite difficult with the second (Bowden, 2005b).

4.2 Path risk and the underlying welfare criteria

Based on the value indicator, the primary welfare criteria for long term risk management might be cast in terms of a terminal distribution of unit value. However, the system dynamics can entail a sequence of conditional distributions for one distribution of the terminal value. In this case, path risk arises as a consequence of

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9 Suppose that returns are generated by the continuous time geometric process $dV_t = \mu V_t + \sigma V_t dB_t$, where $B_t$ is a standard Brownian motion process. As $\text{Var}(B_t) = t$, it follows that $\sqrt{t} \log V_t$ can be decomposed into a trend element of order $\sqrt{t}$ and a zero mean detail $\sigma B_t / \sqrt{t}$, which has constant variance (energy, in the wavelet context). Thus one indication that the log value series is behaving like the classic geometric accumulation process is that once normalised by $\sqrt{t}$, the details should show no obvious expansion in amplitude over time.
path dependence. The inter-temporal structure generated from the interim dynamics of a path can entail secondary welfare effects, which are reflected in the time inseparability of objective functions. The need for two welfare effects could arise for a variety of reasons. Taking the fund investment as an example, it might be a consequence of dual responsibility, as in the distinction between investors and management, each with specific objectives in addition to those shared. Alternatively, the distribution functions of the primary objective might be easier to estimate, mainly because they are asymptotic in nature. However, a specified primary objective value could be consistent with a variety of different formulations for the sequential conditional distributions, which may be difficult to specify and estimate with any exactness. The following remarks intend to elaborate on the general idea, particularly on defining the metrics that could be used for comprising possible adverse effects from alternative path histories.

For any given path history \( \omega \), let the vector \( x_t(\omega) \) represents the portfolio choice at time point \( t \). Denote by \( X_t = \{ x_\tau ; \tau \leq t \} \) the history of portfolio settings up to time point \( t \). A static or passive portfolio policy is a special case in which \( x_t \) is determined at the outset and is constant thereafter. The process \( V_t = f(X_t, t, \omega) \) will be called the ‘unit value history’, as it represents the accumulating portfolio value per dollar of initial capital invested. For simplicity, all proceeds are assumed continually reinvested, such as no external dividends until terminal time \( T \). Finally, there is a utility function \( U \) defined on the alternative path histories, which can remain general at this point.

Time inseparability in the utility function is taken to mean that at any intermediate time \( 0 < t < T \), conditionally expected utility from that point on is a function not only of \( V_t \), but also of the entire history of unit values to that point. In other words, fund managers take into consideration the evolutionary pattern of unit values. For instance, the managerial utility function could reflect the number of investors in the fund. Withdrawing funds by investors as a consequence of poor unit value performance will impact adversely on managerial income or even employment. This would require explicit modelling of investor responses to unit value histories, which is not something that the typical fund manager would want to attempt. A less formal approach is to recognise that some types of path history are going to be more exposed to investor unhappiness and exit (in this example). Such paths – essentially
portfolio choices $x_t$ might then attract a penalty, and their reward element must correspondingly be compensated. As with any form of risk analysis, there are two aspects, first to recognise the path exposures, and second to provide a metric that will penalise paths that are more exposed.

There is a great development in the measurement of the temporal nature of the financial variables. However, the reconciliation among exposures measurement, portfolio design methodology and the underlying welfare objectives is not extensive in the literature (Dunn & Singleton, 1986; Eichenbaum & Hansen, 1990; Constantinides, 1990; Detemple & Zapatero, 1991; Novales, 1990; Heaton, 1993). Most of these existing studies incorporate time dependence into the utility function by specifying the objective variable as a linear function of current and past observation. An alternative to this specification is the spectral utility function, which has been developed by Bowden (1977) and Otrok (2001) to accommodate time dependent aspects of managers’ preferences. With a spectral utility function, economic agents’ preferences in terms of the conditional characteristics of a variable over time are constrained to its overall auto-covariance value, which can be summarized as the spectrum of frequencies. Spectral analysis provides a possible framework of this kind, while interpreted broadly in the current context to cover Fourier analysis and wavelet decompositions. As Fourier analysis only transforms data in frequency dimension while wavelets decompose data in both time and frequency domain, the latter has a wider application than the traditional spectral utility.

Furthermore, the techniques to be developed could be regarded as dynamic analogues of mean variance analysis. In a long term mean-variance framework, expected terminal utility could be written as $U(\mu_T, \sigma^2_T); U_1 > 0, U_2 < 0$, where $\mu_T$ and $\sigma^2_T$ are the mean and variance of terminal unit value respectively. Now time domain path variability is expressible in terms of the Cramèr representation (stationary Fourier analysis) or the scaling detail decompositions in the case of wavelet analysis. Correspondingly, let $E$ stands for a spectral density for Fourier analysis, or detail energy for wavelet decompositions. Suppose that $G$ is the set of available energy measures for the application in hand. The required path variance analogue would then be a positive definite function $\phi: G \to \mathbb{R}_+$. In addition to overall energy, the function $\phi$ is chosen to assign penalty weightings to the frequency or detail elements. The weightings are designed in accordance with managerial
perceptions of secondary exposures or some other type of path exposure. In the case of wavelet decompositions, the mean element can be translated into the highest order approximation, which remains once most of the details have been removed (see Chapter 5). Portfolio efficiency in this extended framework requires maximising the expected reward element, subject to acceptable values of the path risk metric $\varphi$.

In what follows, path risk will be defined operationally as a weighted sum of spectral power energy, of the form

$$
\varphi = \sum_k w_k E_k; \quad w_k \geq 0, \sum_k w_k = 1.
$$

(4.1)

As discussed earlier, the weights $\{w_k\}$ can be set by the user in accordance with perceptions of path exposures.

The following discussion illustrates some of the possible considerations in choosing whether to penalise at the longer or shorter run end of the power spectrum; specific implementation is contained in Chapter 9.

### 4.3 VaR duration and the empirics of investment value

Path preferences could be set in something as simple as a general preference for smoothness. Paths that soar can also plunge, which welfare benefits and costs might not compensate: investors are more apt to anxiety on extended downturns than happiness on upturns. As to the choice of weighting function, much would evidently depend on the advertised stance of the fund. High long term energy (variation) would more likely be a danger to a fund that advertised itself in terms of stable balanced growth.

A more structured approach to manage path welfare costs might run in terms of a dynamic version of VaR, or in this context, VaR duration. The idea is that paths should not spend too long below a comparator or benchmark path that could be interpreted as a moving VaR critical value, incorporating investor regret. Figure 4.1a depicts path histories generated by two alternative static portfolios, both valued in terms of US dollars.
Figure 4.1a: Path risk from benchmark violations

Portfolio A is an accumulation total return index for New Zealand stocks, while B is made up of an Australian equity total return index portfolio and a US treasury bond index in equal proportions. The stock indexes are collected from the MSCI while the treasury bond index is from Federal Reserve Bank. In addition, path C is a benchmark portfolio representing investment in US treasury stock held to maturity, using the 10 year yield at inception time (6.01% at May 1993) as a proxy for the entire period 14 year rate. All unit value series are measured in logs, hence the straight line for the benchmark.

The two subject paths A and B are not widely different as to the variation of monthly returns. Note, however, that path B is uniformly above the benchmark C, but path A spent almost three years below (see the double headed exposure duration arrow). Unit holders might tolerate a short period of discomfort, but not long term losses. Even though path A is a little superior on a primary welfare objective taken as terminal unit value, its adverse performance on the penalty element would likely see managers prefer path B. In terms of *ex ante* portfolio selection, portfolios such as A, viewed as more subject to long swings, would carry the burden of higher path risk. To attract risk averse investors, additional compensation would be required in the trade-off with the primary welfare objective of this path. Moreover, a point such as *P* has different welfare consequences for path A versus path B. In a dynamic portfolio...
problem, different portfolio choices could be derived from then on, as a manifestation of utility inseparability.

The role of the weighting function (4.1) is illustrated more explicitly in figure 4.1b, which reallocates the total detail energy of path A towards the finer of the 7th detail levels (wavelet conventions and methodology are exposited in Chapter 5 below).

![Figure 4.1b: Energy structure and benchmark exposure duration](image)

Total detail energy of path A is 11.984, of which the four bands spanning two years and longer contribute 11.574 for the original series. Series B is a reconstruction of A that redistributes the total detail energy so that only 4.830 remains at the longer fluctuations. The result is evident in the form of greater short run variation. Therefore, the duration of the adverse exposures of path B (below path C) are transitory, never lasting longer than two to three months at a time. Given the implied preference as to reward and path risk, B could be regarded as path-preferred to A. It might not be so for an alternative choice of the weighting function in equation (4.1) for path risk. For instance, short run fluctuations might well be viewed as less attractive for some particular class of fund. The portfolio investment decision may vary by the managerial choice for paths. In the empirical application part (Chapter 9), it will be shown how the weighting function approach can be built into a portfolio selection procedure.
4.4 Conclusions

This chapter has developed the objective welfare function for managers with concerns on long term risk management. Over a long interval, values of financial variables are generally exposed to both low and high frequency variation. These variables might display different multi-scale behaviour corresponding to various value paths. Although the measurement of path exposures has been largely developed, the related decision theories are limited. With the strategic fund management as a particular example, the present chapter proposes a multi-scale approach to enable a better reconciliation between portfolio measurement methodology and underlying welfare objectives. The methods that result comprise a primary objective of the long period expected value and a secondary welfare effect of path risk. The comparison of investment outcomes is based on the value paths rather than the return at one particular point. Risk regarding the value paths has points of contact with a dynamic version of VaR, specifically the duration of VaR. In this respect, the present chapter extends the application of the VaR theory to the long term risk management.

The path exposures are measured in the form of a function of wavelet energies at different scales. Application of this proposed long term fund management methodology is provided in Chapter 9. With the aid of wavelet analysis, the approach is implemented in the form of band pass portfolios, which enable fund managers to choose the variation at designated scales. The consequent approach are well adapted to asset accumulation in an economic environment that creates longer run, or macro-scale, variation along with which is superimposed shorter run fluctuation arising from market noise and similar disturbances.

Econometric implementation of wavelet analysis in the particular context of currency variation and market efficiency will be discussed in the following chapter. The exchange rate market efficiency is controversial, and such a problem is exceptionally important in currency risk management as the value of hedging largely depends on the degree of market efficiency. To examine the currency market efficiency, the time series will be decomposed in a wavelet framework along both time and frequency. Such decompositions enable users to identify variation patterns of the time series on a scale by scale basis. In other words, the wavelet provides a multi-dimensional view of the data. The wavelet multi-scale decomposition on exchange
rate variations further indicates the patterns of currency value, which are valuable in deriving the exchange rate forecasting model.
Understanding whether the market is efficient and how the currency fluctuates with time is essential for developing an optimal currency risk management framework. In an efficient market, the market price of financial assets reflect all the available information and thus corporate managers cannot outperform the market by actively estimating and managing the exchange rate risk. In examining currency variation and exchange rate market efficiency, a first issue is to study the forward unbiasedness hypothesis in the exchange rate market. When the forward rate is an unbiased predictor of the future spot rate, the currency market is efficient and the optimal risk management under a GVaR welfare function is simply a complete hedging with forwards, as was shown in Chapter 3.

A linear regression is generally employed to test the unbiased hypothesis. However, it should also be recognised that the unbiased hypothesis is a sufficient but not a necessary precondition for an efficient exchange rate market. The degree of bias of forwards can be explained by reasons other than market inefficiency. The following discussion further examines the currency variation with wavelet analysis. The wavelet decomposition of the exchange rate conveys information on the nature of the variation pattern in the currency market. Such findings may provide evidence for market inefficiency and motivate development of the exchange rate forecasting model.

In addition to investigations on the nominal two-way exchange rates, the econometric examination of the currency variability will also be based on a constructed absolute exchange rate. Since the quoted exchange rate corresponds to the value of two countries’ respective currencies, the variation in one exchange rate pair may be caused by fluctuations in either currency. A multinational company encounters the currency risk when it converts foreign income, expenses or other cash flows back to home currency at a volatile exchange rate. If the exchange rate fluctuation originates mainly from the foreign country, the currency exposure could be diversified by trading in different countries. On the other hand, if the home currency is volatile in its own right, then the currency risk for a trading company is
systematic and could not be geographically diversified. The comparison of the systematic exchange rate exposure in different countries assists the choice of the application of the financial hedging framework developed.

The chapter is organized as follows: Section 5.1 discusses the market efficiency and the corresponding regression tests on the exchange rate market. Section 5.2 introduces the analytical tool – wavelet analysis and its use in measuring the variation in the currency value. Section 5.3 presents the construction of currency reference rates and discusses the variation patterns in a variety of countries’ different reference rates. Section 5.4 concludes.

5.1 Currency market efficiency

5.1.1 Market efficiency conditions

Since Fama (1965, 1970) initiated the concept of the efficient market, there has been a long-standing debate about the market efficiency. An efficient market is usually defined as the market where the current security prices should reflect all the available information. In such a market, the present price would be a good estimate of the future value and no speculators can expect consistent profits based on the current information. In these circumstances, the deviation of the spot value from the current estimate will only arise from unexpected news. Among the existing empirical work on testing the efficient market hypothesis, a variety of approaches have been developed for investigating the market efficiency. One common method is to test whether the asset prices follow random walks over time in an efficient market (e.g. Samuelson, 1965). Alternatively, one can perform the technical analysis to examine whether there is pattern in historical price and whether trading rules relying on these patterns will lead to excess returns (e.g. Neely & Dittmar, 1997). Other methods for testing the market efficiency include event studies, which focuses on the test of whether new information is rapidly incorporated into asset prices (e.g. Fama et al, 1969).

Furthermore, when the discussions focus on the exchange rate changes, impacts of interest rate differential on the currency values have to be taken into account. The return on one foreign investment is dependent not only on the currency changes during the investment period, but also on the absolute interest returns in the target country. Investors’ required rate of return on holding one currency will therefore be

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10 Section 5.3 is based on Bowden and Zhu (2007b).
affected by the interest rate in the corresponding country. Given relations between the exchange rate and the interest rate, the test for market efficiency can be fulfilled by investigating the Uncovered Interest Parity (UIP), which suggests that the interest rate differential should simply reflect anticipated currency movements. It is consistent with exchange rate market efficiency literature to start the test with the UIP (e.g. Hansen & Hodrick, 1980; Goodhart, 1988; Frankel & Chinn, 1991).

The UIP can be written as:

$$ s_{t+1}^r - s_t = i_t^* - i_t, $$

(5.1)

where where $i_t^*$ and $i_t$ represent the interest rate in term currency and commodity currency country respectively. The symbol $s_t = \log S_t$ and $S_t$ denotes the spot exchange rate at time $t$ (1 unit of commodity currency = $S$ units of term currency). The symbol $s_{t+1}^r$ stands for the rational expectation of the future log spot rate at time $t+1$, given information available at time $t$. As market expectations of future exchange rates $s_{t+1}^r$ are not easily observed, it is difficult to test the uncovered interest parity directly. Therefore, the uncovered parity has usually been tested jointly with the rational expectation hypothesis, which implies

$$ s_{t+1} = s_{t+1}^r + \epsilon_{t+1}, $$

(5.2)

where the error term $\epsilon_{t+1}$ has zero mean and is serially uncorrelated. Combining equation (5.1) and (5.2), the relation between spot exchange rate and interest rate differential can be expressed as follows:

$$ s_{t+1} = s_t + i_t^* - i_t + \epsilon_{t+1}. $$

(5.3)

Given the above equation, the exchange rate can be said to follow a random walk with drift model (Taylor, 1995), the drift being provided by interest rate differential. While a more complicated auto-regressive model will be provided in later chapters, the current discussions start with a conventional approach, namely a simple linear equation as follows

$$ s_{t+1} - s_t = \beta_0 + \beta_1(i_t^* - i_t) + \epsilon_{t+1}. $$

(5.4)

If the exchange rate does follow the random walk with drift model, the regression estimates of $(\beta_0, \beta_1)$ should not deviate from $(0, 1)$.

The test on the exchange rate market efficiency can also be associated with the covered interest rate parity. In a currency market, the covered interest rate parity
theory states that the difference between the market forward rate and the current spot rate should reflect the interest rate differential in two corresponding countries. This parity suggests that a higher interest rate in the home country should relate to a forward discount in the domestic currency. Because investors can construct a synthetic forward by borrowing money in one currency and lending money in another currency, the parity has to be generally true as there will otherwise be arbitrage opportunities between the forward and spot markets. According to the Covered Interest Parity (CIP), the forward exchange rate can be written as:

\[ F_t = S_t(1 + i_t^*) \left(1 + i_t^* \right)^{-1}, \]  

(5.5)

where \( F_t \) is the forward exchange rate determined at time \( t \) and \( f_t = \log F_t \). Since \( i_t^* = \log(1 + i_t^*) \) and \( i_t = \log(1 + i_t) \) when \( i_t \) and \( i_t^* \) are small, equation (5.5) can be written as:

\[ f_t - s_t = i_t^* - i_t. \]  

(5.6)

The equation is exactly true for continuously compounding variables and approximately right in the discrete case, provided that the interest rates are not too large. The currency market efficiency test can thus be based directly on the unbiased hypothesis of the forward rate with respect to the future spot rate. It leads to another version of the regression equation that can be written as:

\[ s_{t+1} - s_t = \alpha_0 + \alpha_1 (f_t - s_t) + \epsilon_{t+1}. \]  

(5.7)

The efficiency of the currency market can be assessed by testing whether \( (\alpha_0, \alpha_1) = (0,1) \).

A finding of \( (\alpha_0, \alpha_1) = (0,1) \) or \( (\beta_0, \beta_1) = (0,1) \) will imply that the market is efficient. However, findings such as \( (\alpha_0, \alpha_1) \neq (0,1) \) or \( (\beta_0, \beta_1) \neq (0,1) \) do not necessarily lead to the conclusion that the market is inefficient. The deviations may be explained by the risk premium regarding the holding of one currency relative to another currency, as explained later.

### 5.1.2 Regression test results on UIP

The market efficiency has been tested using ordinary least squares with Eview 5 software package. The application relates to monthly exchange rates of NZD/USD as provided by MSCI. The interest rate time series contains the US 1-month CD (Certificate Deposit) rate from the Federal Reserve Bank and NZ 30 day bank bill rate
from the New Zealand Reserve Bank. The data spans the period from June 1985 to August 2007.

The regression test on this data could be based either on equation (5.4) or (5.7). Some tests are employed to detect whether there is a unit root in both the dependent and independent variables. Table 5.1a shows that both Augmented Dickey-Fuller (ADF) tests (Dickey & Fuller, 1979) and Phillips-Perron (PP) tests (Phillips & Perron, 1988) reject the unit root hypothesis for the exchange rate return at a 99% significance level.

### Table 5.1a Unit root test outcomes

<table>
<thead>
<tr>
<th>Test statistic</th>
<th>Exchange rate return</th>
<th>Exchange rate return</th>
<th>Interest rate differential</th>
<th>Interest rate differential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Augmented Dickey-Fuller test</td>
<td>-15.4825</td>
<td>0</td>
<td>-3.2727</td>
<td>0.0172</td>
</tr>
<tr>
<td>Augmented Dickey-Fuller test</td>
<td>1% level</td>
<td>-3.4548</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Augmented Dickey-Fuller test</td>
<td>5% level</td>
<td>-2.8722</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Augmented Dickey-Fuller test</td>
<td>10% level</td>
<td>-2.5725</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phillips-Perron test</td>
<td>-15.4891</td>
<td>0</td>
<td>-2.6394</td>
<td>0.0864</td>
</tr>
<tr>
<td>Phillips-Perron test</td>
<td>1% level</td>
<td>-3.4548</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phillips-Perron test</td>
<td>5% level</td>
<td>-2.8722</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phillips-Perron test</td>
<td>10% level</td>
<td>-2.5725</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

On the other hand, ADF and PP tests for interest rate differential exhibit different evidence for the unit root. The ADF test rejects the null unit root hypothesis while the PP test shows that the hypothesis could not be rejected at a 95% confidence level. Under rational expectations, if the spot rate change is stationary while the interest rate differential is not stationary, the efficient market hypothesis must be rejected. The reason lies in the fact that $\varepsilon_{t+1}$ in equation (5.3) must be a white noise under the rational expectations, which implies that the $s_{t+1} - s_t$ must have the same order of integration as $i_t^* - i_t$. In other words, it shows some variations that are occurring in the interest rate differential have not been reflected in the currency changes. However, Baillie, Bollerslev and Mikkelsen (1996) and Engle (1996) claim that such variations could be attributed to the risk premium. In this respect, the market efficiency might still hold even when $s_{t+1} - s_t$ is stationary while $i_t^* - i_t$ is non-stationary.
Since an ADF test rejects the unit root hypothesis for the interest rate differential at a 95% confidence level, the linear regression is still used and the result is provided in table 5.1b below.

**Table 5.1b UIP test outcomes**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>0.0039</td>
<td>1.3922</td>
<td>0.1650</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>-1.5390</td>
<td>-2.8441</td>
<td>0.0048</td>
</tr>
<tr>
<td>Durbin-Watson statistic</td>
<td></td>
<td>1.9513</td>
<td></td>
</tr>
<tr>
<td>F-statistic</td>
<td></td>
<td>0.0048</td>
<td></td>
</tr>
<tr>
<td>adjusted $R^2$</td>
<td></td>
<td>0.026</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1b provides an estimate of the parameters shown in equation (5.4) and the tests on equation (5.7) present similar results. The estimated values of the Parameters show that the hypothesis of market efficiency should be rejected, as the interest differential coefficient $\beta_1$ is negative as well as significant. A chow type test shows no significant evidence for any structural changes during the mid-1990s. The result implies that under market conditions of high interest rates, currency tends to move up rather than come down. This is contrary to UIP theory.

The Q-statistic tests on the standardized residuals and residuals squared are shown in Appendix F, table F.1b. The Q-tests reveal significant serial correlation on the squared residual for the above linear regression. This suggests that there could be heteroskedasity components in the error term. The relation between exchange rate changes and interest rate differential can thus be examined in the context of GARCH. Detailed discussions on a GARCH model will be provided in the following exchange rate forecasting chapter, which will also show that a rise in the domestic interest rate is generally followed by an appreciation in that country’s currency. Such a situation is consistent with the empirical research done by Hodrick (1987), Froot and Thaler (1990), Lewis (1995), Engel (1996), Meredith and Chinn (1998), Wang and Jones (2002), and Bhar et al (2001).

Many researchers have attempted to explain the puzzle that empirical tests commonly reject the unbiased hypothesis for the forward exchange rate. Lewis (1995) and Evans (1995) attribute the reason for the forward puzzle to the learning and peso problem, which implies that market players determine the price on the basis of an expectation of some improbable event. Frankel and Rose (1994) argue that irrational expectations and speculative bubbles could cause such deviations of forward rate
from the expected future spot rate. Fama (1984) and Nieuwland et al. (2000), among many others, align the anomalous empirical findings with the market efficiency by describing the forward bias as the risk premium. Therefore, the phenomenon that the high interest rate in New Zealand is accompanied by the appreciation in New Zealand currency may indicate that global market investors require a higher return from holding NZ dollars compared with holding US dollars.

Assume $i^*$ stands for the US interest rate and $i$ represents the NZ interest rate. The exchange rate is expressed in terms of US dollar so that the NZ dollar is the commodity currency. The expected return of investment in NZ for a US investor is the sum of NZ interest rate and the expected currency fluctuations. It can be written as

$$r_{NZ}^e = i + (s_{t+1}^e - s_t).$$

At the same time, the home investment return for a US investor is $r_{US} = i^*$. Risk averse US investors tend to require a return on the NZ investment as follows:

$$r_{NZ}^c = r_{US} + \pi_p,$$

where $\pi_p$ represents the risk premium. The equation leads to the result

$$i + (s_{t+1}^e - s_t) = i^* + \pi_p.$$

By rearranging the equation, the expected exchange rate can be expressed as

$$s_{t+1}^e = s_t + i^* - i + \pi_p.$$

In a covered interest arbitrage free world, the equation can be reduced to

$$s_{t+1}^e = f_t + \pi_p.$$

A constant $\pi_p$ may be used to explain why the constant parameter ($\beta_0$) is different from zero but it cannot explain why the coefficient ($\beta_1$) between spot exchange rate changes and interest rate differential in NZ and US is significantly negative (see table 5.1b). However, the risk premium $\pi_p$ may vary over time and may be positive or negative. The characteristics of currency risk premium and impacts of risk premium on the currency value changes will be further investigated in Chapter 6 where a GARCH exchange rate forecasting model is explored. In the proposed GARCH model, the risk premium is expressed under the assumption of rational expectation as the spot forward basis ($s_{t+1}^e - f_t$). The GARCH forecasting model, with which the UIP can be examined, will further facilitate corporate managers in detecting any potential inefficiency exhibited in the exchange rate market.
5.2 Wavelet Analysis

Currency variation and market efficiency can also be investigated with the aid of the wavelet analysis. Although firstly used many years ago (as ‘wave packets’ in quantum physics), empirical wavelet analysis developed rapidly following a burst of activity in the early 1990s directed at new representations and computational techniques by authors such as Mallat (1989), Daubechies (1988, 1990, 1992), Coifman et al (1990), Cohen et al (1992). Wavelet analysis has also been applied extensively in the fields of mathematics, quantum physics, electrical engineering, and seismic geology since 1980s. Further applications of wavelets in economic and finance issues can be seen in recent literature. For useful reviews of the use of wavelet analysis in economics, see Ramsay (1999), Schleicher (2002), or Crowley (2005). Wavelets have also been applied to finance, e.g. Capobianco (2004), Lee (2004) and Fernandez (2004).

5.2.1 An introduction to wavelet analysis methods

This introduction to wavelet analysis starts with a discussion on spectral analysis. More technical description of wavelets are provided in Appendix A. Spectral analysis techniques are used to decompose a given series into the sum of sinusoids of different frequencies (a process called ‘complex demodulation’). When comparing the amplitudes or power of these sinusoids, the fact that one frequency has more associated power than others, suggests that much of the variance in a given series can be explained in terms of a well defined cycle at this or an equivalent frequency. However, these elementary sinusoids themselves do not change over time, either in their frequency or their amplitude. This is one of the limitations of spectral analysis, although from time to time suggestions have been made as to developing time varying spectra (e.g. Priestley, 1965). However, even in that approach, the changes had to be very slow over an extended period of time.

Limitations of this kind were effectively removed by the recent development in wavelet theory and practice. Wavelets analysis now decomposes data into different components along both time and scale (or frequency). A wavelet is rather like a sinusoid localised at a particular point in time, so that its power drops off rapidly on either side of that time point (see Appendix A). Wavelets come with a variety of scales. Thus one might have a scale representing a 6-12 month fluctuation (loosely, one cycle), another for a 1-2 year fluctuation, a third for 3-5 year fluctuation and others. Moving through time, one fits a succession of wavelets for each scale. Each
time point contains contributions from wavelets of the same ‘scale’ (quasi-frequency) but centred at neighbouring points. This feature enables one to model cycles that do not remain constant in amplitude, so that, in this respect, wavelet analysis overcomes the limitations of ordinary spectral analysis.

**Discrete wavelet transform**

The wavelet transform can be either continuous or discrete. Figure 5.1 is a schematic description of a discrete wavelet transform (DWT) outcome. Level 1 is the smallest scale or highest quasi-frequency, so D1 represents the cycle at this highest level of detail. The given series is then split into D1 and A1, where A1 is the series once the highest frequency fluctuations have been removed. Levels 2, 3,... contain successively less high frequency complexity. When more fluctuations have been extracted, the residual series becomes broader time-frame approximations, which reveal longer run cycles and ultimately the trend. An ‘average period’ constructed for a given level of detail D can be derived by finding the sinusoid where the period most closely matches that of the wavelet fitted at any point in time, suitably adjusted for its scale (see Appendix A).

![Figure 5.1 Decomposition into successive details and approximations](image)

The overall effect can be viewed as an operational generalisation of the schematic time series decomposition that is familiar within economic literature:

\[ \text{Series} = \text{irregular} + \text{seasonal} + \text{cycle} + \text{trend}. \]
The successive details in the above would be *irregular* = $D_1$, *seasonal* = $D_2$, *cycle* = $D_3$, while the trend would correspond to the subsequent remaining low frequency variations, namely approximations. However, the decomposition can be more refined than this. There are further levels of cyclical detail, and the approximation itself can take the form of a sum. The highest approximations, for example, $A_7$ in figure 5.1, can be considered the trend, as it shows no residual cyclical character. The wavelet decomposition is a dyadic data analysis process and thus the scale of sample determines how many levels of decomposition can be made. For a data set with 128 (=2^7) observations, the wavelet decomposition can be achieved up to level 7.

The technique has become popular because the methodology relies on few assumptions yet is powerful in its ability to analyse patterns in data. In applications of wavelet analysis, exposures can be linear or non-linear, stationary or non-stationary. Those assumptions usually required for a normative model are not necessary for utilising wavelet analysis. Short and long term exposures are decomposed and examined at different scales. The overall effect is rather like adjusting more and more exactly the focus of a microscope. One of the more celebrated images in wavelet exposition is that of Madame Daubechies’ eye as viewed from successively closer up. At long range one sees only the general features, while the rest as blurred. These appear like $D_6$ or $D_7$ plus the $A_7$, although in this case two dimensional. Moving closer, one sees higher level details of the iris, ultimately up to $D_1$. Instead of a limited one dimension, wavelets allow users to examine multi-dimensions of data.

**Wavelet functions**

A large number of wavelet functions have been developed for the wavelet transform. Collectively across different scales, the existing wavelet functions are flexible enough to allow for asymmetric local cycles of rather arbitrary form, no longer requiring regular sinusoidal patterns. Although all are chosen from the same generic family, wavelets are normalised to refer either to the cycles (‘mother wavelets’) or long term trend or quasi trends (‘father wavelets’). The mother wavelets integrate to zero while father wavelets are normalised to integrate to 1. The results of fitting mother (cyclical) wavelets of different scales are the details ($D_1, D_2, \ldots$) and they are additive in their effect. The progressive sums, by adding more details, are the approximations ($A_1, A_2, \ldots$). A more detailed discussion on the wavelet function is given in Appendix A.
which also depicts the wavelet family used in the present study, namely the ‘coif5’ wavelets.

**Wavelet variation metrics**

By decomposing the time series into orthogonal components, the variance of components at different scales can be derived using wavelet methods. Though this is a local concept that differs over periods, one can compute the average variance over the given time horizon. For time series \( X_t, t=1, 2, \ldots, N \), the decomposed wavelet details at scale \( j \) are indicated as \( D_{j,t}^{(X)} \), which should have a mean of zero as the integral of mother wavelet is 0. The wavelet variance at scale \( j \) can then be calculated with the following equations:

\[
Var_X(j) = \frac{1}{N} \sum_t \left( D_{j,t}^{(X)} \right)^2.
\]

With a discrete wavelet transform up to level \( J \), the total volatility of the variable can be expressed as the sum of all the detail volatilities and the remaining volatility at level \( J \) approximation.

The variance decomposition enables users to quantify how time series differ in terms of multi-scale variation behaviour. For time series with a range of persistence characteristics, the wavelet variance analysis may disclose the main contributors to the overall volatility (Percival & Walden, 2000). For instance, a random walk process should exhibit the concentration of variance at the highest approximations and details.

On the other hand, the variance of a white noise series would mainly come from the smallest level detail, such as \( D_1 \). For a long memory process that has longer persistence of shocks than ARMA (autoregressive moving average) class of time series but shorter than a random walk, Percival and Walden (2000) show that a plot of log of wavelet variance versus log of scale exhibits approximately linear variation.

In addition to the wavelet variance, the wavelet correlation can also be constructed on a scale basis. For example, if the original signal \( S \) is composed of \( X, Y \), the covariance and correlation indicators at a specific scale can be expressed as follows:

\[
Cov_{X,Y}(j) = \frac{1}{N} \sum_t D_{j,t}^{(X)} D_{j,t}^{(Y)}
\]

\[
Corr_{X,Y}(j) = \frac{Cov_{X,Y}(j)}{\sqrt{Var_X(j)Var_Y(j)}}.
\]
This decomposed correlation can be particularly useful in economics and finance as the component of the exposures might have different correlations for low frequency and high frequency variations. For instance, the components might move in different ways over a short term period, as the high frequency variations usually reflect market transients that are hard to predict and vary from market to market. However, as most of financial variables are affected by macroeconomics fundamentals, they may still show a close relationship with each other in the long run.

The above wavelet variance and related indicators are mainly constructed through the discrete wavelet transform. However, the wavelet variance can be estimated more effectively with the maximal-overlap discrete wavelet transform (MODWT) (Percival, 1995; Gencay, Selcuk & Whitcher, 2002). Unlike DWT, the MODWT is not orthonormal and not subsampling the filtered output (details see Appendix A). The MODWT is invariant to circularly shifting the original time series. The size of sample that the MODWT can handle is thus not limited to a multiple of $2^n$.

The MODWT extends the sample by assigning the observable values to those unseen as if it were periodic, e.g. the unobserved samples $X_{-1}, X_{-2}, \ldots, X_{-N}$ are assumed the same as the observed values $X_N, X_{N-1}, \ldots, X_1$.

As Percival and Walden (2000) argue, the wavelet variance, covariance and correlation in terms of MODWT coefficients result in better statistical interpretations than DWT. However, the MODWT can be biased as it introduces artificial circularity into the sample data. The coefficients involving the beginning and end data thus need to be excluded from the above formula to get the unbiased variance. In the current context, excluding the boundary coefficients, such as variation over a five year to ten year frequency will rule out the outcomes of interest. Therefore, the following empirical section focuses on the DWT based variance, in which the variance over different scales is the sum of squared details. The biased MODWT type wavelet variance will be constructed only for verifying the results.

### 5.2.2 Wavelet decomposition results

Figure 5.2 is wavelet decomposition for the (log) USD/NZD exchange rate. The scale of the vertical axis of each individual graph in figure 5.2 can be taken as a rough indication of the energy at each band. A more precise answer can be obtained by reading off the energy decomposition in table 5.2.
Calculations are based on monthly data from Jan 1986 to Feb 2007. Exchange rate data are end-month mid rates collected from Global Trade Information Service via Thomson Financial Datastream, and expressed in logs. The maximum number of scales is limited by the available data. The maximum scale recognizable is of order $2^k$; thus scale 7 would require 128 months but scale 8 would need 256 months. In this case with 254 monthly observations, the highest scale is chosen as 7. As shown in table 5.2, one can then recognise cycles up to about 15 years. The wavelet function used for the decomposition is ‘coif5’. However, the findings were checked using the symmlet and Daubechies wavelet functions (‘sym10’ and ‘db6’) with similar results, in particular as to the existence of long term scaling behaviour.

![Approximations and Details at various scales](image_url)

**Figure 5.2 Details and approximations for log(USD/NZD)**

<table>
<thead>
<tr>
<th>Period centred at (years)</th>
<th>Energies</th>
<th>Detail energy as % all detail energy</th>
</tr>
</thead>
</table>

**Table 5.2 Energy decomposition for log(USD/NZD)**
Chapter 5 Exchange Rate Econometrics I: Currency Variation and Analytical Tools

<table>
<thead>
<tr>
<th>A7</th>
<th>Long-term</th>
<th>0.2645</th>
</tr>
</thead>
<tbody>
<tr>
<td>D7</td>
<td>15.5</td>
<td>0.6156</td>
</tr>
<tr>
<td>D6</td>
<td>7.7</td>
<td>2.9673</td>
</tr>
<tr>
<td>D5</td>
<td>3.9</td>
<td>0.4278</td>
</tr>
<tr>
<td>D4</td>
<td>1.9</td>
<td>0.1690</td>
</tr>
<tr>
<td>D3</td>
<td>1.0</td>
<td>0.0683</td>
</tr>
<tr>
<td>D2</td>
<td>0.5</td>
<td>0.0590</td>
</tr>
<tr>
<td>D1</td>
<td>0.2</td>
<td>0.0357</td>
</tr>
</tbody>
</table>

It can be seen from figures 5.2 and table 5.2 that the exchange rate USD/NZD has highest energy for details at level 6, which is related to a 7.7 years cycle. Cyclical behaviour at a 4-year interval is also apparent, as the level 5 detail energy is higher than the details at level 1 to 4. It implies that the variation persistence may more probably be captured over a long period, e.g. 4-8 years. The variation is not apparent for high frequency data.

The interior peak in the detail energy shows fluctuations at the 7.7-year frequency is the main contributor to the currency value movement. Since the wavelet decomposition of interest rate differential between the US and NZ reveals no interior peak at level 6, this implies that uncovered interest parity cannot be used to explain the cyclical behaviour of the exchange rate. The results do suggest that the cyclical pattern found in the USD/NZD might have a relation with business cycles. There is some support among New Zealand economists for a business cycle of about 7-8 years, partly as a result of the commodity/exchange rate cycle (e.g. Kim et al, 1995; Hall & McDermott, 2006). The potential relation between currency movement and the business cycles indicates some possible structural connections between currency changes and economic fundamental variations. Such structures will be further examined in the subsequent chapter.

5.3 Measuring variation in the currency reference rate

The measurement of the exchange rate variability in foregoing discussions is based on a quoted exchange rate USD/NZD, which is inherently bilateral. However, currency movements in this sense may be the result of new information from either of the two corresponding countries. At one point, the home currency may appreciate against one foreign currency but depreciate against another currency. Given the bilateral exchange
rate USD/NZD, it is hard to answer questions such as whether the NZ dollar is inherently more variable than the US dollar or the UK pound. A more partner-neutral reference rate can be developed for a set of bilateral rates. Empirically, the partner is now based upon the entire set of bilateral exchange rate, so that this becomes a more ‘absolute’ exchange rate.

Such a measurement helps corporate managers to analyse the currency exposure and the corresponding exchange rate risk management. For example, if the volatility in the exchange rate between home currency and foreign currency arises mainly from the fluctuations in foreign countries, an exporting company with business partners in a range of countries is less exposed to the currency risk, as long as currency variations in these countries are not closely correlated. In this respect, the currency risk could be hedged by diversifying the exporting geographically, if it was practical. On the other hand, if the volatility in the bilateral exchange rate arises mainly from the variation in the home currency, the currency risk is considered a systematic risk to the company and thus difficult to be hedged by diversifying the exporting counterparties. Shifting the production or even the headquarters overseas may eliminate this sort of risk effectively but it could be restricted by the availability of natural resources, especially for commodity producers. In such circumstances, it is worthwhile for the corporate to attribute more resources to the treasury department for developing effective financial risk management strategy, in which the exchange rate risk is hedged by financial instruments.

A comparison of the variation in the absolute value of various currencies is helpful in choosing which country the empirical discussion should be applied to. Companies located in a country with highly volatile currency generally face more significant systematic exchange rate exposures. A structured financial risk management approach developed in the current context would be more valuable to such an economy than the one with relatively stable currency.

5.3.1 Construction of currency reference rate

Any exchange rate always has to be a relative price, the price of one thing in terms of another. To extract an absolute rate for the NZ dollar, the US dollar or the UK pound from the bilateral exchange rates between them and possibly other currencies, the respective trade weighted index (TWI) is commonly used but this leads to two general difficulties. The first is that TWI refers only to one aspect of foreign exchange rate
transactions, generally those based on the current account of the balance of payments. But for many countries, the bulk of foreign exchange trading is done on the capital account side, and it is increasingly apparent that capital flow has an important effect on the economy, as well as trade flow. Capital flow weightings by country partners do not use identical trade weights as current account. The second difficulty is that TWI’s cannot be reconstructed into bilateral exchange rates, at least without arbitrage being possible, since different countries may have various trading partners and corresponding trading weights. The TWI’s are then not transactionally consistent with one another. This can mean that variations in countries are not comparable. To avoid this, one could set the required absolute country rates just as the bilateral rates against the US dollar leaving the absolute rate for the US dollar as unity and constant over time. The US dollar would then become the model of stability but this is far from true in practice given the volatility of the US currency.

No-arbitrage absolute rates, in the desired sense, can be constructed against a reference basket of world currencies. Each bilateral exchange rate can then be measured relative to this basket. All no-arbitrage reference rates can be constructed in this way. There is considerable freedom in the choice of reference bases, indeed the weights may be negative as well as positive, and there are useful analogies with portfolio analysis. The reference base construction needs to be resolved in an economically meaningful way, depending on the context and use.

A little structure is useful at this point but more explicit proofs are given in Appendix B. Let $S_{ij}$ be a bilateral exchange rate with currency $i$ as commodity currency and country $j$ as terms currency, so that 1 unit of country $i$ currency is worth $S_{ij}$ units of country $j$ currency. Write $s_{ij} = \log S_{ij}$ and let $S = ((s_{ij})) ; i, j = 1, \ldots, n$ be the matrix of bilateral log exchange rates. Ignoring bid-ask spreads, no-arbitrage will ensure the existence of a set of country-specific prices $\{A_i\}$, or currency reference rates, such that $R_{ij} = \frac{A_i}{A_j} ; s_{ij} = a_i - a_j$ , where the $a$’s are the logs. For $N$ currencies there are only $(N-1)$ independent reference rates, so there is one degree of freedom in

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11 Hovanov, Kolari and Sokolov (2004) also provide a currency value index that is independent of base currency choice. However, the index they developed is still originated in the goods market while ignoring other factors, such as capital flows.

12 In terms of the development that follows, suppose $a_1, a_2, a_3$ are a set of reference rates in a three-country world. The no-arbitrage matrix of bilateral rates will be of the form
choosing them.

A simple way of constructing a set of mutually consistent currency reference rates (CRR’s) is to take the bilateral rates with respect to a chosen reference currency, typically the US dollar. This is how the market avoids arbitrages in practice. If US is the country $n$, the $n$'th column of $S$ would be taken as the vector of CRR’s, namely $a = s_n$. This type of choice has the advantage that adding another country to the set will not disturb the existing CRR’s for existing countries. As already described, however, this process has the disadvantage of resulting in a constant absolute rate for the US dollar.

Alternatively, the currency reference rates could be chosen as any weighted combination of the columns of $S$ in the form of:

$$a^w = \sum_{j=1}^{n} w_j s_j; \sum_j w_j = 1.$$  \hspace{1cm} (5.8)

Choosing currency $n$ (e.g. the US dollar) as base would amount to setting $a = s_n$, which in turn is equivalent to setting $w = e_n$ the $n$th column of the identity matrix. A more general choice is

$$w = w_1 e_1 + w_2 e_2 + ... + w_n e_n.$$  \hspace{1cm} (5.9)

One is now replacing a single currency as the reference variable by a basket of world currencies with weights given by the vector $w$. This could be called a ‘reference basket’ or ‘reference basis’ for the system. The CRR $a_i^w$ for country $i$ is the bilateral rate for that currency with respect to the reference basket, viewed as though it was a currency in its own right.

In fact, all no-arbitrage CRR vectors can be constructed in this way, i.e. as some weighted average of the bilateral rates as in equation (5.8). There is no particular need to have all weights $w_i$ semi-positive – it is quite possible to think of a reference basis that is short in some currencies and long in others, rather like a portfolio, with which

$$S = \begin{bmatrix} 0 & a_1 - a_2 & a_1 - a_3 \\ 0 & 0 & a_2 - a_3 \\ 0 & 0 & 0 \end{bmatrix}, \text{ with the elements below the diagonal filled in from } S' = -S. \text{ But this matrix contains the same information as } \begin{bmatrix} 0 & b & c \\ 0 & c - b & 0 \end{bmatrix} \text{ with only two independent elements } b,c.$$

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it has some features in common\textsuperscript{13}. The matrix $S$ of log bilateral rates has to be skew-symmetric $w'Sw = 0$, from which it follows that $\sum_j a_j^w w_j = 0$. This looks rather like a balance of payments scenario where the commodities being bought or sold in quantities $w_j$ are the currencies at their respective prices $a_j$. In fact, the only requirement is that the bilateral rate of the reference base against itself has to be zero.

5.3.2 Centred rate

The problem then boils down to the best choice of the reference basket $w$ to fit the particular circumstances. A simple choice is to set $w_j = \frac{1}{N}$ for all $j$. The reference basket is a simple average of the world currencies, so that no one country is singled out for special weight. The resulting CRR vector is given by

$$a^0 = \frac{1}{n} S 1,$$

where $1$ denotes the unit vector (all elements =1). This is the simple average of the bilateral rates. The intuition is that taking an equally weighted basket creates a stable portfolio of currencies, so one might as well measure the variation of the individual currencies relative to a stable base. One can call the resulting CRR’s the ‘centred rates’. If $a$ is any other CRR vector, then

$$a_i = a_i^0 + \bar{a},$$

which means that any alternative CRR can always be represented as the centred CRR plus a common factor that adjusts for the mean.

The centred rates can be computed in a very simple way. Start with the bilateral rates against any numeraire such as the US dollar, and then correct them by subtracting the average:

$$a_i^0 = s_i - \bar{s}.$$  

Note that the absolute log exchange rate of the US dollar reference is $a_n^0 = -\bar{s}$ which

\textsuperscript{13} For instance, one could choose the elements of $w$ as proportional to the current account balances, collectively $x$, of the respective countries, measured in their own currencies. In an entire world it should be true that $a'x = 0$; countries that run a positive current account finance those with a negative one. But this is also the condition for a reference basket price against itself. For such a choice of $w$, the USD would be short in the reference portfolio or basket, and the JPY long.
is no longer always zero.

In addition to a simple average of world currency, Bowden and Zhu (2007b) suggest several alternative choices for the reference basket, each with its own meaning. The reference exchange rate can be constructed from the point of view of a particular country, biasing the reference basket to suit a country of primary interest. Furthermore, the currency reference rate can be easily generalised to forward rate or real exchange rate.

5.3.3 Wavelet decomposition: comparative results

The wavelet is again employed as the analytical tool to measure the variation in a variety of absolute exchange rates. Table 5.3 illustrates the wavelet decomposition results for 13 centred log exchange rates. The reference basket is equally weighted in all currencies included. The choice of currencies for the present study was influenced by the following considerations:

(a) A reasonably free exchange rate between Jan 1986 and Feb 2007. Over this period many central banks did try to smooth their currencies in some way. Japan is an example of a fairly tightly managed float, while New Zealand was perhaps the world’s most free float, at least up to 2005. Smoothing was allowed provided the motive was judged to be simple stabilisation and not currency fixing.

(b) Absence of any major structural shift that might have affected the currency, especially conversion during the sample period from fixed to floating. The one exception to this was Germany. It was desirable to include a post 1999 Euro zone currency, and the largest European economy was chosen, notwithstanding a potential impact from German reunification in the earlier part of the period.

(c) A reasonable geographical coverage. Chile is included as the most structurally stable South American currency, in spite of doubts about whether the Chilean peso is a truly floating currency. South Africa also makes the list. Countries from Scandinavia are limited to just Sweden and Norway, the latter being an oil currency and therefore different.

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14 The Chilean central bank has been using a reference rate against a basket of currencies but this is adjusted from time to time and the band limits at any time are also fairly generous.
In the following table, the exchange rate used is the log of the end-month mid rate. The data is from Global Trade Information Service via Thomson Financial Datastream.

Table 5.3: Energy table for nominal centred reference currency rate

<table>
<thead>
<tr>
<th></th>
<th>A7</th>
<th>D7</th>
<th>D6</th>
<th>D5</th>
<th>D4</th>
<th>D3</th>
<th>D2</th>
<th>D1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average period (Years)</td>
<td>15.4700</td>
<td>15.4667</td>
<td>7.7333</td>
<td>3.8667</td>
<td>1.9333</td>
<td>0.9667</td>
<td>0.4833</td>
<td>0.2417</td>
</tr>
<tr>
<td>New Zealand</td>
<td>1.3509</td>
<td>0.0766</td>
<td>0.8275</td>
<td>0.1118</td>
<td>0.1151</td>
<td>0.0354</td>
<td>0.0372</td>
<td>0.0236</td>
</tr>
<tr>
<td>Canada</td>
<td>0.5497</td>
<td>0.1227</td>
<td>0.4309</td>
<td>0.1316</td>
<td>0.0814</td>
<td>0.0385</td>
<td>0.0203</td>
<td>0.0146</td>
</tr>
<tr>
<td>Chile</td>
<td>22.2248</td>
<td>2.0604</td>
<td>0.2727</td>
<td>0.4093</td>
<td>0.1383</td>
<td>0.1077</td>
<td>0.0334</td>
<td>0.0289</td>
</tr>
<tr>
<td>Germany</td>
<td>1.8846</td>
<td>0.2849</td>
<td>0.0657</td>
<td>0.2121</td>
<td>0.0966</td>
<td>0.0313</td>
<td>0.0139</td>
<td>0.0106</td>
</tr>
<tr>
<td>Japan</td>
<td>6.3069</td>
<td>0.2009</td>
<td>1.0678</td>
<td>0.3736</td>
<td>0.1311</td>
<td>0.1181</td>
<td>0.0300</td>
<td>0.0317</td>
</tr>
<tr>
<td>Australia</td>
<td>0.2914</td>
<td>0.0368</td>
<td>0.1480</td>
<td>0.1918</td>
<td>0.1106</td>
<td>0.0422</td>
<td>0.0398</td>
<td>0.0188</td>
</tr>
<tr>
<td>South Africa</td>
<td>39.0692</td>
<td>0.1480</td>
<td>0.4729</td>
<td>0.6553</td>
<td>0.1947</td>
<td>0.1392</td>
<td>0.0843</td>
<td>0.0526</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.2929</td>
<td>0.0257</td>
<td>0.2922</td>
<td>0.2162</td>
<td>0.1221</td>
<td>0.0405</td>
<td>0.0193</td>
<td>0.0131</td>
</tr>
<tr>
<td>Switzerland</td>
<td>2.9432</td>
<td>0.0623</td>
<td>0.1643</td>
<td>0.2675</td>
<td>0.1459</td>
<td>0.0480</td>
<td>0.0230</td>
<td>0.0167</td>
</tr>
<tr>
<td>UK</td>
<td>1.4976</td>
<td>0.1943</td>
<td>0.4730</td>
<td>0.0702</td>
<td>0.0692</td>
<td>0.0341</td>
<td>0.0165</td>
<td>0.0147</td>
</tr>
<tr>
<td>Singapore</td>
<td>5.1380</td>
<td>0.1224</td>
<td>0.2200</td>
<td>0.1720</td>
<td>0.0205</td>
<td>0.0232</td>
<td>0.0161</td>
<td>0.0066</td>
</tr>
<tr>
<td>Norway</td>
<td>0.4001</td>
<td>0.0563</td>
<td>0.0542</td>
<td>0.1391</td>
<td>0.0645</td>
<td>0.0250</td>
<td>0.0165</td>
<td>0.0142</td>
</tr>
<tr>
<td>US</td>
<td>0.8801</td>
<td>0.7652</td>
<td>0.5993</td>
<td>0.2773</td>
<td>0.0534</td>
<td>0.0413</td>
<td>0.0261</td>
<td>0.0114</td>
</tr>
</tbody>
</table>

For many countries, the bulk of the power (energy) is contained in the quasi trend approximation A7, simply because log exchange rates are non-stationary, or otherwise may be related to problematic inflation rates. It is no surprise to find economies with hyper-inflation like Chile and South Africa having currencies with a significant trend. On the other hand it is notable that Japan also shows a clearly visible albeit less pronounced trend in the tabulated results.

Although cyclical variation generally has lower power, cycles (in the sense of generalised wavelets) are fairly substantial for some currencies. Two outstanding countries are Japan and New Zealand both having sharp peaks at the D6 level that is the 7-8 year band. The UK and the US are also variable, while South Africa has a lot of variation in the shorter D5-D3 bands between 1-4 years.

Table 5.4 below summarises in terms of the sum of the detail energy over all cyclical bands. Based on total cyclical variation, Chile was the most unstable currency, followed by Japan. The most stable currencies were Singapore and Norway. Despite the fact that over this period some Australian corporations have been experiencing major issues related to the currency risk, the Australian dollar stands out as a comparatively stable currency in terms of both cyclical and total energy. It has
much lower energy than does its trans-Tasman neighbour New Zealand in the long cycles and only moderate variation in the shorter bands as well. This result came as a surprise, given that the Australian dollar is usually viewed as a commodity currency. The Canadian dollar is appreciably more stable than its close neighbour the US dollar, though the US dollar does have a stable band at D4 which is two years or so.

Table 5.4: Total detail energies

<table>
<thead>
<tr>
<th>Currency</th>
<th>Nominal centred CRR</th>
<th>Total energy</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>New Zealand</td>
<td>1.3121</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Canada</td>
<td>0.7224</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Chile</td>
<td>2.8490</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>0.7914</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>1.8508</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Australia</td>
<td>0.6759</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>South Africa</td>
<td>1.8114</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Sweden</td>
<td>0.8622</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.7942</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.8670</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Singapore</td>
<td>0.4421</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Norway</td>
<td>0.4052</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>United States</td>
<td>1.3880</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Among explanations offered for such high volatility in the NZ exchange rate, Bowden (2006c) points out that a narrowly based monetary policy employed recently by the Reserve Bank of New Zealand (RBNZ), in the latter part of the period studied, contributed significantly to the current variation. To obtain a stable financial and monetary system, monetary policy in New Zealand primarily aimed to keep the inflation rate in a target band, currently between 1% and 3%. Since 1999, instead of a money supply based policy, the Official Cash Rate (OCR) has been adopted by the RBNZ as the principal instrument for implementing the monetary policy. The use of OCR adjustments to conduct monetary policy gives the market and investors a signal for future market interest rate as well as currency changes. As Bowden (2006c) explains, exchange rate movement is one channel that monetary policy works through. In a country with a freely floating currency, such as New Zealand, where international capital is able to flow into or out of the country without barriers, higher interest rate induces a more expensive home currency. In the face of a high consumer
price index, the Reserve Bank could be expected to adopt a tight monetary policy indicating a rise in interest rates.

Global hot money or international hedge funds flow into New Zealand in pursuit for higher interest rates when a rising inflation rate threatens to break the target, as indicated earlier. The funds are waiting for a double win in both the short term interest rate market and the currency market. The large capital inflow consequently drives up the exchange rate. On the other hand, when a loose monetary policy is expected by the market, hedge funds might dump the New Zealand dollars. The corresponding effect on the currency could be a significant depreciation in the New Zealand dollar. The frequent international flows amplify the volatility of the New Zealand currency.

5.4 Conclusions

A statistical inspection of the market efficiency can provide some indications about whether managers could enhance a firm’s wealth by incorporating into hedging decisions their private information about the exposures. In a market where the unbiased forward hypothesis holds, some simple hedging rules, such as those always employed in a complete hedging, could out-perform any other sophisticated strategies. This chapter has examined the efficiency in the exchange rate movements with a linear regression as well as wavelet analysis. Both tests suggest rejection of an efficiency hypothesis.

With a multi-dimensional decomposition of currency exposures, wavelet analysis additionally allows corporate managers to distinguish the risk with respect to short interval cyclical movements from that caused by long term business cycle related fluctuations. In other words, wavelet analysis assists in the decomposition of global as well as localised aspects of the underlying time series. Wavelet analysis has been applied to both nominal quotable exchange rates and synthetic absolute exchange rates. The absolute exchange rate enables the company to compare the currency fluctuation in one country with another country. The finding that New Zealand has one of the most volatile currencies in the world implies that corporate as well as financial investors in New Zealand are facing significant exchange rate risk. It motivates the later application to concentrate on currency risk management for New Zealand trading companies and strategic fund managers. The econometric method that was introduced in this chapter, and in particular the use of wavelet analysis, will be
broadly employed as a tool in Chapter 7 for measuring corporate exposures and in Chapter 9 for designing an optimal long term investment portfolio.

The wavelet decomposition of the NZ dollar against the US dollar also exhibits apparent intermediate cyclical pattern, which may imply the potential impacts of economic fundamental variables on the currency movements. These findings motivate the development of exchange rate forecasting models. The subsequent chapter will accordingly focus on developing a directional exchange rate forecasting model over a one-year term and a mean-GARCH model for much shorter interval prediction. The proposed directional forecasting model differs from a traditional categorical model by accounting for the non-linearity of the relationships as well as the incompleteness of the market information. Such models are constructed in an aim to improve the hedging effectiveness with superior knowledge about the future currency movements.
Chapter 6 Exchange Rate Econometrics II: Exchange Rate Forecasting

In a market where the information has not been fully reflected in the current price, corporate managers may improve the hedging performance of the corporate by actively using such information to estimate the future movements of the applicable variables. However, when available information is limited and incomplete, as is commonly found in practice, managers are unlikely to have any precise structural prediction model. In such circumstances, assigning directions rather than actual values is a natural response. For instance, business cycle commentators are usually willing to call only the basic direction of movement in the face of insufficient information. Directional forecasting is concerned with calls as to whether a given series will move up, stay the same, or move down. This sort of forecasting is preferably by means of assigning numerical probabilities to each possible outcome.

The conventional categorical model, such as Probit or Tobit models may not be suitable in a market where information is imprecise or incomplete. Variables observed from such a market could sometimes incorporate conflicting information, with some variables indicating up while others signifying down state. In addition, the choice for explanatory variables in determining dependent variables can change from one period to another, especially when there is a structural break in the variable movements. Some upward movement indicators may not exhibit significant information for downward changes. A successful directional forecasting model should allow for some filtering process to convert these coarse indicators to meaningful information for predicting purposes. To account for such imperfection in the market, this research develops a non-homogeneous multinomial directional forecasting model that includes neural nets and fuzzy membership function.

The proposed categorical directional forecasting model explores the potential causal relationship between the exchange rate and other fundamental economic indicators. However, the macroeconomic variables usually take time to have an effect and their power of prediction could be lower over a shorter period, as will be discussed later. In other words, the deterministic structure of financial variables could vary according to the estimation period. As a result, the model for a short period prediction could differ from that for estimation over a longer period. Moreover, lots of
empirical evidence shows volatility clustering at short intervals. Monthly exchange rate forecasting will thus rely on the traditional M-GARCH model.

The chapter is organized as follows: Section 6.1 introduces the motivation for developing this directional exchange rate forecasting model. Section 6.2 explores the methods of defining the categorical outcome. Section 6.3 defines the probability regarding the directional output. Section 6.4 extends the model to account for dynamic persistence. Section 6.5 looks at operational issues including regime delineation, model identification and estimation procedures. Section 6.6 describes some forecasting results. Section 6.7 turns to a shorter term context, exploring forecast methods based upon GARCH model. Section 6.8 Concludes.15

6.1 Motivation: Limited information and directional forecasting

As outlined above, directional forecasting is often a response to limited but still useful information. One problem with limited information is that it may be partial. For instance, economic variables used for forecasting can be classified as up indicators or down indicators, with the up indicator applicable when the economy appears to be emerging from a recession, and the down indicator applying when the economy appears to be slowing down. Although the up indicator can have some limited relevance in detecting a slow-down, the focus of attention has shifted. In fact, under these conditions they are only partial indicators. They can conflict, so that an ‘up’ signal could potentially appear at the same time as a ‘down’ indicator. Decision theorists would say that such signals lack 100% validity. The observer has to choose which seems the most probable based on judgements, such as the relative strengths of the signals, or by some simple randomisation device. The lack of a unique functional relationship mapping between the signal space and the outcome space over a longer time period is what makes the information incomplete. The word ‘incomplete’ can also be used with respect to missing data in the estimation phase (see below).

Given limited information, directional calls are often more successful than precise value prediction over a relatively longer term. As Engel (1994) found the regime switching model does not outperform the random walk model but it is superior in forecasting the direction of exchange rate movements. Other researchers such as Levich (2001), Christofferson and Diebold (2004), Pesaran and Timmermann (2004),

15 This chapter is largely based on Bowden, Zhu and Cho (2007).
Leung, Daouk and Chen (2000) also find evidence of success in directional forecasting, irrespective of whether that value forecasting is inefficient.

### 6.2 Defining the output zones

The initial objective in directional forecasting and risk management is to attach probabilities to categorical outcomes. The outcome space can be either two or three dimensional. In the former, the categories are simply ‘up’ or ‘down’. In practice, however, it is useful to have a middle category, which can be described as ‘stable’, ‘no change’, or ‘same’. Given that most economic or financial times series exhibit noise or normal volatility, one would be reluctant to accept smaller movements as a truly up or down trend. Technical analysis of markets uses a similar idea in the form of a ‘break out’ zone. Both two and three dimensional outcome spaces are considered below.

Forecasting is based on a series of economic signals and particular attention is usually focused on signals that might indicate a significant up or down movement. There are several reasons for this. One is that busy managers are more likely to focus on ‘ground breaking’ information, or real news. Another is that economic models tend to be more convincing in describing significant changes in state. Most commentators would accept that an unexpected announcement of a record current account deficit is likely to lead to a down movement in the home exchange rate. Another instance is that a bullish housing market in a small economy is likely to lead to an up movement as capital is transferred in from abroad to fund mortgages. In general, the informational content of economic signals is greatest in describing up or down movements. This suggests that the natural way to proceed is to adopt a two dimensional signal space and a two or three dimensional output space. The signals can be taken as sufficient statistics for a larger number of more elementary economic indicator variables, in a manner to be described. However, the signals themselves can be observed only with noise, so they are latent variables.

It is helpful to begin by imagining that there are two signals $I_1$ and $I_2$, each observable. One can be called the up signal and the other the down signal. However, these are only indicative signals. There is signalling noise involved so that it is quite possible for an up signal to be followed by a down outcome. The two categories of signals can also sometimes indicate conflicting outcomes, with a significant up signal
accompanied with a notably down signal. These possibilities should be taken into account to ensure the signals have validity in directional forecasting.

The signals each have two states, namely ‘on’ and ‘off’. In figure 6.1 these are marked with (+) and (0) signs. Technically this would mean that in general there are four possible signal configurations which can be mapped into either a three outcomes model or a two outcome model. Consider first the three outcome model. With two signals there are four possible combinations. It seems clear enough that if the up signal is on and the down signal is off, the outcome should be an up state, and vice versa. If both signals are off, then one would be confident in assigning the ‘no change’ outcome. Suppose however, that both the up and down signals are on. There is no automatic or obvious way to resolve the conflict (marked with crossed cavalry swords). In principle, the outcome could be any one of the three possible outcomes $R_i$ as marked, though this is not to say that the strength of the linkages need to be the same. In this sense, the model is incomplete. Although figure 6.1 has similar appearance as the trinomial mean-reverting process for illustrating the evolution of rates or prices (e.g. Hull & White, 1994), it is of the nature of a logic diagram. It could be reworked as a neuronal net: the observable economic variables ($Z$’s) feed forward to the combinations, with ongoing links (some of zero strength) to the $R$’s, and hence to the outcomes.

Figure 6.1 Three and Two Outcome Incomplete Models
In terms of the two outcome model, one does not observe the ‘no change’ state. For instance, with continuous observations one might argue that ‘no change’ represents an outcome of precisely zero change, and therefore has zero probability. However, it is useful to keep the three original outcomes, but to squash them down to two. The original three could now be referred to as regimes, and in the absence of direct observation, they are latent. Denote them as $R_1$ (up), $R_2$ (down) and $R_3$ (no change). Collectively they form a hidden feed forward layer, in the language of neuronal nets. Their incompleteness arises from the fact that given $R_3$, there is no automatic way to assign the final up or down outcome.

Figures 6.2a, b show by way of contrast how complete models would look in each case. The two-outcome model would be complete only if the two signals were mutually exclusive, so that an up state in one automatically means a down state in the other.

![Figure 6.2a Complete Three Outcome Model](image_url)
Whether it is best to specify a two or three outcome model is a matter of context and purpose. Economic variables, especially financial ones, are intrinsically noisy or volatile, so that one might be unwilling to identify smaller movements with up or down changes. Instead there would be a range such that if the movement exceeded this, it would be labelled as either an up or a down change. One could think of it in terms of accepting or rejecting the null hypothesis of no change and the regime boundaries as the critical points. This would indicate a three regime model, and require either using the data to determine the critical points or defining the regimes in terms of fuzzy membership functions. Operational procedures are considered further below.

### 6.3 Introducing probabilities

The resulting model can be regarded as a sequence of multinomial trials (3 outcomes) or binomial (2 outcomes), one trial for each time period. At each time period the probabilities of outcomes change in accordance with economic conditions, so that these are non-homogeneous multinomial models. The probabilities themselves have to be derived by mapping the indicators into probability densities. This can be done by using standard distribution models such as the logistic or normal probit, supplemented with rule-based judgment calls where the signals can conflict. There are useful commonalities with the neuronal (neural) network literature, which involves a similar compression process (e.g. McCulloch & Pitts, 1943; Rosenblatt, 1957, 1958; or the reviews by Kuan & White, 1991; Turban, 1995; Tkacz & Hu, 1999). Most of the models can be cast as neuronal nets, with several layers. The three outcomes become

![Figure 6.2b Complete Two Outcome Model](image-url)
collectively a hidden feed forward layer if the primary objective is a two outcome model, so the parallel with neuronal nets becomes even closer. However, a greater degree of prior structure is imposed than would be normal with the neuronal net models, with the purpose of contributing data economy within a macroeconomic context.

In this respect, the basic task is to determine the marginal or conditional probabilities attached to the three regimes. Even for the two outcome model, the hidden layer regime membership variables $R_i \ (i = 1, 2, 3)$ are regarded as sufficient statistics, so that for any observable exogenous indicator variables $Z$, and outcomes $u$ (up) or $d$ (down), $P[u|R, Z] = P[u|R_i] P[R_i|Z]$, similarly for $d$. Hence, the essential task in either model is to make probability statements about the regime membership variables $R_i$.

To assist in this task it is assumed that two sets of observable economic variables are available, denoted $Z_u$ and $Z_d$, sometimes collectively as $Z$. The two sets are oriented respectively towards up and down outcomes, e.g. a higher value of a $Z_d$ variable would suggest a higher likelihood of a down movement. They can have elements in common, provided that model consistency or identification is preserved. For example, a given economic series could appear both as an up indicator variable and with a negative sign, as a down indicator variable.

Where directional movements are concerned, as suggested earlier, it is common to focus on variables that are adapted towards significant movement, rather than those that might specifically indicate a stable or no change outcome. The latter is more naturally thought of as being associated with the absence of indications as to strong upward or downward pressures. Hence, one problem is to explain how just two sets of observable indicator variables can explain three outcomes in a natural manner. Note also that the orientation of any indicator variable is by no means perfect. It implies wrong indicator signals can be given on occasion. For many purposes, it is convenient to think in terms of linear combinations $\beta_u Z_u$ and $\beta_d Z_d$ of these variables as producing a closer link to the respective regimes or observable outcomes.

There are just two index functions, denoted $I_1$ and $I_2$ for the indicator signals. These will depend upon their respective indicator variables $Z_u$ and $Z_d$ but these will often be suppressed for simplicity. Each has two alternative symbolic values $I_1^+$ and $I_1^-$; the ‘+’ indicating that the signal is switched on, and the ‘0’ indicating that it is
switched off. The first task is to determine the probabilities \( P[R_i|Z] \) to be attached to the three regimes.

Probit-style switching models provide a useful illustration. Let \( \varepsilon_u \) and \( \varepsilon_d \) be two random variables with mean zero and unit variance, and further let \( W_1(Z) \) and \( W_2(Z) \) be \( P[I_1 = I_1^+ | Z] \) and \( P[I_2 = I_2^+ | Z] \) respectively. If it is assumed that \( \varepsilon_u \) and \( \varepsilon_d \) were independently normal, \( W_1(Z) \) and \( W_2(Z) \) might be specified as:

\[
W_1(Z) = P[\varepsilon_u \leq \beta_u^i Z_u | Z] = \Phi(\beta_u^i Z_u);
W_2(Z) = P[\varepsilon_d \leq \beta_d^i Z_d | Z] = \Phi(\beta_d^i Z_d)
\]  

(6.1)

where \( \Phi(\bullet) \) denotes the standard normal distribution function. Hereafter, \( W_1(Z) \) and \( W_2(Z) \) is abbreviated as \( W_1 \) and \( W_2 \) for simplicity.

Probabilities such as \( W_1 \) and \( W_2 \) will sometimes be denoted the ‘raw scores’ in what follows. Alternatively the logistic distribution might have been chosen, used in many studies of categorical model. A user who prefers this option could let \( W_1 \) be

\[
\frac{1}{1 + \exp(-\beta_u^i Z_u)}.
\]

(6.2)

Similarly for \( W_2 \). Under the assumption that \( \varepsilon_u \) and \( \varepsilon_d \) are uncorrelated, there are four possible combinations of signals:

\[
(I_1^+, I_2^0) \text{ unequivocally indicating up, with probability } W_1(1-W_2);
\]

\[
(I_1^+, I_2^+) \text{ unequivocally indicating down, with probability } W_2(1-W_1);
\]

\[
(I_1^0, I_2^+) \text{ conflict zone both indicating up and down, with probability } W_1 W_2; \quad (6.3)
\]

\[
(I_1^0, I_2^0) \text{ agree on ‘no change’ outcome, with probability } (1-W_1)(1-W_2).
\]

More generally, if \( \varepsilon_u \) and \( \varepsilon_d \) are correlated,

\[
P[(I_1^+, I_2^0) | Z] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} n(\varepsilon_u, \varepsilon_d) d\varepsilon_u d\varepsilon_d,
\]

where \( n(\varepsilon_u, \varepsilon_d) \) denotes the joint density with the required correlation, \( \rho \) say. Similar expressions hold for the other three combinations.

### Three regime probabilities

The four probabilities associated with combinations have to be compressed to the three regime probabilities \( P[R_i|Z] \), \( i = 1, 2, 3 \). It will be apparent from figure 6.1 that,
for instance, $P[R_i|Z]$ is at least $W_i(1-W_2)$, but it could be more, as mass from point C still has to be distributed. Thus it could be written as:

$$P[R_i | Z] = W_i(1-W_2) + f_i(W_i, W_2)W_1W_2,$$

where $f_i(W_i, W_2)$ is a value between zero and unity indicating the fraction of the unresolved probability mass $W_iW_2$ to be redirected towards regime 1. There are corresponding statements for $P[R_2|Z]$ and $P[R_3|Z]$. The redistributive semi-positive fractions $f_i$ have to add up to unity and should be symmetric, i.e. $f_2(W_1, W_2) = f_1(W_2, W_1)$.

A conditional probability framework is a useful way of generating redistribution fractions with the required properties. Suppose the third combination in condition (6.3) holds, i.e. conflict. The fractions $f_i(W_i, W_2)$ could be interpreted as the conditional probability $P[R_i | (I_1^+, I_2^+), Z]$ of $R_i$, given signal conflict. A convenient specification is

$$f_1(W_1, W_2) = P[R_1 | (I_1^+, I_2^+), Z] = W_1 / (1 + W_2),$$

$$f_2(W_1, W_2) = P[R_2 | (I_1^+, I_2^+), Z] = W_2 / (1 + W_1),$$

$$f_3(W_1, W_2) = P[R_3 | (I_1^+, I_2^+), Z] = 1 - W_1 / (1 + W_2) - W_2 / (1 + W_1).$$

It is easy to show that $0 \leq f_i(W_1, W_2) \leq 1$ and $\sum f_i = 1$. If the raw score $W_i$ for the up indicators is greater than that for the down indicators, it is more likely that any conflict would be resolved in favour of up. If both $W_1$ and $W_2$ tend to unity, the first two conditional probabilities (6.4) tend to 1/2, indicating that one or other of the strong opinions must be correct. If $W_1$ and $W_2$ are both 1/2, then the three conditional probabilities have value of 1/3 each, so the mass in the conflict zone is spread equally.

Expressions in the equation (6.4) are not the only possible way to divide the conflict mass; for instance, it could have been chosen that $P[R_1 | (I_1^+, I_2^+), Z] = W_1 / (1 + 2W_1W_2)$ and still achieved conditional probabilities all lying between zero and unity. The latter would say that when $W_1$ and $W_2$ are both equal to unity, then the mass is distributed 1/3 each across the three regimes.

The conditional probability framework captures the common sense of credible judgments: if signals conflict, one or both of them must be wrong and one has to weigh up which, in the light of all available evidence. However, it does introduce a non-Markovian element to the logic, for the indicator variables $Z$ can now reach
forward to influence signal credibility as well as signal probabilities. Combining (6.3) and (6.4) it can be obtained that:

\[
P[R_1 | Z] = W_1[1 + W_2(W_1 - W_2)]/(1 + W_2) \\
P[R_2 | Z] = W_2[1 + W_1(W_2 - W_1)]/(1 + W_1) \\
P[R_3 | Z] = 1 - P[R_1 | Z] - P[R_2 | Z].
\] (6.5)

Squashes like equation (6.5) obey the elementary symmetry axiom if the raw scores \( W_1 \) and \( W_2 \) are equal, then the up and down probabilities are likewise equal. As noted earlier, this is by no means the only way of generating the multinomial probabilities required. However it is a simple way to do so, and can readily be adapted for other distributions thus the logistic specification could replace the Probit specification in the above.

It may be remarked that the role of the economic indicator variables \( Z_u \) and \( Z_d \) is to assist in discriminating between up/down/stable directions. One might think of extending their informational role into values as well as directions, in a manner parallel to Tobit models, with a statement like

\[
E[Y | Z] = (\beta_u Z_u)P[R_1 | Z] + (\beta_d Z_d)P[R_2 | Z],
\]

assigning the value zero to the expected value given \( R_3 \). There are two problems with this. First, the coverage of \( Z_u \) and \( Z_d \) may not extend as far as forecasting actual values since they are simply direction indicators. Second, the probability structure is incomplete, so that the above expectation expression lacks a sound foundation in probability theory. Expressions such as these will not be used in what follows.

**Two Outcomes probabilities**

A further squashing to a two outcome model, if desired, can again use conditional probabilities. The ‘stable’ outcome can be divided into half each to the up and down outcomes: \( P[u | R_3] = P[d | R_3] = \frac{1}{2} \). Also set \( P[u | R_1] = 1 \) and \( P[d | R_1] = 0 \), similarly for the \( R_2 \) conditionals. Then

\[
P[u | Z] = \frac{1}{2} + \frac{1}{2}(P[R_1 | Z] - P[R_2 | Z]) \\
P[d | Z] = \frac{1}{2} + \frac{1}{2}(P[R_2 | Z] - P[R_1 | Z]).
\] (6.6)

One could allow for the mapping between regimes and outcomes to be less precise, for example by writing:

\[
P[u | Z] = P[u | R_1] \times P[R_1 | Z] + P[u | R_2] \times P[R_2 | Z] + P[u | R_3] \times P[R_3 | Z] \\
= 1 \times P[R_1 | Z] + 0 \times P[R_2 | Z] + \frac{1}{2} \times (1 - P[R_1 | Z] - P[R_2 | Z]) \\
= \frac{1}{2} + \frac{1}{2}(P[R_1 | Z] - P[R_2 | Z])
\]
The first expression in (6.6) is replaced by

\[ P[u \mid Z] = \frac{1}{2} + (\pi - \frac{1}{2})(P[R_1 \mid Z] - P[R_2 \mid Z]), \]

with a similar second expression. The effect of \( \pi < 1 \) is to allow the connections between regimes and outputs to be noisy.

Condensing from the three outcomes to the two implies that more or less stable and minor fluctuations are in effect treated the same as significant movement. In a two outcome model, a lot of market noise has to be allocated in one way or the other to the influence of the up or down economic indicators, diluting the credibility of the latter. However, there may be other contexts where the economics would always indicate either a clear up or a clear down outcome, and in this case the two outcome model may be useful. The empirics reported in the following section covers only the three outcome model.

### 6.4 Dynamic persistence models

Dynamic elements can appear in the above models via lagged values of dependent variables. If the objective is a directional forecast of exchange rates over the coming period, the last period’s actual value could appear among the explanatory variables \( Z \).

Alternatively the dynamics can be more intrinsic to the model, and govern the way that regimes themselves evolve. Recalling that these regimes describe directional changes, it could be that changes are persistent from one period to the next. Thus if the current direction is up, it is more likely that next period it will also be up. In such a persistence model, signal-output conditionals such as \( P[u \mid R_i] \) and \( P[d \mid R_i] \) remain constant, but the regimes themselves obey transition probabilities, of the form

\[ P[R_{ij} \mid R_{j-1}, Z_j] = P[R_i = R_j \mid R_{i-1} = R_j, Z], \quad (6.7) \]

where the symbol \( R_t \) is used to describe a regime-valued random process. The state probabilities evolve according to

\[ P[R_{ij} \mid Z^{(t)}] = \sum_{j=1}^{3} P[R_{ij} \mid R_{j-1}, Z_j] P[R_{j-1} \mid Z^{(t-1)}], \quad (6.8) \]

starting from \( P[R_{i,0} \mid Z_0] \) as initial marginal distributions, where \( i = 1, 2, 3; \ t = 1, 2, \ldots; \) and \( Z^{(t)} \) represent the history of the \( Z \) process up to time \( t \). In some applications
the transition probabilities might depend upon ‘surprises’ in the $Z_t$, i.e. the part that could not have been predicted from $Z^{(t-1)}$.

Equation (6.7) defines a Markov process, or a hidden Markov process (Baum & Petrie, 1966) where the regimes are obscured. If the economy is now in a given state $R$, then it should most likely just stay there unless an economic event occurs. But if the exchange rate (for example) is currently in a stable ‘no change’ state and a very bad current account figure is announced, it should change to a down state with a greater probability. Thus the transition probabilities have to be made functions of the economic indicators $Z$, the latter now being reinterpreted as influences that will produce changes in state. The resulting Markov processes become non-homogeneous.

At first sight, estimation of the equation (6.7) presents a formidable task, as there are now 9 transitional probability densities to specify and estimate. Note, however, that there are only 6 independent probabilities, as the columns of the transition matrix must sum to one. Further, the key entries are ‘turning points’ ($P[R_{i,j} | R_{j,i-1}, Z_t], P[R_{2,j} | R_{i,j-1}, Z_t]$) and perhaps also ‘breakout points’ ($P[R_{i,j} | R_{3,j-1}, Z_t], P[R_{2,j} | R_{j,j-1}, Z_t]$). This suggests that it might be possible to get away with just 2-4 non-homogeneous probability densities leaving the others constant, just as they are for standard homogeneous Markov models.

Alternatively, the regimes above could be reinterpreted as referring to levels rather than rates of change. Suppose the object to be forecasted was known to stay within a stable band over time. One could distinguish three levels: high, middle, and low. A transition probability such as $P[R_{i,j} | R_{j,i-1}, Z_t](i \neq j)$, would now be interpreted as the probability of a transition from level $j$ to level $i$.

**6.5 Operational matters**

This section looks at some issues of regime delineation, model identification, and estimation procedures. In what follows, the forecasting objective concerns the change in an economic state variable of interest. Past values of $Y$ can be observed but not precisely modelled. Instead the task is to estimate a categorical model governing the categorical regimes of change $R_i$ into which the values of $Y$ fall.

**6.5.1 Regime delineation**

A connection between the expectation-maximisation (EM) algorithm and the literature on fuzzy membership functions (Zadeh, 1965) is useful to define the regime.
As indicated earlier, there is often value in adopting three outcomes, not just two, with a middle zone reserved for the lack of any significant up or down tendencies. However, for continuous variables, absent precisely zero outcomes (as with most continuous variants), the operational problem is then how to define the ‘no change’ or ‘stable’ outcome. The boundaries may be treated as either known or unknown, and the data densities that arise shall be considered in each case. In the unknown case, the boundaries separating categories can be regarded as fuzzy, and the parameters of the membership function estimated from the historical data. A probabilistic interpretation can be given in terms of the regime boundaries as hidden variables. One replaces the unobservable regime membership functions with their expected value, given the available information, in a fashion similar to the EM algorithm (Hartley, 1958; Dempster, Laird & Rubin, 1977). This establishes a link between the use of fuzzy regimes and likelihood methodology with incomplete data, potentially useful in other forms of categorical data analysis.

**Known boundaries**

This approach assumes that the investigator has established preassigned boundaries based on such considerations as a normal level of volatility, or what would constitute a breakout zone. With no real loss of generality, for a known number $\delta$, the regimes are manifested by:

- **Regime 1** $Y > \delta$
- **Regime 2** $\delta \leq Y \leq -\delta$
- **Regime 3** $-\delta < Y \leq \delta$.

The index functions associated with these sets will be denoted as $r_i(Y, \delta)$. For example, $r_1(Y, \delta) = 1 \Rightarrow Y > \delta$; $= 0$, otherwise. In the above, the observable $Y$ plays the role of an observable signalling variable, just as it does in Probit analysis.

The likelihood element associated with an observation $Y$ is a straightforward generalisation to three regimes of the standard probit model. It can be expressed in compact form as:

$$l_R(Y, Z; \delta; \beta) = \sum_{i=1}^{3} r_i(Y, \delta) P(R_i \mid Z, \beta). \quad (6.9)$$

Although it contains $Y$ as an observable argument, the likelihood equation (6.9) refers to regime-valued events, emphasized by means of the subscript $R$. The effect is to single out the regime $R_i$ that is actually observed. Thus if regime $i = 1$ is observed
then \( r_1(Y, \delta) = 1 \), while \( r_2(Y, \delta) = r_3(Y, \delta) = 0 \). The probability \( P(R_i | Z, \beta) \) can be ended up with the data density element that applies for this particular period. The classic Probit likelihood function is precisely of this form. It has just two regimes with probabilities \( p(R_1 | Z, \beta) \) and \( p(R_2 | Z, \beta) = 1 - p(R_1 | Z, \beta) \) and the likelihood function over \( t = 1, 2, ...T \) is divided into those periods where regime 1 and regime 2 apply. In both models, equation (6.9) could be described as a likelihood element generator function.

**Unknown boundaries**

Another approach is to model the regime boundaries and membership functions as fuzzy in nature. The original idea (Zadeh, 1965; Zadeh & Bellman, 1970) was that the investigator might have some idea of where the boundaries might be, but less so as any more precise placement. One can assign values between zero and unity to the degree of membership of each regime, so that a given observation \( Y \) is not allocated absolutely to just one or the other.

An alternative interpretation in the present context can be constructed from more classical probability ideas. Here the boundary markers in any given period are sharp, but they can vary across different time periods, simply because some periods are naturally more volatile than others. The boundaries for any single period cannot be observed, but it may be possible to model and estimate their probability distribution. The latter then effectively defines the fuzzy membership functions. Put this way, it becomes clear that the regime boundaries are in the nature of hidden variables, or incomplete data. The EM method potentially applies to the subsequent maximum likelihood solution for the model as a whole. What follows describes this approach.

To link with the known boundary case, suppose that the firm boundary marker parameter \( \delta \) is replaced with a random variable \( \xi \), and it can be assumed that \( \xi \sim N(\delta, \sigma^2) \). It is also assumed that \( \xi \) is statistically independent of \( Y \) and \( Z \); the boundary markers are simply noise parameters. The regime index functions now become the form \( r_i(Y, \xi) \), where the parameters \( \delta, \sigma \) are suppressed for brevity. For instance,

\[
r_i(Y, \xi) = 1 \iff Y > \xi,
\]

so \( R = R_i \) in this case and regime 1 applies. Also,

\[
E_{\xi}[r_i(Y, \xi)] = \int_{-\infty}^{Y} n(\xi; \delta, \sigma^2) d\xi = \Phi(Y, \delta, \sigma^2),
\]

(6.10)
where \( n(\xi; \delta, \sigma^2) \) denotes the normal density function and \( \Phi(Y; \delta, \sigma^2) \) the corresponding distribution function.

Similarly, take expectations of the other two index functions \( E_\xi[r_2(Y, \xi)] \) and \( E_\xi[r_3(Y, \xi)] \), and let \( \varphi_i(Y; \delta, \sigma) \) denote the respective results. This can result in

\[
\begin{align*}
\varphi_1(Y; \delta, \sigma) &= \Phi(Y; \delta, \sigma^2) \\
\varphi_2(Y; \delta, \sigma) &= 1 - \Phi(Y; -\delta, \sigma^2) \\
\varphi_3(Y; \delta, \sigma) &= 1 - \varphi_1(Y; \delta, \sigma) - \varphi_2(Y; \delta, \sigma).
\end{align*}
\]  

(6.11)

Figure 6.3 below plots these functions, which can be called the fuzzy membership functions. They are mutually symmetric, centred at zero, though other forms could be devised that are not necessarily symmetric, e.g. because the upper and lower boundary markers are not symmetric about zero. Note that for all \( Y, \varphi_i(Y) \geq 0 \), and \( \sum_{i=1}^{3} \varphi_i = 1 \).

There is a width or location parameter \( \delta \) and a sharpness parameter \( \sigma \), which could potentially be estimated along with the substantive model parameters \( \beta \).

The boundary markers \((-\xi, \xi)\) cannot be observed, so that observations \( Y \) cannot be allocated with certainty into one or other of the regimes \( R \). However, a fuzzy likelihood element can be obtained by replacing the exact membership function \( r_i(Y, \delta) \) in equation (6.9) by the membership functions \( \varphi_i(Y; \delta, \sigma) \) to obtain:

\[
I_R(Y, Z; \beta, \delta, \sigma) = \sum_{i=1}^{3} \varphi_i(Y; \delta, \sigma) P(R_i | Z, \beta).
\]  

(6.12)
Comparing with equation (6.9) for the known boundary case, it is easy to demonstrate that
\[ l_r(Y, Z; \beta, \delta, \sigma) = E_\xi \{ l_r(Y, Z, \xi; \beta) \}. \]
The effect of equation (6.12) is to give probability mass to all regimes rather than just one, to a degree depending on the strength of membership of observation \( Y \) in the respective regimes. It can be viewed as the marginal probability of regime-observation combinations, once the unobservable boundary uncertainty \( \xi \) has been integrated out. Operationally, the device of replacing the unobservable index variables \( r_i(Y, \xi) \) by their expected values \( \varphi_i(Y; \delta, \sigma) \) corresponds to the EM methodology, wherein hidden or incomplete data are replaced by their expectations, conditional upon what data is available. The quasi-likelihood equation (6.12) will apply no matter whether the boundary parameters \( \delta, \sigma \) have been specified in advance.

**Regime Differentiation**

It may be of interest to test whether the regimes are differentiated in terms of their connections, causal or otherwise, with the different economic indicators. Suppose, for instance, that there was really no need to differentiate between the regimes, e.g. because a simple linear model was operative between the \( Z \) variables and the output variable to be forecasted. Within the framework of model (6.1) above, this would correspond to a nesting where \( \beta_d = -\beta_u \) and a correlation between \( \epsilon_u \) and \( \epsilon_d \) of -1. The model becomes automatically complete and there are only two regimes possible, up or down. Alternatively, setting \( \beta_d = -\beta_u \) but continuing to assume \( \epsilon_u \) and \( \epsilon_d \) uncorrelated would drive an uncertainty wedge between two basically identical regimes, allowing for a stable or no-change band in between, a ‘threshold for change’ type of effect. At a minimum, parameter commonality can be tested by first consolidating the \( Z \)’s to a common set and then testing \( \beta_d = -\beta_u \), or equivalently setting \( Z_d = -Z_d \) and testing \( \beta_d = \beta_u \). Likelihood ratio tests can be used on all or some of the parameter pairs.

**6.5.2 Estimation procedures**

The estimation procedures described in this section are based on maximum likelihood (ML). As previously noted, if the boundary markers are not precisely known, it has more of the character of the EM variant of ML. In addition, it may be necessary to use overlapping data, especially for macroeconomic work. Extensions such as quasi
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maximum likelihood (QML) can be encompassed and comment below on implications.

For the static model, the quasi-likelihood function is simply the product of the one period elements as given by equation (6.12):

$$L_R((Y, Z)^{(T)}; \beta, \delta, \sigma) = \prod_{t=1}^{T} \tilde{l}_R(Y_t, Z_t; \beta, \delta, \sigma),$$  \hspace{1cm} (6.13)

where \((Y, Z)^{(T)}\) denotes the history of observations from \(t = 1, 2, \ldots T\). Equation (6.13) refers to the more general case where neither of the boundary parameters \(\delta, \sigma\) are known; otherwise these parameters can be suppressed.

In this work, the regime ‘smearing’ parameter \(\sigma\) will be preset as a known number, say \(\overline{\sigma}\), while the position parameter \(\delta\) will be estimated along with the \(\beta\) parameters. The probability elements \(P[R_i | Z_t; \beta]\) will be specified by the normal distribution function as in equation (6.1) and (6.5). The parameter estimates are collectively given by:

$$\hat{\theta} = \arg \max_{\beta, \delta} \{-T^{-1} \sum_{t=1}^{T} \ln \tilde{l}_R(Y_t, Z_t; \beta, \delta, \overline{\sigma})\},$$ \hspace{1cm} (6.14)

where \(\hat{\theta}\) stands for \((\hat{\beta}', \hat{\delta}')\). For brevity in what follows, the equation (6.14) can be written as \(T^{-1} \sum_{t=1}^{T} \ln \tilde{l}_R(Y_t, Z_t; \beta, \delta, \overline{\sigma}) = \bar{L}_R(\theta; \overline{\sigma})\).

As earlier mentioned, the quasi-likelihood function is not necessarily the likelihood function corresponding to the true underlying data generation process. First, the conditional regime likelihood function, given \(Z_t\), can be different from \(\tilde{l}_R(Y_t, Z_t; \beta, \delta, \overline{\sigma})\). If so, the information matrix inequality does not hold. Secondly, the given QML function ignores any dynamic persistence, and possibly leads to inefficient QML estimator (QMLE) and an incorrect information matrix or efficiency bounds. On the other hand, it can be valid in this respect if all serial correlation can be captured among the pre-determined explanatory variables, \(Z_t\). Thirdly, the fuzziness in regime boundaries implies an EM type formulation.

Nevertheless, the given likelihood function is flexible in the sense that it is specified by focusing on available economic information, and some of the technical difficulties arising from model incompleteness can be resolved. Under suitable conditions, it can be claimed that \(\hat{\theta}\) as given by equation (6.14) has the almost sure limit \(\theta^* = \arg \max_{\theta} E[\bar{L}_R(\theta; \overline{\sigma})]\) (see assumption I, Appendix C). In such a
formulation, the quasi-likelihood function and convergence limit $\theta^*$ do not necessarily conform with exactly how the data is generated. The limit $\theta^*$ can be defined as the parameter explaining how closely the data would be generated according to the hypothesized model in terms of likelihood. Specifying a quasi-likelihood function gives rise to a different asymptotic distribution from standard maximum likelihood estimators. It can still be claimed that the QMLE obeys asymptotic normality. Under Assumptions I and II in the Appendix C, it follows that:

$$\sqrt{T} (\hat{\theta}_T - \theta^*) \overset{d}{\sim} N(0, (-A^*)^{-1} B^* (-A^*)^{-1}),$$  \hspace{1cm} (6.15)

where $A^* := E[\nabla^2_{\theta} \overline{L}_T (\theta^*; \sigma)]$, $B^* := \text{avar}(\sqrt{T} \overline{L}_T (\theta^*; \sigma))$ and ‘avar’ stands for the asymptotic variance of the given argument. Note that $(-A^*)$ is not necessarily the same as $B^*$, which should hold if the likelihood function is correctly specified. The metrics $A^*$ can be estimated by applying the strong uniform law of large numbers to $T^{-1} \sum_{t=1}^{T} [\nabla^2_{\theta} \ln(l_{R_t}(\hat{\theta}_T))]$ and the convergence in probability of $\hat{\theta}_T$ to $\theta^*$. Estimation of $B^*$ is based on the Newey and West’s (1987) method. 

Experience thus far is that identification or maximisation difficulties can arise in the case where regime boundaries have to be estimated. For instance, if the smearing parameter $\sigma$ becomes large, the regimes become indistinguishable, the more so if the $\beta$ parameters are poorly identified. This is why a preset $\sigma = \overline{\sigma}$ has been used in the empirical work reported below. Likewise, the parameter $\delta$ must be restricted to be positive and if allowed to become too large, all the data will be attributed to just the one regime, namely the stable zone. Operationally, one looks to the existence of an interior maximum within an interval that will attribute mass from all three regimes to the sample observations.

### 6.6 Directional exchange rate forecasting results

The selected application of the intermediate term directional exchange rate forecasting is to the exchange rate of NZD/USD. Table 6.1 summarises the variables used in what follows for the currency forecasting and their timing conventions.
### Table 6.1 Economic variables used for yearly forecasting

<table>
<thead>
<tr>
<th>Variables</th>
<th>Definition and Timing Conventions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exchange rate</td>
<td>(\ln(\text{NZD/USD})_{t+4} - \ln(\text{NZD/USD})_t)</td>
</tr>
<tr>
<td>Current account balance as % of GDP</td>
<td>((\text{CAB/GDP})_{t-8})</td>
</tr>
<tr>
<td>House price annual change index</td>
<td>((\text{HPI}<em>{t} - \text{HPI}</em>{t-4})/\text{HPI}_{t-4})</td>
</tr>
<tr>
<td>Exchange rate as smoothed level</td>
<td>(\frac{1}{7} \sum_{k=0}^{7} \ln(\text{NZD/USD})_{t-k})</td>
</tr>
<tr>
<td>Relative share price index NZ/US</td>
<td>(\ln(\text{SPI}<em>{\text{NZ}}/\text{SPI}</em>{\text{US}}) - \ln(\text{SPI}<em>{\text{NZ}}/\text{SPI}</em>{\text{US}})_{t-4})</td>
</tr>
<tr>
<td>Relative GDP ratio index NZ/US</td>
<td>(\ln(\text{GDP}<em>{\text{NZ}}/\text{GDP}</em>{\text{US}}) - \ln(\text{GDP}<em>{\text{NZ}}/\text{GDP}</em>{\text{US}})_{t-4})</td>
</tr>
</tbody>
</table>

Based on preliminary testing, the forward rate has not been included in the table as one of the dependent variables. The efficient capital markets hypothesis would say that the only informative forecast, based on publicly available data, is the forward rate if the risk premium for the exchange rate is assumed to be zero. The poor forecasting performance of the forward rate is well known in the foreign exchange literature (e.g. Hodrick, 1987 for a review; Froot & Thaler, 1990; Lewis, 1995; Engel, 1996; Meredith & Chinn, 1998; Wang & Jones, 2002; Bhar et al., 2001). The low predictive power of the NZD/USD forward rate over an intermediate run such as one year is also found in the current context.

On the other hand, Bowden (2004) has noted a close connection between the NZD exchange rate and the housing market, in the first instance created by the inflow of offshore funding for mortgages (the ‘hoovering effect’), with a further influence on the balance of payments capital account being provided by the relative fortunes of the NZ and US stock markets and business cycle. Adjustments in the exchange rate tend to follow major movements in these variables, and occur at times when the exchange rate is already at historic highs or lows, suggesting error correction elements in the level of the exchange rate. Sharp corrections are also seen as more likely when the balance of payments is seen as weak, e.g. as the build up of large current account imbalances. Unlike the US dollar, the NZ dollar is not seen as a major reserve currency. Taken together, the above suggests that exchange rate forecasting can be episodically successful, based on unusual movements or exposures, and provided one’s ambitions are limited in the first instance to picking the direction of the movement, rather than necessarily the new equilibrium value.

Some exchange rate forecasting models including Meese and Rogoff (1983a, b), use concurrent information in the process of forecasting. This is not really ex-ante forecasting as realised explanatory variables at \(t+1\) have been employed to forecast
exchange rate at time $t+1$. The current method is consistent with Evans and Lyons (2005) that relies on an ex-ante forecasting. By estimating exchange rates at time $t+1$ using the predictor variables at time $t$, this thesis discusses exchange rate prediction in the true sense.

The endogenous variable is the NZD/USD exchange rate log changes, as shown in table 6.1. The regime model is taken as static, meaning that changes in the exchange rate are assumed not to have persistence, apart from that derived from the dependent variables ($Z$). To obtain regime probabilities, the normal version of the model was used. Three fuzzy outcomes were employed, corresponding to up, down and no movements of the exchange rate. Allowing both the position $\delta$ and spread $\sigma$ as free fuzzy parameters led to indications that the spread parameter was poorly identified relative to the beta parameters. Based on a priori assessments as to regions of doubt for the boundaries, the fuzzy spread parameter $\sigma$ was preset as 0.001, leaving the mean boundary delimiter $\delta$ to be estimated as the empirical regime marker of primary interest.

The estimation method was a quasi-maximum likelihood based on equation (6.13) using the computational routine ‘fmincon’ from Matlab 7.3.0. The data is quarterly, with 72 observations from Q3 1989 to Q2, 2007. The data for the exchange rate comes from the global trade information services. The house price index is from Reserve Bank of New Zealand. Both the NZ and US share price index are collected from MSCI. The data source for NZ gross domestic product (in current dollars) and NZ current account balance is Statistics NZ. The indicator of US GDP, also in current dollars is collected from International Monetary Fund. Official GDP and house price data is available only quarterly. In order to focus on annual changes, use of overlapping data became necessary, though not in the one-quarter based forecasting. Potential inefficiencies or biases due to the use of overlapping data have not been explored. Year on year changes are often used in the exogenous variables to improve the signal to noise ratio.

Criteria for inclusion or exclusion of indicator variables were based primarily on asymptotic $t$ ratios and likelihood ratio tests from the QML estimation phase, and secondarily on contributions to the consequential hedging performance. Table 6.2 gives the estimated beta values and their asymptotic $t$ ratios for the up and down indicators as listed and defined in Table 6.1. Three significant up indicators are retained, namely house prices, relative share prices and GDP ratios. Two down
indicators were retained. The smoothed level has a primary impact when high, indicating the riskiness of exposure to the NZD at higher levels. The current account balance is important for similar reasons.

### Table 6.2 Directional Indicators and Parameter Estimates Annual\(^{17}\)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Role</th>
<th>Estimates</th>
<th>t ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>House price annual change index</td>
<td>Up</td>
<td>4.0537</td>
<td>1.7008*</td>
</tr>
<tr>
<td>Relative share price index NZ/US</td>
<td>Up</td>
<td>7.3907</td>
<td>3.5167***</td>
</tr>
<tr>
<td>Relative GDP ratio index NZ/US</td>
<td>Up</td>
<td>21.931</td>
<td>1.6709*</td>
</tr>
<tr>
<td>Exchange rate as smoothed level</td>
<td>Down</td>
<td>1.5791</td>
<td>2.4438**</td>
</tr>
<tr>
<td>Current account balance as % GDP</td>
<td>Down</td>
<td>-17.7958</td>
<td>-2.3706**</td>
</tr>
<tr>
<td>Fuzzy boundary ((\delta))</td>
<td></td>
<td>0.0556</td>
<td>65.3327***</td>
</tr>
</tbody>
</table>

In addition, a likelihood ratio (LR) test is applied to test whether all the indicator coefficients (\(\beta\)) are zero, so that they provide no predictive information. Under the null hypothesis, it follows from expressions (6.1) and (6.5) that:

\[
P[R_u, Z; \beta] = P[R_{2t}, Z; \beta] = P[R_{3t}, Z; \beta] = 1/3.
\]

The resulting LR statistic decisively rejects the null hypothesis that they are zero. Finally, the regimes are indeed quite different. Consolidating the \(Z\)'s as suggested in the previous sections, followed by \(\beta_d = -\beta_u\), was rejected at 5% significance level, with all beta coefficients significantly different. This excludes alternative models such as simple linearity.

The directional model has also been examined in a short term context, e.g. just one quarter ahead. It was found that the computed \(t\) ratios are not significant as in the annual data case. Evidently, short run noise is obscuring the effect of the economic variables to a greater extent than with the longer forecasting horizon.

Both the in-sample and out-of-sample effectiveness of the forecasting will be examined in the following chapter with the hedging performance as the welfare criteria.

### 6.7 Short term forecasting using GARCH model

Preceding discussions have shown the proposed directional forecasting model is not effective over a short interval. In general, it is difficult to forecast a short term currency return with a structured economic model, as Cheung and Chinn (1999) found that economic fundamentals are more important at longer horizons. In a very short

\(^{17}\) LR statistic testing all zero coefficients =50.306. * 90% significant, ** 95% significant, ***99% significant.
interval, such as weekly or monthly, the correlation between expectation of asset return and the explanatory variables can be insignificant and the fluctuations in asset prices are mainly affected by random effects.

In addition, there is empirical evidence to show that many financial time series have periods of high volatility that can be followed by a time of low volatility, known as volatility clustering, in particularly for short run changes. Given this characteristic, the ARCH model and its generalizations – GARCH model have been commonly employed for deriving short term exchange rate estimation (Baillie 1991, Yang & Allen 2005). The GARCH model, which was originally introduced by Engle (1982), accounts for the conditional heteroskedastic process in volatility. Although the generalised ARCH model can also take into account the time dependence in mean, these models have been mainly developed for comprising the dependence structure in volatility.

In the current context, a GARCH-in-Mean model is established for predicting the monthly conditional mean and variance:

\[
Y_t = X_t'\theta + \lambda \sigma_t^2 + \varepsilon_t;
\]

\[
\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2;
\]

\[
e_t \sim N(0, \sigma_t^2),
\]

where \(Y_t\) is a dependent variable and matrix \(X_t\) represents the predictor variables, which include both exogenous variables or the lagged value of \(Y_t\). The conditional variance is also included in the mean equation (Engle, Lilien & Robins, 1987).

Model specification and estimation results
A Mean-GARCH model to be developed is for forecasting the short term USD/NZD changes. The monthly spot and forward exchange rates span from June 1985 to June 2007. The series for one month interest rates are taken as the US CD rate, the 30 day bank bill rate for NZ. Exchange rate data is from MSCI and interest rate data is sourced from the central bank in each country. All data has been collected via Thomson Financial Datastream.

Symbol \(S_t\) indicates the spot exchange rate at time \(t\) and \(\tilde{F}_t\) represents the actual one month forward rate that is determined at time \(t\). One month interest rate is denoted with \(r\). The dependent variable is specified as the one month exchange rate return. The choice of independent variables has several features of interest. The first is the use of

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the spot-forward differential or basis as an explanatory variable in the mean part. The spot-forward differential describing the excess returns to currencies was referred to as the currency risk premium in Chapter 5. In addition, cointegration between spot and forward rates potentially introduces predictability, appearing in the form of an error correction element as represented by the one-month spot-forward basis or differential, which is described in Chapter 5 as the currency risk premium. For empirical evidence see Baillie and Bollerslev (1989), Ngama (1992), Evans and Lewis (1995), and Luintel and Paudyal (1998). In addition, following Wu and Zhang (1996), positive and negative values of the spot forward basis differential were entered separately in the mean equation, representing asymmetry in their effect. A second feature is that the exchange rates usually show longer run mean-reverting tendencies at a constant level or a trend. Hence, the level of the current exchange rate also appears as a longer term error correcting element, though the effects may not be significant in a short run forecasting.

The number of lags, namely the value of $p$ and $q$ for model GARCH($p,q$) are determined according to the Akaike information criteria. The parameters are estimated with Eviews 5. The model to be fitted reads as follows:

$$
R_{t} = c + \alpha_{s} R_{s,t-1} + \alpha_{s_2} R_{s,t-2} + \alpha_{s_3} R_{s,t-3} + \alpha_{s_4} R_{s,t-4} \\
+ \beta_{s_1} B_{s_1,t-1} + \beta_{s_2} B_{s_1,t-2} + \beta_{s_3} B_{s_1,t-3} + \beta_{s_4} B_{s_1,t-4} + \gamma R_{t-1} + \epsilon_{t}
$$

$$
h_{t} = \alpha_{h_1} h_{t-1}^{2} + \alpha_{h_2} h_{t-2} + \alpha_{h_3} h_{t-3} + \alpha_{h_4} h_{t-4}, \hspace{1cm} u_{t} = \sqrt{h_{t}} \nu_{t}.
$$

where:

$$
s_{t} = \log(S_{t}); \hspace{0.5cm} R_{s,t} = s_{t} - s_{t-1}; \hspace{0.5cm} \tilde{f}_{t} = \log(\tilde{F}_{t});\hspace{0.5cm} B_{s,t} = \left[ s_{t} - \tilde{f}_{t-1} \right]; \hspace{0.5cm} B_{-} = \left[ s_{t} - \tilde{f}_{t-1} \right];\hspace{0.5cm} rd = r_{n0} - r_{nl}; \hspace{0.5cm} sl = \log(S_{t-1}).
$$

Tables 6.3a and 6.3b contain the fitted parameters and significance levels. Table 6.3c includes the relevant statistic measurements. Some detailed diagnostic test results tables can be seen in Appendix F. All results are from Eviews 5.

**Table 6.3a: Mean equation parameters USD/NZD**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>0.4965***</td>
<td>0.1289</td>
<td>3.8516</td>
<td>0.0001</td>
</tr>
<tr>
<td>$C_s$</td>
<td>0.0045</td>
<td>0.0043</td>
<td>1.0497</td>
<td>0.2939</td>
</tr>
</tbody>
</table>
The mean equation estimation outcomes show that the interest rate differential in two countries is a significant predictor variable for forecasting the exchange rate over a short term. The parameter has a negative value, indicating that a high interest rate in NZ is followed by an appreciation in NZD. The results reject the hypothesis that forward rate is an unbiased predictor for future spot rate, as discussed in Chapter 5. The negative parameter estimation for the previous spot rate level implies a mean-reversion statement in the currency value movement. The spot-forward basis is indicative of future spot rate but it is limited to a positive spot-forward basis only. The coefficient estimation regarding the spot-forward basis implies that a negative excess return in holding the NZD in the spot market relative to the NZD forward may
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indicate appreciation pressure on the NZD. On the other hand, a positive excess return in holding the NZD may not indicate depreciation in the NZD. The variance estimation outcomes in the table 6.3b show significant evidence of volatility clustering in the exchange rate changes. Some model diagnostic tests (see Appendix F) show there is no residual serial correlation and no GARCH in the error term. The effectiveness of these forecasting on hedging outcomes will be tested empirically, in particular, within Chapter 8.

6.8 Conclusions

The current chapter has developed a categorical forecasting model for an incomplete market. In such a market, information might be useful, but usually limited. Variables directly observed from the market could signify contradictory outputs, with some showing upward movement while others indicating downward movement. To filter out the valuable signals from the coarse information, the developed forecasting model incorporates methods analogous to neural nets as well as fuzzy sets. Four possible combinations of two pairs of raw indicators (up and down indicators) can be compressed by the model into three outcomes (up, down and stable) or two outcomes (up and down). Fuzzy membership functions facilitate the classification of three outcomes regarding the currency movement, up, down and no change. The consequent categorical directional forecasting framework employs implicitly a binomial or trinomial step process to derive non-homogeneous multinomial directional probabilities over longer time intervals. In such a categorical model, house prices, relative share prices and GDP ratios, the current exchange rate level, and the current account balance has been found as significant explanatory variable. Turning to the short term context, the estimation outcomes based on GRACH model show significant variation persistence across different time periods.

The exchange rate forecasting framework developed in the current context is mainly tailored for the hedging purpose. Categorical forecasting on future currency movement is worthwhile in the hedging context as knowing the direction of currency movement enables corporate managers to determine whether to hedge or not. For instance, an exporter expecting a weaker home currency tends to remain unhedged, or at least partly unhedged against its exchange rate exposure. Information regarding volatility in the short term will also assist corporate managers in determining the optimal hedging ratio. The value increased by hedging could be exaggerated when the
market becomes more volatile. The effectiveness of these forecasting will be assessed against the benchmark of hedging outcomes in Chapter 8. A forecasting is valuable only if the conditional hedging that relies on this anticipation does improve the company’s hedging performance.

The following chapter commences the empirical implementation of the hedging decision rule by introducing operational matters with respect to corporate hedging, including exposure measurement, hedging horizon, hedging instruments as well as a simple hedging rule. To capture the most sensitive exposures faced by a corporate, an indicator called corporate terms of trade is developed in the form of a net profit margin. By decomposing the composite exposure indicators with the aid of wavelet analysis, the next chapter reveals the cyclical pattern, the origin and the causal influences of risk profiles encountered by New Zealand dairy farmers. The wavelet decomposition outcomes provide valuable information in determining an appropriate corporate risk management framework. In addition to the discussions on corporate exposures, the following chapter also introduces a smoothing hedging strategy which staggers the cover over a variety of forward maturities in order to smooth the exposure.
Part IV Empirical Hedging Algorithms for Corporate

Chapter 7 Operational Aspects of Corporate Hedging

The empirical application of the developed decision theory and exchange rate forecasting model is implemented within a New Zealand context. It commences with the analysis of some operational features regarding a structured hedging framework. The first task is to establish an indicator for measuring corporate exposures. An appropriate exposure indicator should account for the most critical exposures faced by the corporate. Total cash flows can fulfil this purpose but this measure is infeasible in practice due to difficulties of acquiring the necessary data. The current chapter instead develops a composite index to represent the main exposures encountered by corporates. Based on this developed exposures indicator, the variation of the exposure will be examined with the aid of wavelets. The wavelet decomposition outcomes such as the multi-scale pattern of exposures assist corporate managers in determining the length of a hedging horizon. A good understanding of variation and origination of exposures further aids corporate managers in determining what key exposures to hedge and when to hedge.

This chapter also introduces a smooth hedging strategy. In such a strategy, a series of forwards instead of a single forward is employed in managing currency risk. The smoothing hedging strategy contributes to the reduction of the volatility on the hedged portfolio. Another operational aspect of the corporate hedging discussed in the current chapter is the choice of hedging instrument. The advantages and disadvantages of using forwards and options in currency risk management will be explored in a variety of circumstances.

This chapter is organized as follows: Section 7.1 describes methods of constructing the appropriate corporate exposure indicator. Section 7.2 describes the decomposition of corporate exposure using wavelet analysis. Section 7.3 discusses the choice for the hedging horizon. Section 7.4 introduces the smoothing hedging strategy. Section 7.5 discusses the advantages and disadvantages in using forwards and options for corporate currency risk management. Section 7.6 concludes.18

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18 Section 7.1 and 7.4 are based on Bowden and Zhu (2006a). Section 7.2 is mainly based on Bowden and Zhu (2007a).
7.1 Measurement of corporate exposures

A good understanding of the exposures faced by firms assists corporate managers in establishing an effective risk management framework. To examine the corporate exposures, an initial task is to establish an appropriate welfare indicator which is suitably adapted to the statistical metrics and techniques employed for analysing the given exposure. For simplicity, most hedging literature (e.g. Harris & Shen, 2003; Myers & Thompson, 1989) has been based on the assumption that the corporate faces only one risk, which is the exchange rate in the current context. However, a complete risk management strategy may have to deal with multi risks. It would thus be useful to find ways to condense these into a single exposure index, for which the welfare function discussed earlier is still applicable.

For corporates, an indicator to measure all the future expected payoffs would entail total free cash flows. Operationally, this is a formidable problem for a number of reasons. Firstly, data may be inadequate for measuring all the cash flows. Secondly, there may be the issue of normalising the variables to express them on a common basis, e.g. avoiding the problem of hedging stationary with non-stationary time series data. For instance, it is expected that profits or accumulated cash flows grow every year and could be non-stationary. However, a hedging instrument, such as the forward exchange rate, might fluctuate around a historical average and appears more stable than the total profits.

The present discussion adopts a key indicator approach that isolates the most critical exposures in the form of the corporate terms of trade, which effectively constructs a price of output or income relative to the price of inputs. Adjusting this for productivity gives a single-valued measure. Take the exporter as an example. A schematic decomposition is as follows:

$$\text{profitability index} = \frac{\text{output price} \times \text{exchange rate} \times \text{output quantity}}{\text{expenses price} \times \text{input quantity}}$$

$$= \left[\frac{\text{output price} \times \text{exchange rate}}{\text{expenses price}}\right] \times \left[\frac{\text{output quantity}}{\text{input quantity}}\right]$$

$$\sim \text{terms of trade} \times \text{productivity.}$$

The above profitability index (PI) can be regarded as the ratio of operating revenue to costs. A more standard accounting ratio is the net profit margin (NPM), which is the ratio of revenue net of costs to total revenue. The relationship between the two is
The net profit margin is a widely used measure of managerial performance. It is a possible target for bonus fixing in the case of large scale farming operations, which rely on a professional farm manager.

For the profitability index, productivity is treated as exogenous in what follows, though this should not serve to minimise the longer term necessity to develop productive capacity as a defensive response to adverse price variations. This leaves the objective variable for hedging as the corporate terms of trade and only price risk is being hedged. Representing the denominator with the consumer price index, the terms of trade actually accounts for the real income variation. The terms of trade can also be regarded as an indicator for the real exchange rate:

\[
\text{terms of trade} = \frac{\text{output price}}{\text{expenses price}} \times \text{exchange rate},
\]

where the exchange rate is adjusted with the ratio of foreign and domestic consumer price indices. In the form of real income or the real exchange rate, the corporate terms of trade might appear to be stationary or approximately stationary, if the Purchasing Power Parity holds to some extent. The stationary nature makes it possible to apply the conventional decision theory to determine the optimal hedging strategy. A further advantage is that using the log terms of trade allows a log linear separation of the variables. Operationally, therefore, the hedge object will be taken as:

\[
X = \log \left[ \frac{\text{output price}}{\text{expenses price}} \times \text{exchange rate} \right]. \quad (7.1)
\]

Specification of the terms of trade as the ratio of output and input price index enables the indicator to reflect net exposures, which encompasses the potential effects of companies’ abilities to pass on risk to others in some cases. For instance, a monopoly exporter in one country could pass on negative effects to suppliers by means of decreasing the supplier price. Also, an importer, such as an oil importing company, may transfer rising costs due to a weak home currency on to its customers through raising the sale price. Since the composite index, denoted in the form of the terms of trade, comprises variations in both exchange rate and output or input price, the index is able to capture the corporate’s net exposures to financial market risk.
7.2 Wavelet decomposition of corporate exposures

A decomposition of the corporate exposure with wavelets reveals what comprises the natural exposure and how that exposure fluctuates over time. This sort of information assists corporate managers in understanding cyclical behaviour of companies’ exposures, structural changes in the cycles, and what components mainly contribute to changes. As will be discussed later, such understanding enables corporate managers to identify the key exposures and determine how and when to hedge.

7.2.1 Selected application: New Zealand farmer terms of trade

The empirical application is specifically selected for New Zealand farmers. One reason for the selection is that the high percentage of foreign sales exposes New Zealand farmers to a significant currency risk. From a practical point of view for the industry itself, understanding the currency risk it is exposed to may assist farmers and producer organizations to appreciate the sources of exposure, the relative importance for variation in farmer incomes or cash flow, and what can be done to manage the risk. Moreover, the agriculture industry plays an important role in the New Zealand economy as a lot of export income is generated from offshore sales of commodities. Consequently, currency exposures of the agricultural sector in turn feed through into cyclical booms or busts of the economy as a whole. Hence, to understand how agriculture is impacted by the macroeconomic environment is a first step in understanding the business cycle as a whole.

In terms of components of exposures faced by New Zealand farmers, exchange rates are only part of the story, because economic outcomes such as commodity prices, input prices, or interest rates are sometimes of equal impact on farmer incomes. The New Zealand dairy industry is heavily exposed to the external economic environment. The bulk of output is sold offshore on freely traded commodity markets (powder, butter, cheese, casein, and other products). Thus it inherits a heavy exposure to both world commodity prices as well as the NZ dollar, with an additional exposure to costs at home (such as labour, interest rates and animal health costs). All this adds up to a substantial risk management problem for New Zealand farmers, along with an ongoing preoccupation with long term viability trends.

Therefore, for the purpose of examining the trend and cyclicity, the chosen context, namely the New Zealand dairy farmer terms of trade, provides an excellent case study. There are obvious cycles in the New Zealand dairy industry. The cycles in
this context, however, are not of regular periodicity, or of constant amplitude, nor are there invariant phase differences among components. As a result, one cannot use spectral analysis or ARIMA-type models to capture the cyclicity. Our solution is to employ wavelet analysis, which can encompass cycles that change through time, as described in Chapter 5. When the original time series is decomposed along the frequency and time trend, long cycles or short run data pattern can be observed separately. The method can be employed to examine the cyclical behaviour of time series over different time scales.

The interest in such wavelet decompositions is more than just historical, though even this aspect has its own use as a graphic reminder to policy-makers and prospective participants that agriculture is a risky business. A better understanding of what causes the cycle will help farmers to manage the risk, either in terms of their own personal stabilization policies, or else knowing just what key exposures to hedge.

The wavelet function ‘coif5’ is employed to examine the exposures faced by New Zealand dairy farmers. The wave form ‘sym10’ is also used to check the findings. Given the similarity between the results derived from the two functions, discussions in the rest of this chapter are mainly based on the wavelet function ‘coif5’. A simple discrete wavelet transform as well as a maximal overlap discrete wavelet transform is employed to construct the wavelet variance or correlation. It is found that the former leads to the almost same results as the latter. The main findings provided in the following text are on the basis of a discrete wavelet transform.

The application is to monthly data, spanning from Jan 1986 to Feb 2007. The mid spot exchange rate is sourced from Thomson Financial Datastream and the dairy price data is obtained from ANZ Bank data series. The quarterly farmer expense price index provided by Statistics NZ is interpolated linearly to derive the monthly index.

7.2.2 General findings

Figure 7.1 depicts the history of the farmer terms of trade over the sample period.
The most obvious thing is that there is indeed cyclical behaviour, at least prior to 2004. New Zealand dairy farmers have had to live with and adapt to cycles of boom and bust, which at times have been severe enough to constitute a survival hazard. But it is not obvious that the cycle is regular. A spectral analysis does show a very slight peak at 3.5 years, but too minor to accept as statistically significant. Moreover, there are indications of some sort of change occurring around 2000. Wavelet analysis would be useful to deconstruct these cycles into their component elements.

Figure 7.2 is a wavelet decomposition of the log of farmer terms of trade (\( \text{ftt} \))\textsuperscript{19}. As earlier indicated, the approximation at level 7 is treated as the trend. As successively more detail is added vertically upwards, the approximations approach closer and closer to the original \( \text{ftt} \) series. In practice, the fitting moves vertically downwards (‘deconstruction’), removing more and more of the detail; a vertical upwards movement is referred to as ‘reconstruction’.

\textsuperscript{19} Farmer terms of trade is indicated by ‘FTT’.
Figure 7.2 Actual decomposition of $ftt$ up to level 7

The respective powers or energy of the details can effectively be gauged from the scales on the vertical axis. A more formal energy measure can be derived that is analogous to the variance of each detail level, expressed as a percentage of the overall energy. As table 7.1 shows, apart from the longer term trend component, most cyclical power in $ftt$ is allocated to the level 5 detail, which corresponds to an ‘average period’ of 3.87 years, but neighbouring levels 4 and 6 also attract some weight. It is also noted that the longer cycle (level 6 detail) does show some evidence of growing amplitude over time. The causes of this are explored in what follows.
### Table 7.1 ftt power decomposition

<table>
<thead>
<tr>
<th>Approx. &amp; Details</th>
<th>A7</th>
<th>D7</th>
<th>D6</th>
<th>D5</th>
<th>D4</th>
<th>D3</th>
<th>D2</th>
<th>D1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy %</td>
<td>20.01</td>
<td>5.28</td>
<td>25.30</td>
<td>31.54</td>
<td>12.70</td>
<td>3.34</td>
<td>1.01</td>
<td>0.82</td>
</tr>
<tr>
<td>Corresponding periods (years)</td>
<td>longer period</td>
<td>15.5</td>
<td>7.7</td>
<td>3.87</td>
<td>1.9</td>
<td>1</td>
<td>0.5</td>
<td>0.2</td>
</tr>
</tbody>
</table>

**7.2.3 Long term behaviour**

Figure 7.3 depicts the trend component (the level 7 approximation) for the ftt and also for its components, namely the commodity price, the exchange rate, and the input price or expense index. These are logs in each case. Note that the input price component has been illustrated with a *negative* sign; thus to get the ftt series simply add the component graphs vertically.

It can be observed that the marked negative trend in the ftt series in earlier years derives from steadily increasing expenses in the form of the input price series. Until the mid-nineties, Prebisch (1960) and Singer (1950) were evidently right (as to the Prebisch-Singer thesis): the terms of trade for these particular primary producers were indeed declining. Only secular growth of about 2.4% per annum in dairy farm productivity (Dexcel, 2004) kept the industry competitive. But in more recent years, stronger commodity prices have helped to increase the farm profitability.
7.2.4 Cyclical behaviour and its components

Once the trend has been removed, levels 4 to 6 provide most of the cyclic power. The respective levels can be added to give a composite level:

\[ D_{4-6} = D_4 + D_5 + D_6. \]

The underlying orthogonality of the wavelet decomposition means that the composite level 4-6 can be treated as a detail in its own right. Figure 7.4 depicts the composite level 4-6 detail for the \( ftt \) series and also for its constituent components, namely the commodity price, the exchange rate and the input price.

![Figure 7.4 Detail 4-6 for \( ftt \) and its components](image)

Several things are evident from figure 7.4. The first is that, at a cyclical level, input prices are relatively unimportant: almost all the action comes from commodity prices and the exchange rate. The second is that the two major peaks, 1995 and 2001, have quite different causes. The 1995 peak is due to commodity prices, which are dragged down by a high NZD. The 2001 peak is a major one because the two components no longer conflict: a very weak NZ dollar is reinforced by high commodity prices. A smaller peak in 1992 derives from the same effect.

7.2.5 Has there been a structural break?

Details at levels 1 and 2 can be used to examine the issue of structural breaks in the cycle. One sign of a break is that high resolution levels show a violent fluctuation around any break point. Intuitively, when the major cycle suddenly changes character,
too much burden is placed on the higher level (lower resolution) wavelets centred around this point. The poorness of local fit is then transferred through to appear as more violent short run fluctuation in the lower level details. In figure 7.5a and 7.5b, both details $D_1$ and $D_2$ show increased volatility around 2001-2002, suggesting a break.

Figure 7.5a Details at level 1

Figure 7.5b Details at level 2
One hypothesis is that prior to 2000, it was commodity prices that generated the fluctuation in the \( ftt \). This could happen either directly or indirectly via the lagged dependence of the NZD exchange rate on commodity prices, of which dairy prices are the largest by value. After that date, commodity prices are accompanied by an independent influence from the exchange rate.

To test this, the exchange rate and input prices are projected on the commodity price and the respective residuals, isolated as the components of each that could not be attributed to any causal influence from commodity prices. In order to concentrate on longer run cycles, this was done using the composite detail level 4-6 with overlapping semi-annual time intervals for each lag. The projection is done with least squares in the form of a one-sided distributed lag, in which the exchange rate or input price is potentially affected by current and prior commodity prices. Thus if \( L \) is the backward lag operator (e.g. \( Ly_t = y_{t-1} \)), one fits a lag structure of the form

\[
D_{4-6}^{\text{exch}} = \alpha(L)D_{4-6}^{\text{comm}} + e_{\text{exch}}^{\text{exch}}
\]

\[
D_{4-6}^{\text{input}} = \beta(L)D_{4-6}^{\text{comm}} + e_{\text{input}}^{\text{input}},
\]

where the lag structure is extended back for eight semi-annual periods. The residuals \( e_{\text{exch}}^{\text{exch}} \) and \( e_{\text{input}}^{\text{input}} \) represent the effects independent of the commodity price. Finally the constructed series is formed as:

\[
\tilde{ftt}_{4-6}^{C} = (1 + \alpha(L) - \beta(L))D_{4-6}^{\text{comm}}.
\]

The projection \( \tilde{ftt}_{4-6}^{C} \) can be taken to represent the sum total of commodity price causality, and the object is to see how well it succeeds in tracking the \( ftt \). Figure 7.6, which plots \( ftt \) against \( \tilde{ftt}_{4-6}^{C} \) is instructive. Up to 1997 the tracking is reasonably good. But after that date it becomes poor, suggesting a stronger independent influence for the exchange rate. Chow-type tests of coefficient stability for the distributed lags also indicate that the period pre-1997 was not the same as post-1997. However, such tests are no more than suggestive, as the underlying dependent variables \( D_{4-6}^{\text{exch}} \) and \( D_{4-6}^{\text{input}} \) themselves represent constructed data.
The above findings as to the greater independence of the exchange rate can be attributed to structural changes in the economy as a whole, which in turn reflect the increasing globalisation of world capital markets. The year 1997 marked the Asian crisis, which resulted in a sudden correction to an overvalued NZ exchange rate. From mid-1997, the NZ dollar lost a third of its value against the USD, with a minor recovery in 1999, but thereafter resuming a downward slide until late 2001. The economy as a whole fell into a mild recession, but commenced a recovery in 2000, supported by rising commodity prices and a NZ dollar at historically low levels and continuing to fall. This is why the farmer terms of trade rose so dramatically around 2001.

However, the recovery spread into the asset markets, most notably the housing market, and it came to be financed by large scale borrowing by banks through the Eurobond and Uridashi markets, either directly so, or indirectly via the swap market. After 2001 the trend accelerated and became a virtual offshore inflow of money to support home mortgages and consumer lending (Bowden, 2004). At the same time, the Reserve Bank of New Zealand, concerned at inflationary pressures, started a cycle of monetary tightening which also attracted short term ‘hot money’ from offshore. The result was a soaring NZ dollar at a time when the US dollar itself was coming under attack. The $f_i$ was under exchange rate pressure.
7.2.6 The natural buffer and its prognosis

Wavelet analysis can be used to break the component effects down to contributions from different periodicities. It becomes apparent in the table 7.2 that the commodity price effect has impact in two different bands, while the exchange rate has its primary impact in only one. Figure 7.7a displays the component details for level 5, with an average period of 3.87 years, while figure 7.7b shows what happens for level 6 detail, with an average period of 7.7 years.

Table 7.2 FTT component power decomposition

<table>
<thead>
<tr>
<th>Approx. &amp; Details</th>
<th>A7</th>
<th>D7</th>
<th>D6</th>
<th>D5</th>
<th>D4</th>
<th>D3</th>
<th>D2</th>
<th>D1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Commodity price</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>energy %</strong></td>
<td>21.96</td>
<td>12.35</td>
<td>21.65</td>
<td>30.67</td>
<td>8.81</td>
<td>3.71</td>
<td>0.51</td>
<td>0.33</td>
</tr>
<tr>
<td><strong>Exchange rate</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>energy %</strong></td>
<td>5.74</td>
<td>13.36</td>
<td>64.41</td>
<td>9.29</td>
<td>3.67</td>
<td>1.48</td>
<td>1.28</td>
<td>0.77</td>
</tr>
<tr>
<td><strong>Input price</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>energy %</strong></td>
<td>88.13</td>
<td>7.32</td>
<td>1.45</td>
<td>2.50</td>
<td>0.54</td>
<td>0.04</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Corresponding</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Periods</strong></td>
<td>long</td>
<td>15.5</td>
<td>7.7</td>
<td>3.87</td>
<td>1.9</td>
<td>1</td>
<td>0.5</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Figure 7.7a Details at level 5 – components
A fair amount of shorter run fluctuation over a 4-year cycle is apparent throughout the whole period. Commodity price is the largest contributor to variations at this level, although a little out of phase in the early years. The exchange rate variance also has big impacts on $ftt$ level 5 fluctuations, although its power is not apparent during mid-1990s. The impacts of input price remain minor through the time.

Exchange rates in addition come into their own at the longer cycles corresponding to level 6 detail. Commodity prices are also powerful in this band, but up until 1998, the two had a clear tendency to neutralise each other. This amounted to a buffering effect: whenever commodity prices became strong, the NZD exchange rate also became strong. Table 7.3 shows that the two components have a relatively high correlation in this band.

### Table 7.3 FTT component wavelet correlation (level 4-6)

<table>
<thead>
<tr>
<th>Approx. &amp; Details</th>
<th>A7</th>
<th>D7</th>
<th>D6</th>
<th>D5</th>
<th>D4</th>
<th>D3</th>
<th>D2</th>
<th>D1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commodity price VS Exchange rate</td>
<td>-0.73</td>
<td>-0.98</td>
<td>-0.72</td>
<td>-0.04</td>
<td>0.39</td>
<td>-0.28</td>
<td>-0.27</td>
<td>-0.03</td>
</tr>
<tr>
<td>Commodity price VS Input price</td>
<td>0.82</td>
<td>0.75</td>
<td>0.46</td>
<td>0.25</td>
<td>0.40</td>
<td>0.11</td>
<td>0.19</td>
<td>-0.09</td>
</tr>
<tr>
<td>Exchange rate VS Input price</td>
<td>-0.21</td>
<td>-0.82</td>
<td>-0.56</td>
<td>0.01</td>
<td>0.19</td>
<td>0.03</td>
<td>-0.14</td>
<td>-0.08</td>
</tr>
</tbody>
</table>
Because they were slightly out of phase, there was a resultant effect on the fit but a relatively minor one. Things changed between 2000 and 2002. In the latter case, the NZ dollar was exceptionally weak coming out of the Asian crisis. Commodity prices were not strong, but on the other hand not weak enough to neutralise the beneficial effects of the weak NZ dollar. The level 6 buffering effect failed. Because the bulk of the exchange rate effect accumulates in this band, the fit appeared to dislocate as a whole.

The post 2002 period in figures 7.7a,b gives an early indication that things might well restore themselves to normal patterns. The level 5 detail is much as before. At level 6, however, where the exchange rate is important, commodity prices and the exchange rate looked to be once again on opposing tacks, just as they were prior to the Asian crisis. If this indeed turns out to be the case, something like the normal cycle might be restored.

On the other hand, there seems to be potential for a more inter-run volatile NZ dollar. In addition to the short end volatility associated with monetary policy, there are exposures arising from the growing use of offshore funding to raise debt for mortgages, consumer spending and other purposes. It is unclear whether the more or less natural buffer of early years will continue to apply, or with the same force. As a result, the fit might face more instability in the future.

### 7.2.7 Risk management implication

In general, knowledge about the New Zealand dairy farmer exposures may assist the average farmer or farm management adviser to know how to formalise the ups and downs of profitability and to recognise when the cycle is at a historic high or low. Information of this kind can be an important input to the estimation of likely recovery periods for prospective farm development programmes or decisions about herd replacement.

Among the three exposure components, fluctuations in the exchange rate and commodity price index account for the most part of the variation in NZ dairy farmer terms of trade. Perceptions of a need to shield dairy farmers against foreign exchange and commodity prices exposures have on occasion caused more harm than good. In late 1997, the NZD was at a historically high rate of 71c US, while milk solids and other dairy prices had taken a turn for the worse. As discussed in Chapter 2, the NZ Dairy Board thought that farmers were at risk of serious financial harm should the NZ
dollar strengthen further, and instituted a hedging strategy, which resulted in a large amount of opportunity costs.

The implicit buffer between the two main contributors, as it appears in figure 7.7b, makes it fairly clear (with the advantage of hindsight) that there was little need to protect against foreign exchange and commodity price exposures in a long term context. Over a term long enough to threaten survival for the typical dairy farmer, the one exposure tended to balance out the other. The wavelet analysis suggests that if formal hedge instruments were to be employed, the instrument maturities should be fairly short. Essentially, one would be aiming to hedge the level 4-5 variations. Since such exposures are cyclical over a period of 2-4 year, the hedging horizon could thus be set as half of the cycle, e.g. 1-2 years, or even shorter. But because dairy commodities do not possess well developed forward markets, this in turn suggests that market based hedges could be only dependent on currency forwards or options.

In the particular case of the NZ dollar, there are good reasons (based on expected values but not risk) for a bias towards using currency forwards, arising from the chronic forward discount on the NZ dollar associated with high NZ interest rates, as will be discussed in the next chapter. However, as some case studies in Chapter 2 show, it can go wrong on occasions. Also, the existence of a forward bias does not preclude the more adventurous dairy company treasury from seeking to take advantage of what might be seen as unduly high or low foreign exchange rates, or of attempting to second guess market efficiency. The effectiveness of such an attempt will be explored in the following chapter, with active hedging based on exchange rate forecasting models developed earlier as examples.

A longer-term strategy for the industry as a whole is to seek growth and diversification into developing markets, especially those based on engines of economic growth such as the Chinese economy. There is some evidence that this has been responsible for the trend effects noted above. Likewise, diversification into resurgent economies such as Japan can yield business cycle diversifications.

7.3 Hedging decision horizon

The corporate currency hedging literature mainly focuses on short term exposures and corresponding short range risk management. The reason for ignoring long term currency risk management is that many researchers believe purchasing power parity (PPP) tends to hold in the long run, while could be violated for short time periods
If PPP holds, a change in the exchange rate should simply represent the differential in the inflation rate between the two corresponding countries. As a result, the real exchange rate, which adjusts nominal exchange rates by the corresponding inflation rates, would remain constant and exporters or importers would face little exposures. It is obvious from the expression of the exposure indicator (7.1) that the impact of currency fluctuations on corporate exposures could be offset by changes in the price ratio between the two countries as long as the PPP holds.

On the other hand, some empirical evidence has shown that PPP can be violated even over a relatively long time period, e.g. exceeding three or five years. Rogoff (1996) found the half-life of deviations from purchasing power parity is between 3 and 5 years, while Cashin and McDermott (2003) argued that the half-life of deviations from PPP can last up to 8 or 13 years. Cashin and McDermott compensated for the downward bias by using median-unbiased estimators instead of conventional least-squares-based estimators of half-lives. The violation of these parities over 10 years and even more means the expected return on currency in a very long interval may deviate from zero.

However, even when real currency exposure exists over a long interval, corporates may still ignore long term hedging. The reason lies in the fact that it is easy to hedge the real exchange rate in the short term, but not over a long time period. During a short time interval, fluctuations in real exchange rates mainly arise from sudden falls or rises in demand for a given currency. Currency forward or options with the nominal exchange rate as the underlying asset are able to eliminate such currency risk. On the other hand, unexpected inflation and real interest differentials drive the long term fluctuations in real exchange rates. Currency derivatives are less effective in managing this sort of currency risk. Froot (1993) provided empirical evidence for these objections to long term hedging. He analysed a data set of US financial returns from the perspective of British international investors. The results show that full hedging can reduce return volatility over short intervals, but increases the real return variance of many portfolios over a long term period. In a period of five years or more, the volatility of hedged foreign stock investment is higher than that of unhedged foreign stocks.

In addition, a long term hedging strategy is difficult to put into practice. Firstly, over a long interval, the quantity and price risk for using derivative contracts can be
extremely high. In a volatile world, it is difficult for companies to determine the volume of sales and prices of products over long periods. As a result, it is hard to predict the long term exposures faced by a company. Forecasting errors may lead corporations to over- or under-hedge against natural exposures. Secondly, long term hedging instruments can be very expensive as the corresponding market is small and the required liquid risk premium for the products is high.

Nevertheless, some companies that know exactly what they will receive in the long term might still favour long term hedging. One example is Rolls Royce plc, the second-largest aircraft engine maker in the world. Currency forward covers for this company are long, up to eight years (Paul, 2001). The forwards are in the form of standard foreign exchange contracts in which the strike price depends on future interest rate differentials. Among the reasons for this long term hedging policy, one is that a large proportion (16% in 2001 and 38% in 2005) of signed orders are long term after-market service agreements, with the first seven years’ revenue recorded in the order book. For Rolls Royce plc, long term hedging aligns the hedging period with the horizon of exposures. The exposures are fixed in the contracts and thus the hedging is free of quantity risk.

In the current context, the above wavelet decomposition of the New Zealand dairy farmer terms of trade provide the evidence for the offsetting effects existing among the New Zealand dollar and international commodity prices over a 7-8 years cycle. The buffering implies that companies can abstain natural hedging over long time periods and the focus of risk management should thus be on the reduction of short or intermediate term currency exposure. For protecting the firm from these exposures, a smoothing hedging policy, in which the hedging ratio gradually increases as it gets closer and closer to the maturity of exposures, is chosen in the current context.

### 7.4 Rolling hedge frameworks

Given a specified hedging horizon, academic discussions about hedging decisions are commonly confined to a simple forward-spot hedge, using a single forward contract of pre-set maturity. The whole optimisation problem is simplified as a choice between the spot and the corresponding forward. However, many practical hedging rules are of the rolling hedge type, staggering the cover over a variety of forward maturities in
order to smooth the exposure. To adequately compare hedging outcomes or design optimal hedging rules, some measurement problems must first be resolved.

To illustrate, suppose that a series of forward rate contracts, collectively denoted \( F \), are to be used to hedge a foreign exchange rate exposure, denoted by \( S \). (The precise relationship of this to the objective outcome in the utility function will be discussed later). Consider, for instance, a common rolling quarterly hedge of the following form in foreign exchange risk management: Hedge 100% of Q1 exposure, 75% of Q2 exposure, 50% of Q3 exposure and 25% of Q4 exposure. In other words, one makes sure that the near quarters are hedged more completely than the further out, constantly adjusting the proportions upwards as the distant quarter approaches. This type of hedging policy is consistent with the managers’ aversion to potential quantity risk by means of hedging more nearer exposures.

Once a steady state has been achieved, the hedge portfolio for the exposure at one real time period will consist of equal proportions of one quarter, two quarter, three quarter and four quarter forwards, which have been respectively bought at one quarter, two quarters, three quarters and four quarters ago. In this example, the effective conversion rate \( ECR \) achieved at time \( t \) would be given by:

\[
ECR_t = 0.25F_{t-4,4} + 0.25F_{t-3,3} + 0.25F_{t-2,2} + 0.25F_{t-1,1},
\]  

(7.2)

where \( F_{m,n} \) = forward price for maturity \( n \), bought at time \( m \). The effective conversion rate \( ECR_t \) can then be compared directly with \( S_t \), the spot rate, which provides a way of evaluating the hedge strategy compared with remaining unhedged.

More generally, one could have arbitrary four-quarter rolling hedges of the form

\[
ECR_t = h_1 F_{t-4,4} + h_2 F_{t-3,3} + h_3 F_{t-2,2} + h_4 F_{t-1,1} + (1 - \sum_i h_i) S_t.
\]  

(7.3)

It is assumed that \( h_i \geq 0 \) and \( \sum_i h_i \leq 1 \). This would amount to leaving a proportion of the exposure unhedged, i.e. exposed to the final spot exchange rate \( S_t \). In effect, one is constructing a portfolio of forward rate contracts combined with some unprotected spot exposure.

By using forwards, the corporate can lock the financial variable at a pre-set price and thus eliminate the conditional volatility. However, forwards are not necessary leading to lower unconditional volatility. As Froot (1993) shows, employing long term forwards to hedge against overseas investment eventually increase the variance of the total return. In the current context, it is found that the smoothing hedging
strategy enables the companies to benefit from a lower unconditional volatility in addition to a smaller conditional volatility in cash flow. To plot the cash flow of the firm over time, the smoothing hedge results in a relatively smoother time path.

Take the USD/NZD time series as an example. Figure 7.8 depicts the effective conversion rate for two alternative hedging portfolios, 3-year fully forward hedging and a smoothing hedge with equal weights of 1-year, 2-year and 3-year forwards. The smoothing strategy leads to a historical time series of conversion rates with the volatility as 0.048 while single forward hedging results in an exchange rate history with a variance as 0.062. The rolling strategy reduces the volatility in the historical cash flows by around 30% relative to the single forward hedging strategy.

![Figure 7.8 Comparison of single forward hedging and smoothing hedging for USD/NZD](image)

**7.5 Operational choice of hedging instrument: general consideration**

It can be seen from the figure 7.9a and 7.9b how gains can be obtained by hedging with currency forwards and options. The strike price of a forward and put option is assumed to be the same as the stress point $P$. It is also assumed that there are no transaction costs for simplicity. The corporate manager has the same view of the payoff as the market when $R>P$ but not when $R<P$. The dashed lines $AB$ and $CD$ represent the market view of the payoff on the forward and option respectively. For a forward contract, the market price is zero, as the gains on the $AP$ are offset by the

---

20 Monthly USD/NZD derives from MSCI and spans from July 1990 to Feb 2007.
opportunity costs $PB$ caused by the hedging. The put option price is assumed to be $p$, represented by line $PE$. The solid lines $MP$ in 7.9a and $ME$ in 7.9b stand for the cash flow occurring to the corporation when the financial instruments eliminate the lower tail risk. Since the two lines are above the lines $AP$ and $CE$ respectively, which represent the market view of downside risk, the corporation earns more from using forwards and options than the market average.

![Figure 7.9a Gains from using forward](image1)

Figure 7.9a Gains from using forward

![Figure 7.9b Gains from using options](image2)

Figure 7.9b Gains from using options

Corporates will benefit from the use of either currency forwards or options. The decision regarding whether to utilise forwards or options as hedging instruments could be affected by a variety of considerations. Firstly, it may depend upon the views of corporate managers about possible future currency movements. Companies may prefer using options when they predict that the future asset price may go against a
forward position. That explains why exporters who expect a depreciation of the home currency are inclined to employ options rather than forwards. On the other hand, exporters prefer forward hedging when they believe that the domestic currency is likely to strengthen. In this circumstance, the forward seems to be a cheaper tool for managing currency risk as there are no costs involved in fixing the exchange rate at a favourable level by forwards. As the RBNZ reported (Briggs, 2004), New Zealand exporters decided to increase the forward hedging ratio in 2000 and 2001, when the New Zealand dollar reached a historically low level and was expected to appreciate in the future.

Decisions regarding whether to use forward or option hedging may as well be related to the firm’s risk aversion attitude to the upside gain. When corporate managers agree with the market’s view on the value that is higher than the critical point $P$, the firm should be indifferent to use forwards or options as hedging instruments. The foregone upside profits by employing a forward contract is of the same value as the market price of an option, which implies that the opportunity costs regarding the two hedging instruments are identical. On the other hand, when corporate managers under assess the upside gain relative to the market perspective, forwards tend to be a cheaper hedging tool as the subjective value of forward hedging opportunity cost is lower than the market price of options. The same conclusion was drawn by Albuquerque (2003), although it is contradictory to the conventional statement that options are ideal instruments to hedge downside risk (Stulz, 1996). Albuquerque found that the higher payoffs from forwards for low spot value means that fewer forward contracts than options are required for reducing a certain level of debt default probability.

The use of options can somehow be related to the shape of the implied volatility of the market option price. It has been well known that the implied volatility, as backed out from the Black-Scholes option pricing function, can deviate from the historical volatility in the underlying asset price (e.g. Derman & Kani, 1994). The relation between the implied volatility and the increasing strike price displays differing characteristics in different markets, such as being downward sloping for equity options or U-shaped for currency options (Scott, 1992; Bates, 1996). The latter volatility pattern is known as the volatility smile, which signifies that the implied volatility of an at-the-money option is lower than suggested by the historical volatility while the price of a deep in-the-money or out-of-the-money option entails a relatively
higher implied volatility. This pattern can be employed somehow to guide currency hedging decisions. When the current asset price is well above the distress point $P$, individuals who believe in the historical volatility may regard the put option, with a strike price identical to $P$, as being overvalued. On the other hand, when the spot exchange rate is close to the stress level, an at-the-money option may appear to be a cheaper hedging tool.

Furthermore, some company managers, who are assessed against the benchmark of unhedged positions, are unwilling to lose the potential gain opportunity due to the use of forwards. In these circumstances, options appear to be more attractive as the opportunity costs regarding the forwards are higher than the market value. Corporates in a competitive environment might also favour the use of options as the hedging opportunity costs related with forward contracts might be exploited and thus exacerbated by their rivals’ actions, as discussed in Chapter 3.

Uncertainty regarding the sale volume and price of products exposes companies to the risk of over-hedging, where the hedging losses may not be offset by natural exposures. In such a situation, options become particularly attractive, as the opportunity costs of option hedging are constrained to the premium payments. Given the high volatility in the value of financial variables, the losses relating to the forward position can be very high, especially when the hedging horizon is long. Moreover, options are better hedging tools than forwards when companies encounter both hedgeable exchange rate risks and non-hedgeable price risks. In such circumstances, currency exposures are non-linear and options with a non-linear payoff are thus regarded as the appropriate hedging instrument. Wong (2003), Gay, Nam and Turac (2003) have also come to this conclusion. They find that an optimal hedging position generally is comprised of linear contracts such as forwards. However, when the levels of quantity and price risk increase, the use of linear contracts will decline and non-linear contracts such as options become more effective due to the risks and costs associated with over-hedging.

However, option hedging strategy entails cost of premium paid upfront, while firms usually are reluctant to pay a large amount of money in advance for hedging. In a small open economy like New Zealand, the currency option can be extremely expensive in some periods because of the illiquidity and the high currency risk premium. To relieve pressure on cash flows, companies usually buy as well as sell options for the purpose of self funding, as happened in the cases discussed in Chapter
2. A portfolio of different options enables firms to lock the exchange rate within a band instead of at a particular point, i.e. collar payoff. Option portfolios make firms better off when asset prices remain inside the band, but the consequences are similar to that of forwards when prices move out of the band. This strategy still suffers from opportunity costs when the financial market is volatile. Furthermore, although options give firms more flexibility to manage currency risk, complex payoff profiles for option portfolios can make it more difficult for users to design effective hedging policies. As occurred in the Pasminco case, a misplaced option hedging portfolio might expose users to additional financial risk.

Since discussions on corporate’s cash holdings are beyond the current context, the option which requires cash payment in advance is excluded in the following empirical application. Instead, the forward is chosen as the hedging instrument for describing the hedging algorithms and the consequent hedging outcomes. But it does not rule out the possibility of using options in some circumstances, as discussed above.

7.6 Conclusions

The corporate terms of trade index has been developed in the current chapter to account for the most critical exposures faced to the corporate. Such an index can be regarded as the ratio of operating revenue to costs and in addition has the similarity with the firm’s net profit margin. A wavelet decomposition of New Zealand dairy farmer terms of trade shows that there are strong cyclical patterns over a period of 2 to 4 years, although some structural breaks are detected. The natural buffering effects exhibiting over a 7-8 year cycle further shows that there might be low long term currency exposures for the New Zealand dairy farmers. Such wavelet decomposition outcomes provide the corporate manager at least elementary guidance for risk management decision making, for instance the detection of long term natural offsetting effects in the farmer terms of trade may imply that long run financial hedging is not required in the dairy industry.

The current chapter also discussed some operational aspects regarding a smooth hedging rule. The rule suggests that a series of forwards with various maturities instead of a single forward can be employed as hedging instruments. The consequent hedging policy smoothes the cash flow and so reduces the corresponding volatility. By building up the hedging position along the time, hedging ratios for short term
exposures are higher than those against longer term risk. As a result, this form of hedging strategy leads to lower quantity risk, especially when corporate hedges against forecasted exposure.

Discussions regarding the hedging instrument explain that using either currency forwards or options can add value to the firm. However, the values enhanced by them differ from one situation to another. In some circumstances, such as when the quantity risk is high or corporate managers are highly averse to the opportunity cost of forward hedging, the option is better than forward as the hedging instrument. On the other hand, the currency forward hedging performs better than option hedging when cash is limited or when the spot rate is expected to move in a direction favourable to the firm. Only the forward is employed as the hedging instrument for the later empirical discussion as the premium cash payment of options could be related to other production decision in a firm and such discussions are beyond the current context.

The following chapter applies the risk management framework developed so far to practical problems. The chosen application is to the New Zealand dairy farmers for reasons discussed earlier. The hedging is aimed to maximise the GVaR criterion developed in Chapter 3. The hedging outcome is discussed from the perspective of unconditional hedging and the conditional hedging respectively, according to whether corporate managers vary the hedging ratio when new information becomes available. The hedging performance described in the next chapter explains whether corporate managers can improve the value of the firm by incorporating their private information and judgement on the currency movement into the hedging decision.
Chapter 8 Optimal Corporate Currency Hedging

From a corporate finance perspective, hedging may be designed to increase the upside gain and decrease the risk of extreme downside, as discussed in Chapter 3. The problem is how to establish hedging decision rules that balance the upside and downside. This chapter describes the framework in which a GVaR approach as developed in this thesis can be applied to derive the optimal corporate currency hedging policy. It may give the corporate manager some guidance as to how to incorporate organisational financial risk preferences into the wealth objective function.

Optimal corporate currency hedging algorithms and outcomes are examined in terms of both unconditional hedging and conditional hedging. In an unconditional hedging, the optimal weights for forwards are determined on the basis of the historical data distribution and the hedging ratios do not vary along the time period. The application of an unconditional hedging examines the effectiveness of the smoothing hedging framework developed in the current part. A focus on such a passive hedging also enables the implementation of the sensitivity analysis, which inspects the impacts of risk aversion attitudes on the optimal hedging decision. In a conditional hedging, the optimal hedging decision needs to be continually reviewed and adjusted as new information becomes available. The conditional hedging is based upon the currency movement predictions derived from the exchange rate directional forecasting model developed in Chapter 6. However, because the forecasted outcome is only the direction rather than the quantitative level of currency movements, the general problem is how to map such directional indicators and probabilities into actions. The conditional hedging outcome will also be discussed from a short term perspective when the M-GARCH exchange rate forecasting model is employed.

The structure of this chapter is as follows. Section 8.1 contains revision and specification of the hedging welfare function. Section 8.2 defines the hedge target and object. Section 8.3 describes unconditional smooth hedging outcomes of the dairy farmers’ terms of trade. The optimal hedge outcome, derived from maximising the GVaR objective function, is assessed on the basis of both the “in sample” and “out of sample” analysis. Section 8.4 discusses the potential hedging outcomes in the context of importers. Section 8.5 investigates conditional hedging outcomes with directional forecasting of future currency movements. Section 8.6 examines the hedging
outcomes with short term M-GARCH exchange rate forecasting. Section 8.7 concludes.  

8.1 Hedging welfare function

8.1.1 Review of objective function

The corporate hedging in the current application is constructed to maximise the GVaR welfare criterion developed earlier. The consequent managerial utility function, which is as though the managers have written put options on firm value in favour of third-party claimants (detailed discussions see Chapter 3), can be written in a form of equation (3.5) as:

\[ U(R; P) = R - P + b(R - P)^- \quad (8.1a) \]

where \( R \) represents the underlying exposures. The description of \( R \) will be detailed in later sections.

For convenience, figure 8.1 again shows the objective function, as shown in figure 3.1. Above the critical point \( P \), the utility increases linearly with the return or value outcome. Around the critical point \( P \), the slope increases from unity to \( 1 + b \), indicating a higher disutility as the return sinks below the critical point.

![Figure 8.1 Objective function](image)

The connection with CVaR can be viewed by taking the expected utility:

\[ 21 \text{ Section 8.2 and 8.3 are based on Bowden and Zhu (2006a). Section 8.5 is based on Bowden, Zhu and Cho (2007).} \]
where \( F_R(P) \) is the distribution function of \( R \) evaluated at \( R = P \). If one chooses \( P \) to satisfy a pre-set VaR critical point, then the risk penalty is provided by the GVaR, including VaR term \( F_R(P) \) and the CVaR term \( E[R - P | R \leq P] \). The parameter \( b \) is set as a penalty to the GVaR component relative to the overall or unconditional expectation \( E[R] \); its value will incorporate or otherwise be influenced by the desired VaR critical point (see below). As shown in the equation (3.7), an extension of equation (8.1a) to deal with long tail risk is:

\[
U(R; P) = R - P - b | R - P \gamma SF(P - R), \tag{8.2}
\]

where the constant \( \gamma > 1 \), \( SF(x) = 1 \) if \( x > 0 \), = 0 otherwise. Given \( \gamma > 1 \), the marginal disutility is increasing with larger losses.

It should be stressed at the outset, however, that not all agents will necessarily have this form of welfare function. For instance, some investors are perhaps more consistent with a decreasing marginal utility over upper as well as lower ranges. On the other hand, as discussed in Chapter 3, the importance of bonus-seeking behaviour in managerial life on the upside, coupled with the prospect of being fired on the downside, might produce a risk aversion profile more of the type investigated in the present context.

### 8.1.2 Risk parameter calibration

Risk aversion depends upon the two parameters \( P \) and \( b \), so the user has to calibrate their values at the outset.

(a) The critical point \( P \) could be calibrated by examining the firm’s cash flows, as in cash flow at risk. In terms of the present application on corporate exposure risk management, a given terms of trade or net profit margin implies a cash flow for the operation. Danger-points for the latter generate corresponding danger-points for the former. Alternatively, one can plot the natural exposure (unhedged terms of trade) over the past and isolate the lower 5 or 10% crucial points, identifying these as alternative values of \( P \). This would amount to saying that the past natural or unhedged values constituted a desired range for the hedged as well as unhedged exposures. If that is not true, one could adjust \( P \) upwards, e.g. identifying the desired hedged 5% VaR with the natural or unhedged 10% VaR.
(b) The risk aversion parameter $b$ could be calibrated by rearranging equation (8.1b) as:

$$E[U(R; P)] = w_L E[R - P | R \leq P] + w_U E[R - P | R > P], \quad (8.3)$$

where $w_L = (1+b)F_R(P)$ and $w_U = 1 - F_R(P)$.

Equation (8.3) splits up the expected utility into the conditional expectation contributions less and greater respectively than the critical point $P$ (the ‘bad’ and ‘good’ zones). The bad zone expectation (i.e. the first on the right hand side) is effectively the CVaR. The consequent hedging objective makes a trade off between the expected payoff in the good zone and the expected damage in the bad zone. Simple risk neutrality would correspond to $b = 0$. On the other hand, the manager might wish to weight the two zone expectations equally, setting off the expected gain on the upside against the CVaR. Setting $w_L = w_U$ gives $b = (1 - 2F_R(P))/F_R(P)$, so if $P$ is chosen as the 10% point, then $b = 8$; or if $P$ is the 5% point, $b = 18$. Increasing $b$ beyond these values amounts to overweighting the CVaR relative to the expected gain on the upside, so this is greater risk aversion. The extreme case as $b \to \infty$ is interpreted to mean that the manager is concerned only with the GVaR.

Other methods of calibration are available. For instance, one could compute a risk aversion coefficient such as the generalised Rubinstein risk premium $\theta$ (e.g. Bowden, 2005a) which says that expected utility is certainty-equivalent to an outcome of $\mu_R - \theta$. As $\theta$ depends on the aversion parameter $b$, one can set the value of beta to accord with any desired certainty-equivalent outcome. Appendix D gives the relevant formula for the generalised Rubinstein risk premium in the present context.

Whatever the method used, a reality check is always useful where calibration is concerned. It is suggested to run the optimisation with a range of $b$ values and computing risk diagnostics for each. For instance, there is an inverse relationship between beta and the GVaR, so if the latter is seen as too high, then the remedy would be to increase the value of beta. Likewise, it would seem a useful precaution to explore the effects of different powering constants ($z$) as in equation (8.2).
8.1.3 Smoothing the objective function

In practice, there are reasons for replacing the kink at $R=P$ in figure 8.1 with a smoother utility transition while preserving the VaR motivated point $P$ as a calibration benchmark: (a) If the payoff function resembles that of a put option, as indicated earlier, then one could consider the disutility as in the nature of a cost related to the price of the option, which is inherently continuous in nature. (b) It is rationale to smooth the kink point in practice as the data is usually too noisy to determine precisely the point $P$. (c) Computationally, one often experiences convergence difficulties with the optimisation of expected step or segmented (ramp style) functions, where the sample expectation has to be estimated from a finite number of observations. In the present context, there will be fewer observations in the critical zone ($R \leq P$), and small changes in the trial solution for the proposed hedge ratios may have no effect, leading to the numerical optimisation programme getting stuck. This problem is well known in switching regression theory, where there may be very few observations at or around the chosen switch point.

Two alternative ways of smoothing can be derived, based on fuzzy logic, or drawing on the options interpretation, respectively. The former operates by replacing the implicit step function in (8.1) with a fuzzy membership function (Zadeh, 1965, 1970) similar to a notional probability distribution, adjusting the variance parameter to achieve different degrees of smoothing. Appendix E describes this method in more detail. It is very easy to implement and is correctly centred at the critical point $P$, but has the disadvantage that it can assume a local negative slope for very high values of the aversion parameter $b$.

The options-based method is more appealing in terms of its underlying economic interpretation. Given any value $R$, the implied put options ($b$ of them) are priced by assuming the current physical value is $R$ and the strike price is $P$. Thus if the outcome is $R$, one can think of the manager as being assigned a penalty equal to the value of a put option at that price, with strike set at $P$. The market price of the option is treated as the risk aversion penalty. One could assign the volatility parameter as that of the unhedged position – specifically, expected spot exposures in the current context. In terms of corporate finance theory, the objective would be to limit the value of the options held by third party claimants in the event of bankruptcy. In effect, this becomes the option VaR criteria.
Given an assumed volatility number $\sigma$, the put option pricing model (Garman & Kohlhagen, 1983) is inserted into equation (8.1a) in place of the ramp function element $b \min(R - P, 0)$, to give:

$$U(R; P) = R - P - b \times \pi(R; P),$$

where: $\pi = e^{-rT} \times [P \times \Phi(-d_2) - R \times \Phi(-d_1)]$

$$d_1 = \frac{\ln \left(\frac{R}{P}\right) + (r + \frac{\sigma^2}{2}) \times T}{\sigma \sqrt{T}}, \quad d_2 = d_1 - \sigma \times \sqrt{T}. \quad (8.4)$$

Setting the parameters $\sigma$, $r$ and $T$ depends on how they are interpreted. As suggested earlier, one could assign $\sigma$ as the volatility of the unhedged $R$, $r$ as the market rate and $T$ as the length of a performance evaluation horizon.

On the other hand, if all that is required is a smoothing device, then one could set $\sigma$ as small as is consistent with computational convergence requirements, and might as well fix $r = 0$ and $T = 1$. Figure 8.2 shows what happens, in this case for the subsequent application, choosing $\sigma = 0.025$, $r = 0$ and $T = 1$.

Figure 8.2 Option equivalent approach to the exact utility function$^{23}$

### 8.2 Decision problem

The decision problem which follows mainly relates to currency hedging for New Zealand dairy farmers. To bring the objective function and the hedging strategy together, it is helpful to adapt some terminology from the optimal control literature, of which hedging can be regarded as an instance. A *hedge instrument* refers to a market-based contract that may be bought or sold in the pursuit of risk management. A *hedge*

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$^{23}$To preserve the positivity needed for the logs in the option price, a constant number has been added to the natural terms of trade variable, though $P$ is still formally located as the zero point.
target refers to a variable that the hedge instrument is naturally adapted to. A hedge object refers to a variable that enters the hedging objective function as the ultimate object of control and it will typically depend in some way on the hedge target variable. The hedge object in the current context is the variable $R$ that appears in the objective function of equation (8.1). In turn, $R$ depends wholly or partly on a foreign exchange rate $S$. The latter is a suitable hedge target because of the availability of a set of foreign exchange forward contracts adapted to it. The FX forwards are thus the hedge instruments.

In the application that follows, the decision is to hedge the company’s net profit margin as represented by the log terms of trade. The object $R$, as described in detail in the previous chapter, can be written as the dairy farmer terms of trade:

$$R = \log\left[\frac{\text{dairy product price index} \times \text{effective exchange rate USD/NZD}}{\text{NZ farmer expense price index}}\right]. \quad (8.5)$$

By replacing the spot exchange rate as it appears in the industrial terms of trade with the effective conversion rate, the distribution of the object variable $R$ comes to depend upon the set of hedge ratios $h_i$. The objective is then to choose the optimal hedge ratios $h_i$ in order to maximise the expected value of the objective function.

A major source of uncertainty for the New Zealand dairy industry is the exchange rate. The bulk of NZ dairy products is contracted in terms of US dollars, and the USD/NZD exchange rate is consequently the major single foreign exchange exposure. It has been highly volatile trading over the last ten years in a range of about 1.35 to 2.55 USD/NZD. Dairy commodity prices are also a significant source of variation, as discussed in Chapter 7. In addition, farmers are exposed to cost variations via the local economy and once again through the exchange rate for equipment, fertiliser, chemicals etc. The current application will be concentrated on the management of exchange rate risks, given the reasons discussed earlier. Dairy companies have been highly active in the FX hedging market in coping with the volatility exhibited in the dairy industry, although not always with good results, as discussed in Chapter 2. The design of effective hedging rules is therefore an important task for the industry.

**8.2.1 Unhedged behaviour**

Figure 8.3a shows the history of the dairy farmer log terms of trade over the entire sample period. The variability is self-evident, as also illustrated in Chapter 7. Two
possible discomfort points are marked, corresponding to the 5% and 10% lower quantiles ($P_5$ and $P_{10}$). These points are also marked in figure 8.3a as horizontal lines at $P_5 = -1.8804$ and $P_{10} = -1.8578$. Figure 8.3b depicts the same data in the form of a formal histogram (this should not be taken too literally as the data are not serially independent and do not need to be in what follows). There are appreciable tails both to the left and the right but the two tails are asymmetric. It would make simple variance reduction an economically sub-optimal hedging decision criterion.

Early 1997 was the decision point for the NZ Dairy Board’s ill-fated hedging decision (see Chapter 2), in the course of which dairy farmer bankruptcies were cited by the Board as a possible outcome if the NZD exchange rate strengthened any further. This suggests that the discomfort or pain point for the farmer terms of trade would have been about the 10% point ($P_{10} = -1.88$ in the above diagrams). This will be the maintained assumption, though the 5% point could also be used to illustrate the effect of relaxation, and the 20% point for a tightening.

![Figure 8.3a Unhedged farmer log terms of trade, time series](image-url)
8.2.2 Nature of the hedge

Direct hedges of dairy commodity prices to the NZ industry are not available. Discussions in Chapter 7 show that the two major sources of variability in the farmer terms of trade are the exchange rate and dairy commodity prices, with the input prices more stable, at least in recent years. Areas of major discomfort are where low commodity prices coincide with a strong NZD/USD exchange rate. The effective hedge instrument for the farmer terms of trade is based on the forward exchange rate, which has both a direct relationship with the exchange rate and an indirect relationship with commodity and input prices, given that the NZD is widely regarded as a commodity currency. However, one should be aware that the correlation between the exchange rate and commodity price or input price is relatively weaker over a shorter interval (see Chapter 7).

In a capital importing country like NZ, interest rates can be chronically high in comparison to other OECD countries. This implies that NZD generally has a forward discount and thus New Zealand exporters have a natural bias to use the forward exchange rate, as reflected in the one year forward NZD discount plotted as the forward premium on the USD/NZD in figure 8.4.
Figure 8.4 Forward premium on the USD/NZD

On the other hand, a forward discount on the home currency does not necessarily mean a forward profit, in terms of the actual conversion rate on maturity. Figure 8.5 graphs the difference between the forward rate at maturity and the spot rate as of the same day, presented in the form of a profit to the forward user. Use of the forward would have resulted in a loss at times, as the Dairy Board discovered in their 1997 hedging programme. Moreover, use of longer term forward (not depicted) is even riskier; consistent losses would have resulted between June 1998 and June 2002. Conversely, significant gains would be made where a fall in the USD reinforced the forward rate discount, as happened in the later part of the sample period.
8.3 Unconditional smoothing hedge of dairy farmer terms of trade

The first hedging application is to an unconditional hedge of dairy farmer terms of trade. The optimal hedging ratio is derived from the historical information and assumed to remain constant over time. The discussion on unconditional hedge illustrates the effectiveness of the structural hedging framework in practice.

8.3.1 Smoothing hedging

The smoothing hedging that was discussed in the earlier chapter will be employed as the hedging strategy in this section. The hedged conversion rate at time $t$ can thus be written as:

$$ECR_t = h_4 F_{t-4,4} + h_3 F_{t-3,3} + h_2 F_{t-2,2} + h_1 F_{t-1,1} + (1 - h_4 - h_3 - h_2 - h_1) S_t. \quad (8.6)$$

The decision regarding the weights for a series of forwards with maturity as one year, 9-month, 6-month, and 3-month, and natural unhedged position, needs to be derived.

The effective exchange rate will mean either the actual exchange rate or the effective conversion rate $ECR$, depending on the context. If the actual exchange rate is used, $R$ will be referred to as the natural or unhedged terms of trade exposure. If the $ECR$ is used, the result will be the hedged objective variable. In the latter case it will be convenient to use log forward rates to accord with the desired hedge object. Equation (8.6) can be modified as:
The hedges considered currently are of the smoothing or passive type. The objective is to assist farmer production planning by smoothing out bumps, with special reference to those at the lower end. It will not be considered, in the present section, the issue of active or tactical hedging wherein the hedge ratio might depend upon the current state of exchange rates or commodity prices, with an implicit forecasting agenda. The conditional hedging on the basis of forecasting will be discussed in the subsequent application.

8.3.2 The hedging decision specification

The complete optimisation model may be summarised as follows.

Notation:

\[ S = \text{spot exchange rate USD/NZD.} \]
\[ F = \text{forward rates: for brevity } F_3 = \text{3-month forward executed 3 months prior to spot date,} \]
\[ F_6 = \text{the 6-month forward executed 6 months prior to spot date,} \]
\[ F_9 \text{ and } F_{12}. \]
\[ P_d = \text{price index of product.} \]
\[ P_s = \text{expense price index.} \]
\[ R = \text{objective variable.} \]

The decision problem:

\[
\text{max}_{h_1,h_2,h_3,h_4} \quad E[U(R)] = E[R - P] + bE[(R - P) \times SF_b(P - R)],
\]

where

\[
R = \log P_d + (1 - h_1 - h_2 - h_3 - h_4) \log S + h_1 \log F_3 + h_2 \log F_6 + h_3 \log F_9 + h_4 \log F_{12} - \log P_s,
\]

\[
P = \text{VaR}_{5\%} \{ \log P_d + \log S - \log P_s \};
\]

And either:

The options equivalent model (as in equation (8.8d))

\[
(R - P) \times SF_b(P - R) = [R \times \Phi(-d_1) - P \times \Phi(-d_2)]
\]

\[
\ln\left(\frac{R}{P}\right) + \frac{\sigma^2}{2}, \quad d_2 = d_1 - \sigma;
\]

or: the logistic fuzzy model (as in appendix E)

\[
SF_b(x) = \frac{1}{1 + e^{-x/\lambda}}.
\]
All subject to $h_1 \geq 0$, $h_2 \geq 0$, $h_3 \geq 0$, $h_4 \geq 0$ and $h_1+h_2+h_3+h_4 \leq 1$.

Comments:

(8.8a): For the objective function, $b$ is initially set as 8, for reasons explained in the calibration subsection. However, this parameter is also varied to examine responses to differing degrees of risk aversion including different settings of $P$ (see below).

(8.8b): Derived from equations (8.5) and (8.7). The problem is to find the optimal forward rate weights $h_i$.

(8.8c): The critical parameter $P$ is based on the historical lower 10% VaR point for the natural exposure, i.e. the historical exchange rate $S$ is used in place of any hedged exchange rate. In effect, $P$ is located historically at the most uncomfortable zone for the dairy farmer and $P_\alpha$ is for the historical VaR point at $\alpha$% level. Thus $P_{10} = -1.8578$, with an alternative at $P_5 = -1.8804$ as a slightly more relaxed stance; $P_{20} = -1.8142$ is also employed.

(8.8d): The results reported below are based on the put option equivalent version of the fuzzy utility function. This assists with the convergence of the numerical algorithm, as previously noted. The option pricing model is primarily employed for smoothing the segmented welfare function. Thus, in the above option pricing model, interest rate $r$ is assumed to be zero while the maturity $T$ is assumed to be one. The volatility can be set as a small number $\sigma = 0.025$. The logistic fuzzy minimum with $\lambda =0.01$ was also utilised as a check, with broadly similar results.

The expected values appearing in the above maximisation problem (equation 8.8a) were estimated as sample averages of the corresponding magnitudes. It is not necessary for the observations on $R$ to be i.i.d. over time. It does not require that log exchange rates or commodity prices are necessarily stationary. The only assumption is the law of large number. It implies that the sample values for the objective function converge, as the sample size becomes large, uniformly with probability one in the hedge parameters $h$ in a neighbourhood of the true optimum. Inspection suggests that this is true in the present context.

The above formulation assumes a rolling hedge spanning a year ahead. A longer term hedge horizon out to 3 years ahead – in this case the forwards $F$ are for 1, 2 and 3 years, will also be investigated. Longer horizon is not considered in the current context as the wavelet decomposition of corporate exposures show buffering effects dominate the time series over an extensive period (see Chapter 7).
8.3.3 Data and computation

Data is monthly, reflecting the availability of the commodity prices series. It spans the period Aug 89 to Feb 07, giving 211 monthly observations. Spot and forward exchange rates are obtained from Thomson Financial Datastream as the mid rate in each case. However, 1, 2 and 3 year forwards were constructed synthetically from covered interest parity, rather than being direct market data. For this purpose, interest rate swap rates were used after 1999 when they became available, while before this date swap rates were estimated by adding a credit spread to the government bond rates. The credit spread is derived from the average difference between swap rates and government bond rates from 1999 to 2007. The constructed swap rate was validated by comparing with the post 1999 period. Zero coupon rates bootstrapped up from the swap rates were then used to compute the longer dated foreign exchange forwards. Data for long term hedging was slightly restricted to span July 90 to Feb 07. Dairy price data is derived from ANZ Bank data series used to construct their commodity price indices. The monthly farmer expense price index is interpolated using quarterly data from Statistics NZ, the official statistical agency.

The computational method used to solve the decision problem (8.8) is \textit{fmincon} from Matlab. It was necessary to smooth the objective function as earlier described, and once this is done, the routine converges quickly.

8.3.4 Hedging results

\textit{One-year horizon}

Table 8.1 shows how the optimal weights vary with changing the $P$ and $b$ parameters to indicate different degrees of lower tail risk aversion. Points $P_5 = -1.8804$, $P_{10} = 1.8578$, and $P_{20} = -1.8142$ refer to VaR points of the unhedged distribution as a way of locating discomfort points. Note that these will not necessarily be VaR points of the hedged distributions, and the latter will be given as output diagnostics. The weights $h_i$ are formally expressed as percentages.

As anticipated, low values of the penalty parameter $b$ amount to an absence of effective risk aversion and the result is to use only the one-year forward rate. As $b$ rises, the weight given to the one-year forward diminishes, though it always remains appreciable, and a more diversified portfolio of nearer months appears. A small spot exposure is called for where risk aversion is substantial, while near months are also weighted in the case of $P = P_{10}$ which is treated as the default case in the following.
### Table 8.1 Variation of optimal weights with tail risk aversion

<table>
<thead>
<tr>
<th>$P_5 = -1.8804$</th>
<th>Optimal weights $h \rightarrow$</th>
<th>Spot</th>
<th>3-M F</th>
<th>6-M F</th>
<th>9-M F</th>
<th>12-M F</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b \downarrow$</td>
<td></td>
<td>0</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>100.00%</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>-17.25%</td>
<td>20.12%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>97.13%</td>
</tr>
<tr>
<td>18</td>
<td></td>
<td>0.92%</td>
<td>11.05%</td>
<td>2.84%</td>
<td>1.26%</td>
<td>83.92%</td>
</tr>
<tr>
<td>50</td>
<td></td>
<td>5.68%</td>
<td>9.82%</td>
<td>0.00%</td>
<td>5.27%</td>
<td>79.23%</td>
</tr>
<tr>
<td>100</td>
<td></td>
<td>8.46%</td>
<td>9.92%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>81.62%</td>
</tr>
<tr>
<td>200</td>
<td></td>
<td>7.33%</td>
<td>12.85%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>79.82%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$P_{10} = -1.8578$</th>
<th>Optimal weights $h \rightarrow$</th>
<th>Spot</th>
<th>3-M F</th>
<th>6-M F</th>
<th>9-M F</th>
<th>12-M F</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b \downarrow$</td>
<td></td>
<td>0</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>100.00%</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>9.84%</td>
<td>10.42%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>79.74%</td>
</tr>
<tr>
<td>18</td>
<td></td>
<td>17.74%</td>
<td>8.14%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>74.13%</td>
</tr>
<tr>
<td>50</td>
<td></td>
<td>23.26%</td>
<td>8.11%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>68.63%</td>
</tr>
<tr>
<td>100</td>
<td></td>
<td>25.43%</td>
<td>9.20%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>65.37%</td>
</tr>
<tr>
<td>200</td>
<td></td>
<td>26.05%</td>
<td>10.46%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>63.49%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$P_{20} = -1.8142$</th>
<th>Optimal weights $h \rightarrow$</th>
<th>Spot</th>
<th>3-M F</th>
<th>6-M F</th>
<th>9-M F</th>
<th>12-M F</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b \downarrow$</td>
<td></td>
<td>0</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>100.00%</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>12.34%</td>
<td>17.58%</td>
<td>10.77%</td>
<td>4.19%</td>
<td>55.12%</td>
</tr>
<tr>
<td>18</td>
<td></td>
<td>19.57%</td>
<td>14.55%</td>
<td>9.10%</td>
<td>5.06%</td>
<td>51.72%</td>
</tr>
<tr>
<td>50</td>
<td></td>
<td>24.30%</td>
<td>11.29%</td>
<td>10.27%</td>
<td>2.18%</td>
<td>51.96%</td>
</tr>
<tr>
<td>100</td>
<td></td>
<td>25.48%</td>
<td>10.64%</td>
<td>10.91%</td>
<td>1.07%</td>
<td>51.90%</td>
</tr>
<tr>
<td>200</td>
<td></td>
<td>26.27%</td>
<td>10.14%</td>
<td>11.12%</td>
<td>0.56%</td>
<td>51.92%</td>
</tr>
</tbody>
</table>

Hedge performance can be depicted in several ways. Table 8.2 compares the optimised portfolio for $b=18$ and $P_{10}$ against simple hedges involving just one forward, the unhedged outcome and spot exposures. A simple one-year forward has a
higher mean, as expected, but is more adverse with respect to the VaR, CVaR and expected utility of the hedged distributions.

### Table 8.2 Outcomes for log terms of trade

<table>
<thead>
<tr>
<th></th>
<th>Unhedged</th>
<th>1-Y Forward Fully hedged</th>
<th>Optimised portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>P = P_{10}, b = 18</strong></td>
<td>LOG FTT</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td>-1.6959</td>
<td>-1.6649</td>
<td>-1.6735</td>
</tr>
<tr>
<td><strong>Variance</strong></td>
<td>0.0222</td>
<td>0.0165</td>
<td>0.0152</td>
</tr>
<tr>
<td><strong>10% VaR</strong></td>
<td>-1.8578</td>
<td>-1.8348</td>
<td>-1.8241</td>
</tr>
<tr>
<td><strong>10% CVaR</strong></td>
<td>-1.8879</td>
<td>-1.8697</td>
<td>-1.8481</td>
</tr>
<tr>
<td><strong>E[U]</strong></td>
<td>0.1079</td>
<td>0.1634</td>
<td>0.1796</td>
</tr>
</tbody>
</table>

Alternatively, one can compare the optimally hedged portfolio with the unhedged, natural outcome, taking the latter as benchmark. Figure 8.6 is an ordered mean difference (OMD) plot (Bowden, 2000, 2005a) choosing the optimum portfolio for \( b=18 \) and \( P_{10} \). The height at each value of the benchmark represents the investor or managerial surplus that results relative to the unhedged position. Lower values on the horizontal axis represent a more risk averse investor. The OMD plot is presented with 95% confidence bands by adjusting the value with 1.96 \( \sigma \) on each side. The uniform positivity of the OMD schedule shows that when taken over the entire period, the optimised portfolio would be better to remain unhedged, by any risk averse investor. On the other hand, the optimised technique does not go so far as to stochastically dominate the unhedged position.

![Figure 8.6 OMD plot for optimised portfolio against unhedged as benchmark](image-url)
Superior performance over the entire period does not guarantee superiority over sub-intervals and this issue may be of importance at times. Figure 8.7 is a historical comparison of the optimised farmer terms of trade $R$ (with $P_{10}$ and $b=18$) as the lighter line, and the natural or unhedged outcome as the darker. The optimised portfolio is generally better on both the downside and the upside. However, it would not have recovered as quickly after 1997-1998 as did the natural unhedged position, though the recovery in the latter was short-lived.

![Figure 8.7 Historical time paths, hedged v unhedged](image)

**Three year horizon**

Table 8.3 gives the optimal weights in the three-year hedging context. The predominant influences come from the one-year forward, with a negligible gain from hedging with three-year forward. The reason lies in the correlation between the three components of the term of trade index. Table 8.4 reveals that, though the historical average of three-year forward rate is higher than the one-year forward rate, the regression coefficient for the former against the [commodity price/expense price] is positive while that for the latter is negative. Use of the three-year forward hedged portfolio will therefore weaken the natural hedging existing among the spot rate or short term forward rate and commodity price. By breaking the natural buffering effect, long term forward hedging will expose the corporate to more volatile cash flows as well as higher downside risk. A risk neutral exporter might still prefer three-year forwards to hedge the currency exposure, but risk averters will tend to weight more heavily the one-year forward.
Table 8.3 Variation of optimal weights with tail risk aversion

<table>
<thead>
<tr>
<th>$P_5$</th>
<th>Optimal weights $h$</th>
<th>Spot</th>
<th>1-y</th>
<th>2-y</th>
<th>3-y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.8824</td>
<td>$b \downarrow$</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>100.00%</td>
</tr>
<tr>
<td>5</td>
<td>0.00%</td>
<td>51.62%</td>
<td>0.00%</td>
<td>48.38%</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.00%</td>
<td>58.03%</td>
<td>0.00%</td>
<td>41.97%</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>0.00%</td>
<td>64.37%</td>
<td>5.73%</td>
<td>29.90%</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>0.00%</td>
<td>66.40%</td>
<td>7.90%</td>
<td>25.70%</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>2.12%</td>
<td>65.83%</td>
<td>9.42%</td>
<td>22.63%</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$P_{10}$= -1.8590</th>
<th>Optimal weights $h$</th>
<th>Spot</th>
<th>1-y</th>
<th>2-y</th>
<th>3-y</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b \downarrow$</td>
<td></td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>100.00%</td>
</tr>
<tr>
<td>5</td>
<td>0.00%</td>
<td>61.91%</td>
<td>0.00%</td>
<td>38.09%</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.00%</td>
<td>67.38%</td>
<td>0.00%</td>
<td>32.62%</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>4.73%</td>
<td>71.79%</td>
<td>2.40%</td>
<td>21.08%</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>14.52%</td>
<td>67.81%</td>
<td>8.48%</td>
<td>9.19%</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>17.00%</td>
<td>67.24%</td>
<td>8.81%</td>
<td>6.95%</td>
<td></td>
</tr>
</tbody>
</table>

Table 8.4 Comparisons among forward rates and spot rate

<table>
<thead>
<tr>
<th></th>
<th>spot rate</th>
<th>1y forward rate</th>
<th>2y forward rate</th>
<th>3y forward rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Historical mean</td>
<td>1.7195</td>
<td>1.7793</td>
<td>1.8423</td>
<td>1.8986</td>
</tr>
<tr>
<td>Regression coeff. on commodity price/expense price (brackets are $t$ values)</td>
<td>-9.0272 (-7.5858)</td>
<td>-10.9537 (-10.2858)</td>
<td>-3.4008 (-2.7409)</td>
<td>4.7757 (4.0002)</td>
</tr>
</tbody>
</table>
### Table 8.5 Outcomes for log terms of trade

<table>
<thead>
<tr>
<th>P = P_{10}, b = 18</th>
<th>Unhedged</th>
<th>1-Y Forward Fully hedged</th>
<th>2-Y Forward Fully hedged</th>
<th>3-Y Forward Fully hedged</th>
<th>Optimised portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LOG FTT</td>
<td>Forward</td>
<td>Forward</td>
<td>Forward</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-1.7027</td>
<td>-1.6695</td>
<td>-1.6362</td>
<td>-1.6067</td>
<td>-1.6608</td>
</tr>
<tr>
<td>Variance</td>
<td>0.0222</td>
<td>0.0168</td>
<td>0.0322</td>
<td>0.0489</td>
<td>0.0170</td>
</tr>
<tr>
<td>10% VaR</td>
<td>-1.8590</td>
<td>-1.8381</td>
<td>-1.8697</td>
<td>-1.9185</td>
<td>-1.8268</td>
</tr>
<tr>
<td>10% CVaR</td>
<td>-1.8894</td>
<td>-1.8713</td>
<td>-1.9558</td>
<td>-1.9979</td>
<td>-1.8544</td>
</tr>
<tr>
<td>E[U]</td>
<td>0.1016</td>
<td>0.1598</td>
<td>0.0468</td>
<td>-0.0121</td>
<td>0.1906</td>
</tr>
</tbody>
</table>

The result that hedging with three year forward increase rather than decrease the firm’s risk exposure is consistent with the conclusion drawn by Froot (1993). Froot tests the sample set of US financial returns from the perspective of British international investors and finds that long term currency hedging tends to increase the variance of total return. Similar phenomenon can be observed from table 8.5 that 3-year forward hedged NZ farmer terms of trade has a much higher variance and GVaR indicator than the unhedged exposures. The explanation provided by Froot (1993) for the poor performance of long term hedging also lies in the natural hedging effect which arises from the correlations among return components and prevailing over a long period.

#### 8.3.5 Out of sample performance

Tests of potential robustness separate the estimation period from the evaluation period and can have different forms.

**Parameter stability and its effects**

Separation of the estimation period from the subsequent evaluation period does in general result in different hedge ratios, depending on the incidence of tail events in the chosen sample period. Thus if the sample period is constrained to end in May 1998, one finds that optimal hedge ratio is biased towards 100% one year forward, instead of the more distributed pattern noted in Table 8.1. The reason is that a major event has been excluded, namely the fallout from the Asian Crisis, which saw the NZ dollar plunge over the next two years. Relying exclusively on forward sales of the US dollar would have been unwise. By the same token, one should include events spanning any critical or ‘pain’ point that might otherwise result from the exclusive use
Sequential or embedded time hedges

A second testing procedure for robustness corresponds to that employed by Chan, Gan and McGraw (2003) to examine empirical hedging effectiveness. The out of sample analysis for short term hedging begins with a sample estimation period spanning Aug 1989 to Jan 1999, at which a sufficient number of points below the pain point was included. The estimated optimal weights from the initial sample were used for hedging against the spot exposure at Jan 2000 (in one-year hedging). This was then rolled forward one month at a time. Thus the next estimation was based on the period from Aug 1989 to Feb 1999 and the hedged exposure at Feb 2000. At each step, a longer sample period was used for the weights estimation, the idea being to imitate the way that things might be done in practice.

Table 8.6 shows the diagnostics for the shorter term hedging. The optimised technique is marginally inferior on VaR to one year forward hedging, though it has a better CVaR. Both are markedly superior to the unhedged position.

### Table 8.6 Sequential hedging performance comparison

<table>
<thead>
<tr>
<th></th>
<th>Unhedged</th>
<th>1-Year forward hedging</th>
<th>Optimised hedging (P10 =-1.86, b =18)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>-1.6969</td>
<td>-1.6450</td>
<td>-1.6685</td>
</tr>
<tr>
<td><strong>Variance</strong></td>
<td>0.0344</td>
<td>0.0152</td>
<td>0.0173</td>
</tr>
<tr>
<td><strong>10% VaR</strong></td>
<td>-1.8730</td>
<td>-1.8168</td>
<td>-1.8230</td>
</tr>
<tr>
<td><strong>10% CVaR</strong></td>
<td>-1.9103</td>
<td>-1.8624</td>
<td>-1.8523</td>
</tr>
</tbody>
</table>

Things change with the longer 3-year horizon. The optimised and unhedged positions are now both superior to the straight 3-year forward. The optimised strategy has the smallest VaR and CVaR and is superior on the mean as well. Table 8.7 illustrates.
Table 8.7 Sequential hedging performance comparison

<table>
<thead>
<tr>
<th></th>
<th>Unhedged</th>
<th>3-Year forward hedging</th>
<th>Optimised hedging (P&lt;sub&gt;10&lt;/sub&gt; = -1.85, b =18)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>-1.7227</td>
<td>-1.5375</td>
<td>-1.6361</td>
</tr>
<tr>
<td><strong>Variance</strong></td>
<td>0.0307</td>
<td>0.0559</td>
<td>0.0136</td>
</tr>
<tr>
<td><strong>10% VaR</strong></td>
<td>-1.8799</td>
<td>-1.8673</td>
<td>-1.8059</td>
</tr>
<tr>
<td><strong>10% CVaR</strong></td>
<td>-1.9146</td>
<td>-1.9906</td>
<td>-1.8496</td>
</tr>
</tbody>
</table>

Finally, the bad zone is powered up as in equation (8.2) using the fuzzy logistic smoothing as in Appendix E. As expected, a value of $\gamma = 1.5$ or 2 resulted in a stronger tendency to hold to straight one-year forwards at the expense of any spot exposure.

### 8.4 Unconditional hedging for importers

All the above discussions were explored from the exporters’ perspective. The following discussion will apply the unconditional hedging framework as above to importers. The purpose of this application is to examine whether importers can derive similar hedging decisions as those obtained by exporters, given the fact that they have opposite preference as to the direction of currency movement. The selected application is to New Zealand importers.

Owing to the difficulty of acquiring data regarding the sale price and importing product price for a specific importer, the exposures in the current discussion are constrained to one variable, namely exchange rate. However, it should be noted that the composed indicator, namely corporate terms of trade, as illustrated in Chapter 6, is also applicable to importers. The correlation between the price of imported products and exchange rate changes may play an important role in determining a firm’s whole risk profile. One example is Air New Zealand’s exposure to the risk in oil prices and exchange rates (Air New Zealand Annual Report, 2005). In a competitive environment, Air NZ can not pass the risk to customers. A high oil price to be accompanied with a strong NZ dollar, as happened in 2004-2005, may not affect the company greatly. But the worst thing occurs when a soaring oil price is accompanied with a weak NZ dollar. The hedging against currency movements becomes extremely important in such a case.
8.4.1 Expected managerial utility function

For importers exposed to the currency risk, the objective function can be written in a similar way as that in equation (8.1). The difference lies in the fact that the currency exposure in the current context does not come from revenue but from costs. Corporate managers will prefer lower expected costs and the corresponding adversity option turns to be a call option with the payoff positively correlated with the cost. In order to make the hedging framework derived in the previous discussions also applicable for importers, one solution is to replace the variable \( R \) with the negative exchange rate \( -S \), where 1USD=\$NZD, which is consistent with other applications. Another method is to use the inverse exchange rate with the NZ dollar as the commodity currency, e.g. 1NZD=\$USD. Importers favour a higher \( E \) as a strong NZ dollar makes foreign goods cheaper. The following application is based on the second method, in which exchange rate is represented with \( E \), as it is convenient when there is a need to take the log of the variable \( \log S = -\log E \). Simultaneously, the forward rate should be expressed in the same way as the spot rate, with the NZ dollar as the commodity currency. The optimal hedging decision for importers on the basis of first method should be similar with that obtained by this method.

The critical point \( P \) can be derived from the history of NZD/USD, e.g. the 5% VaR point. Considering the difference between history and future, a 10% lowest historical value can be regarded as the 5% lowest price of future variable. The specification of another risk parameter \( b \) is same as the discussion for exporters’ unconditional hedging. A 5% VaR critical point is related to \( b = 18 \), while a 10% VaR reference level is combined with \( b = 8 \).

8.4.2 Hedging decision

The hedging process still follows a roll-over hedging strategy by spreading the forward maturity from three months to one year. By analyzing the relation between spot and forward rate of NZD/USD, it is expected that the historical average hedging ratio for NZ importers should be smaller than that for exporters. As discussed earlier, there is usually a forward discount on the NZ dollar. Thus, the current spot rate tends to be more favourable than the forward rate for importers. Only when the local currency is likely to substantially decrease in value can, the importers benefit from using forwards. On the other hand, importers averse to the downside risk are still
likely to benefit from the usage of forwards to some extent because forwards protect hedgers from the risk of bankruptcy.

**Decision problem**

The decision for importers can be derived in the same way as exporters except the expression for $R$. Since the term of trade is not applicable in the current context, equation (8.8c) can be modified as:

$$R = (1-h_1-h_2-h_3-h_4) \log E + h_1 \log F_1 + h_2 \log F_2 + h_3 \log F_3 + h_4 \log F_4.$$

The critical point can be set as historical lower 10% or 5% VaR point of the natural exchange rates $E$, with $P_{5} = \text{VaR}_{5\%} = -0.8641$ and $P_{10} = \text{VaR}_{10\%} = -0.7788$. The risk aversion parameter $b$ also varies from 0 to 200 for each choice of $P$. The problem is to find the optimal forward rate weights $h_i$. The put option pricing formula will still be employed to smooth the hedging objective function, with the volatility indicator $\sigma = 0.025$.

**Hedging outcome**

The optimal weights shown in table 8.8 have a different pattern from that for exporters. For a zero beta, the hedgers are assumed to be risk neutral and remaining unhedged turns to be the optimal decision as the historical average spot rate is higher than the forward rate. For importers, the weights given to longer duration forwards increase and the proportions of unhedged position diminish when the risk parameter $b$ increases. On the other hand exporters will tend to increase the unhedged weights as $b$ increases. However, there are also some similarities between importer and exporter hedging outcomes. For both of them, the hedged portfolio tends to be more diversified when the company becomes highly risk averse. Even though there is a chronic forward discount, forwards still play an important role in importers’ optimised portfolio. The hedge ratio is almost 50% when the main concern of the corporate is to avoid the distress risk.
Table 8.8: Variation of optimal weights with tail risk aversion

<table>
<thead>
<tr>
<th>$P_5 = -0.8641$</th>
<th>Optimal weights $h$</th>
<th>Spot</th>
<th>3-M F</th>
<th>6-M F</th>
<th>9-M F</th>
<th>12-M F</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b \downarrow$</td>
<td></td>
<td>0</td>
<td>100.00%</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>70.72%</td>
<td>21.28%</td>
<td>0</td>
<td>0</td>
<td>8.00%</td>
</tr>
<tr>
<td>18</td>
<td></td>
<td>70.83%</td>
<td>14.71%</td>
<td>1.06%</td>
<td>0</td>
<td>13.41%</td>
</tr>
<tr>
<td>50</td>
<td></td>
<td>75.19%</td>
<td>0</td>
<td>4.07%</td>
<td>2.58%</td>
<td>18.16%</td>
</tr>
<tr>
<td>100</td>
<td></td>
<td>74.29%</td>
<td>0</td>
<td>4.06%</td>
<td>2.54%</td>
<td>19.11%</td>
</tr>
<tr>
<td>200</td>
<td></td>
<td>73.09%</td>
<td>0</td>
<td>4.51%</td>
<td>2.65%</td>
<td>19.74%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$P_{10} = -0.7788$</th>
<th>Optimal weights $h$</th>
<th>Spot</th>
<th>3-M F</th>
<th>6-M F</th>
<th>9-M F</th>
<th>12-M F</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b \downarrow$</td>
<td></td>
<td>0</td>
<td>100.00%</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>67.39%</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>32.61%</td>
</tr>
<tr>
<td>18</td>
<td></td>
<td>56.29%</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>43.71%</td>
</tr>
<tr>
<td>50</td>
<td></td>
<td>50.87%</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>49.13%</td>
</tr>
<tr>
<td>100</td>
<td></td>
<td>49.96%</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>50.04%</td>
</tr>
<tr>
<td>200</td>
<td></td>
<td>49.56%</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>50.44%</td>
</tr>
</tbody>
</table>

Table 8.9: Outcomes for log exchange rate

<table>
<thead>
<tr>
<th>$P = P_{10}, \beta = 18$</th>
<th>Unhedged</th>
<th>1-Y Forward Fully hedged</th>
<th>Optimised portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.5526</td>
<td>-0.5836</td>
<td>-0.5662</td>
</tr>
<tr>
<td>Variance</td>
<td>0.0230</td>
<td>0.0211</td>
<td>0.0185</td>
</tr>
<tr>
<td>10% VaR</td>
<td>-0.7788</td>
<td>-0.8029</td>
<td>-0.7998</td>
</tr>
<tr>
<td>10% CVaR</td>
<td>-0.8607</td>
<td>-0.8782</td>
<td>-0.8350</td>
</tr>
<tr>
<td>E[U]</td>
<td>0.0725</td>
<td>0.0059</td>
<td>0.1072</td>
</tr>
</tbody>
</table>

Table 8.9 compares the hedging outcomes of the optimised portfolio with those of benchmarks, including unhedged, forward fully hedged portfolio. It can be seen that the spot rate has a higher mean and lower downside risk relative to the forward rate while the 1-year forward rate is related with the lowest mean and highest extreme value. That is why the unhedged spot position accounts for a large proportion of an importer's optimised portfolio when the main purpose is to increase the expected return. However, managers’ averse attitudes to the downside risk imply that importers
will still retain some spot exposure. The optimal hedging provides the smallest CVaR while the mean is still relatively attractive.

### 8.5 Conditional hedging upon the currency directional forecasting

Corporate managers who have superior information, expertise, or consultants for the financial market movement may seek to improve the hedging outcomes by continuously integrating the new information into the risk management decision and adjusting the hedging strategy over time. This form of hedging, namely conditional hedging, largely depends upon the corporate’s prediction of future financial price or rate changes. Since the exchange rate forecasting algorithm differs according to the forecasting horizon, discussions around conditional hedging outcomes are divided into two sections. The current one considers the hedging upon the currency directional forecasting in a relatively longer term context while the subsequent section describes the hedging outcome on the basis of the exchange rate forecasting in a much shorter term context.

As discussed in Chapter 6, directional calls are often more successful when the time horizon of forecasting and conditional hedging are extended to encompass macroeconomic fundamentals. Therefore, the hedging ratio over a longer time period can be based on the forecasting for the direction of currency fluctuation rather than attempting to predict its precise value. In this case, managers might feel that if a directional probability is high, the value of the movement itself is likely to be appreciable and should be acted upon. However, it may be not true as the probability could be higher simply because the variance is lower, without materially affecting the expected size of the movement. Nonetheless, if the probability of a rise in the home exchange rate is assessed as, for example, 0.9, then export managers would load up on foreign exchange forwards to protect their foreign currency receipts, on the grounds that the size of the movement is also likely to be significant.

The general problem then is how to map directional calls and probabilities into actions. A first step is to establish a suitable welfare or loss function associated with possible outcomes. Loadings can then be devised that weight the directional probabilities according to the welfare consequences of state transitions, so that the resulting hedge ratios reflect not only directional probabilities, but also welfare outcomes. As discussed earlier, the welfare function is chosen as the GVaR criterion that is described in equation (8.1). The welfare function employed is the default case,
with \( b=18 \) and \( P=-1.8578 \), since the current application is proposed mainly to examine the value enhanced by actively using information in hedging.

The resulting decision problem can also be used to inform the statistical inference associated with variable selection in the estimation phase. The link between forecast and action has also been addressed by Leitch and Tanner (1991), Diebold and Mariano (1995), and Leung et al (2000).

### 8.5.1 Hedging framework

With directional forecasting, estimation of outcome probabilities is to be used as an input into the hedge ratios that will protect the firm against an adverse outcome, in this case of the exchange rate, over the coming horizon. Hedging is to be done by buying or selling the foreign currency forward. Let \( h_t \) be the hedge ratio as a decision variable, meaning the proportion of the spot exposure that is to be protected. It is assumed that treasury policy requires \( 0 \leq h_t \leq 1 \). The case \( h_t = 1 \) would correspond to complete forward cover. As mentioned earlier, this kind of hedging policy takes advantage of the usual forward discount on the NZD or the generally resulting premium on the USD. However, on occasions it would have been inappropriate, with unprotected spot as much the better choice (see above unconditional hedging outcome and following conditional hedging results). The issue is whether a better hedge strategy could be designed by allowing \( h_t \) to be variable, constituting an active rather than passive hedge rule.

At the current time \( t \) suppose that the estimated regime probabilities for the coming forecast period (e.g. \( t+1 \) or \( t+4 \)) are

\[
\hat{p}_{j+1} = P(R_{j+1} | Z_t, \hat{\beta}), \ (j = 1, 2, 3).
\]

Similarly with \( t+4 \) for forecasting one year ahead, there are a number of ways of mapping these forecast regime probabilities into the desired hedge ratio \( h_t \).

One method is to fix as parameters, \( q_u \) and \( q_d \), based on the size of the up and down movements. These can be used together with the regime probabilities to mimic a trinomial process, setting the movement for third regime \( R_3 \) as zero. The values of \( q_u \), \( q_d \) can be estimated by a historical least squares fit of the generated series with the actual. Using the tick values and the estimated up and down probabilities, one can then estimate the expected value of the coming spot rate and adjust the hedge ratio according to the difference between the expected future spot rate and the currently quoted forward rate. One would expect this sort of technique to work better for very
short horizon hedging, where the trinomial tick process provides a better approximation.

Better optimisation results, however, can be obtained by making more use of the three possible outcome regimes, in conjunction with other information such as whether the exchange rate is currently high or low relative to history. The latter specification implicitly employs the rule of hedging with disequilibrium. Under such a rule, it is assumed that the asset prices should be consistent with economic fundamentals and any deviation from the fundamentals could be corrected in the near futures. Thus a more effective rule, in this particular context, is based on the observation that historical values of the NZD/USD exchange rate have fluctuated within a broad band, but one without any noticeable trend. Suppose the historical series can be divided into three zones: high, middle and low. The consequences for NZ exporters of the NZD/USD exchange rate moving up are worse if the exchange rate itself is already high. On the other hand, if the NZD/USD is at a historical low, exporters would be less troubled by the prospect of a further up movement, or even if the rate stayed the same. This generalisation needs to be considered along with the chronic forward rate discount of the NZD which would lead to a bias in favour of using the forward rate. One could imagine a $3 \times 3$ matrix of loading weights $\Lambda = \{ (\lambda_{ij}) \}$ proportional to the marginal utilities of movement up, down or stable ($j$), given that the current NZD/USD rate is at a historical high, medium or low level ($i$). For instance, $\lambda_{ij}$ would be the loading if the current state was in the high historical zone and the movement was stable.

The hedge rule would then be of the form

$$h_i = h_{i(t)} = \sum_{j=1}^{3} \lambda_{ij} \hat{p}_{j+1}.$$  \hfill (8.10)

If at time $t$ one observes that the current exchange rate is in historical zone $i$, then one weights the estimated direction probabilities with the loadings appropriate for zone $i$. The hedge ratio is the expected value of the loadings. The loading weights, collectively $\Lambda$, can be determined by maximising the chosen welfare function, assuming that rules reflected in (8.10) had been applied historically.

In order to economise slightly on the number of loading parameters to be estimated, it is assumed that the elements of the last row of $\Lambda$ are all equal. The last row refers to the good state for NZ exporters (low NZD), so one can assume a natural
tendency to simply use the forward rate no matter what the outcome is. The loadings to be estimated are constrained to the range $0 \leq \lambda_i \leq 1$, which will similarly constrain the hedge ratio.

### 8.5.2 Hedging outcome

Table 8.10 shows that the optimised loadings are either unity or zero, except the one regarding the down movement for a middle band spot rate. The differences are most marked in the high zone, where the unhedged exporter is suffering from the high NZ dollar. If the direction call is up, then the exporter would definitely want complete protection; but if down, then the exporter would want to be fully exposed to the spot rate and forego the relatively high forward altogether.

**Table 8.10 Optimised Hedge Loadings Elements**

<table>
<thead>
<tr>
<th>Current state/Movement</th>
<th>Up</th>
<th>Down</th>
<th>Stable</th>
</tr>
</thead>
<tbody>
<tr>
<td>High band spot rate</td>
<td>100%</td>
<td>4.16%</td>
<td>51.84%</td>
</tr>
<tr>
<td>Middle band spot rate</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Lower band spot rate</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Once the hedge ratios are ascertained, the effective conversion exchange rate can be computed for the exports, made up of the spot rate and the forward rate in proportions indicated by the hedge ratio for each time period. In turn, this can be used to calculate the dairy farmer’s hedged terms of trade, on a historical basis. This can be compared with the historical terms of trade without hedging, and also with that derived from using just the forward rate, unconditional hedging framework or logit model based hedging to make the conversion. It is useful to compare the present three regime model with the threshold hedging rule of Leung et al (2000), which is based on two-regime conditional logit or probit model. In the present context, two regimes are selected as ‘up’ versus ‘not up’ for the NZD. Of these, the logit performed better than probit, so comparison is confined to this case. The hedge rule suggested by Leung et al (2000) would be to set the logit estimator $p$ as the probability for NZD to appreciate. If $p > 0.7$, hedge with $h = 1$; if $p < 0.3$, not hedge $h = 0$; otherwise, half hedge $h = 0.5$.

Table 8.11 compares the outcomes using a number of common metrics. The VaR is the lower 10% critical point for the marginal distribution of the terms of trade, as though they all came from a common underlying distribution. The CVaR is the conditional expected value given that the terms of trade is less than the VaR critical value.
point, which is a measure of the mass in the left hand tail. The optimised hedge is superior on all other alternatives in terms of both mean and downside risk indicators.

**Table 8.11 Dairy Farmer terms of trade comparison**

<table>
<thead>
<tr>
<th></th>
<th>Unhedged</th>
<th>One year forward hedge</th>
<th>Unconditional optimised</th>
<th>Logit optimised</th>
<th>Conditional optimised</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-1.6947</td>
<td>-1.6614</td>
<td>-1.6707</td>
<td>-1.6573</td>
<td>-1.6568</td>
</tr>
<tr>
<td>Variance</td>
<td>0.0225</td>
<td>0.0179</td>
<td>0.0163</td>
<td>0.0158</td>
<td>0.0162</td>
</tr>
<tr>
<td>10% VaR</td>
<td>-1.8597</td>
<td>-1.8233</td>
<td>-1.8205</td>
<td>-1.8035</td>
<td>-1.8011</td>
</tr>
<tr>
<td>10% CVaR</td>
<td>-1.8815</td>
<td>-1.8634</td>
<td>-1.8424</td>
<td>-1.8360</td>
<td>-1.8322</td>
</tr>
<tr>
<td>E(U)</td>
<td>0.1214</td>
<td>0.174</td>
<td>0.1869</td>
<td>0.1909</td>
<td>0.2029</td>
</tr>
</tbody>
</table>

Figure 8.8 compares the history of the dairy farmer terms of trade under the three alternatives. The optimised outcome generally tracks the simple forward closely as the 100% hedge rule. However, there is useful divergence over the interval 1997-1999, which saw the NZD crumble in the aftermath of the Asian crisis. It was a bad idea to follow a simple forward strategy at this time, which locked in the high NZD exchange rate prior to the crash. The optimised strategy uses the economic information current at the time and elects to remain partially with the unhedged spot rate. Note that both the pure forward and optimised strategies managed to avoid the adverse effects of the high NZD from 2002 onwards.

![Figure 8.8 Historical comparison, one year hedging horizon](image-url)
Using 100% forward cover is a common recommendation from market efficiency proponents, so it provides a natural benchmark in the above comparisons. However, there remains the possibility that it can be improved on using alternative econometric techniques based on conditional information.

### 8.5.3 Diagnostic test

To assess the forecasts and the resultant hedging strategies, an out of sample test and a rolling estimation test are constructed. The former is to separate the forecasting period from the evaluation period. A rolling estimation test is similar with the one which was employed for testing the robustness of an unconditional optimal hedging. The models will be re-estimated each period when more information becomes available and the consequent hedging decisions therefore reflect all the current information.

#### Out of sample test

As already discussed, the forecast can be evaluated in terms of the consequent economic significance. The out-of-sample robustness of the directional forecasting model is now examined by addressing the issue of whether the forecasting leads to better hedging outcome. The parameters are estimated on the basis of the first 50 observations, which are related to periods from the Q3 1989 – indicating the third quarter in 1989, to the Q1 2002. The model is then applied for all the observations in the evaluation period that is from Q1 2003 to Q2 2007. The hedging decision is made one year in advance and thus the exchange rate forecasting based on the information available up to Q1 2002 is used to derive the hedging decision for Q1 2003. One quarter later, the hedging decision for Q2 2003 is based on the information available to managers on Q2 2002. However, the model would not be re-estimated by incorporating newly released information over the last quarter. The out of sample test results are described in the fourth column of the table 8.12.

#### Rolling estimation test

As Pesaran and Timmermann (1995) pointed out, the rational users of a forecasting model may adjust the specification when information is added along the time. This is why a single constant parameter model, as employed in the above out of sample test is not consistent with reality. A more sensible out of sample test is formulated, in which parameters are predicted on the basis of all the available information. The first model is still derived from the data over the period of Q3 1989 to Q1 2002 and the consequent exchange rate fluctuation direction forecasting facilitates the decision of
hedging against currency exposures on Q1 2003. After one quarter, observations over the Q3 1989 to Q2 2002 are used to forecast currency movement for Q2 2003.

Table 8.12 Sequential hedging performance comparison (Q1 2003 to Q2 2007)

<table>
<thead>
<tr>
<th></th>
<th>Unhedged</th>
<th>1-Year forward hedge</th>
<th>Out-of-Sample optimised</th>
<th>Rolling optimised</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-1.7750</td>
<td>-1.6532</td>
<td>-1.6656</td>
<td>-1.6545</td>
</tr>
<tr>
<td>Variance</td>
<td>0.0114</td>
<td>0.0210</td>
<td>0.0172</td>
<td>0.0190</td>
</tr>
<tr>
<td>10% VaR</td>
<td>-1.8698</td>
<td>-1.8546</td>
<td>-1.8211</td>
<td>-1.8208</td>
</tr>
<tr>
<td>10% CVaR</td>
<td>-1.8951</td>
<td>-1.8826</td>
<td>-1.8413</td>
<td>-1.8409</td>
</tr>
</tbody>
</table>

Table 8.12 illustrates the log of unhedged and hedged dairy farmer terms of trade from period Q1 2003 to Q2 2007. The optimised hedging outcome is compared with the natural exposure and one year forward fully hedging results. Both the out-of-sample test and the rolling estimation test show that the optimal hedging based on the directional forecasting are superior in all the risk measures. In addition, by incorporating new information into the estimation procedure along the time, the rolling optimised hedging outperforms the simple out-of-sample optimised hedging in terms of mean, VaR, and CVaR.

8.6 Short term hedging upon the M-GARCH forecasting

As discussed in Chapter 6, the M-GARCH could be employed for the exchange rate estimation over a short interval. Given the estimated mean and volatility through the M-GARCH model, a data set of 1000 possible future spot rate return is simulated. The optimal hedging ratio will then be derived from maximising the expected welfare function shown in the equation (8.1b), with the modification that only one month forward rate is employed as the hedging instrument. Table 8.13 compares the optimised portfolio for $b=18$, $P_{10}=-1.8578$ against some simple hedges, including natural unhedged exposures, 100% forward hedging, and 50% hedging policy.
Table 8.13 Farmer terms of trade comparison

<table>
<thead>
<tr>
<th></th>
<th>Optimal</th>
<th>Unhedged</th>
<th>1-Month Forward hedge</th>
<th>Half-half</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1.6660</td>
<td>-1.6714</td>
<td>1.6665</td>
<td>-1.6688</td>
</tr>
<tr>
<td>Variance</td>
<td>0.0246</td>
<td>0.0247</td>
<td>0.0244</td>
<td>0.0243</td>
</tr>
<tr>
<td>10% VaR</td>
<td>-1.8358</td>
<td>-1.8432</td>
<td>-1.8359</td>
<td>-1.8436</td>
</tr>
<tr>
<td>10% CVaR</td>
<td>-1.8716</td>
<td>-1.8834</td>
<td>-1.8748</td>
<td>-1.8785</td>
</tr>
<tr>
<td>E[U]</td>
<td>0.1582</td>
<td>0.1369</td>
<td>0.1518</td>
<td>0.1492</td>
</tr>
</tbody>
</table>

The optimised hedging outcome exhibit the highest mean but lowest risk measurement. It marginally outperforms the forward hedging policy, but is significantly superior to spot exposures.

The out-of-sample and rolling regression tests, illustrated earlier in the one year hedging context, are also employed to examine the potential robustness of the mean-GARCH exchange rate forecasting model and the effectiveness of the consequent hedging. Table 8.14 shows the diagnostic test results. Both tests show that the optimised hedging on the basis of the mean-GARCH exchange rate forecasting model is superior in terms of GVaR measures.

Table 8.14 Sequential hedging performance comparison

<table>
<thead>
<tr>
<th></th>
<th>Unhedged</th>
<th>1-Month Forward hedge</th>
<th>Out-of-Sample optimised</th>
<th>Rolling optimised</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-1.7806</td>
<td>-1.7702</td>
<td>-1.7710</td>
<td>-1.7724</td>
</tr>
<tr>
<td>Variance</td>
<td>0.0049</td>
<td>0.0052</td>
<td>0.0051</td>
<td>0.0046</td>
</tr>
<tr>
<td>10% VaR</td>
<td>-1.8699</td>
<td>-1.8395</td>
<td>-1.8395</td>
<td>-1.8395</td>
</tr>
<tr>
<td>10% CVaR</td>
<td>-1.8813</td>
<td>-1.8613</td>
<td>-1.8609</td>
<td>-1.8588</td>
</tr>
<tr>
<td>E[U]</td>
<td>0.0284</td>
<td>0.0646</td>
<td>0.0647</td>
<td>0.0681</td>
</tr>
</tbody>
</table>

8.7 Conclusions

This chapter has examined the empirical application outcomes of unconditional and conditional hedge for New Zealand dairy farmers. The unconditional hedge results indicate that New Zealand dairy exporters can benefit from currency forward hedging as the country has a higher interest rate relative to others and so there is chronic...
forward rate discount in New Zealand. However, the focus on a forward in this circumstance suggests only paying attention to the expected return. To be less exposed to the exceptional welfare losses, corporates favour a hedge distributed over time, including spot exposure as well as a series of forwards with various maturities, rather than relying wholly on forwards. The empirical hedging outcomes also show that long term forwards are not recommended for risk averse hedgers, even though the profits resulting from using forwards might grow along with the forward maturity time. Though hedging originally aims to reduce the downside risk, the empirical results in fact suggest that remaining unhedged to some extent is a response to risk aversion. The inclination to forward contracts instead, arises from a preference for simple expected return. In terms of importers, findings suggest that, even though the forward bias is unfavourable for importers, it is still useful to hedge with forwards to some extent. In this respect, the use of forwards contributes to the reduction in a corporate’s financial distress risk.

This chapter also explored the implementation of the conditional risk management, where corporate managers derive the currency hedging decision from their directional exchange rate forecasting. The optimal hedging ratio has been derived by incorporating the directional calls and probabilities into the chosen welfare function. In a conditional hedging context, the hedge decisions could vary over time when new information becomes available to decision makers. Dairy exporters tend to reduce hedge ratios in the face of a potentially favourable future exchange rate, and increase hedging proportions if the New Zealand currency is expected to go up. The diagnostic tests show that a conditional hedge enables value-improving hedges, especially when there is a significant change in currency movement.

Thus far in this discussion, the optimal hedge framework has been mainly applied to corporates such as dairy industries, for which the objective of hedging is to maximise the expected objective function with respect to a specific time point. As suggested earlier, the hedge decision process for strategic fund managers can be different from corporates. In the following chapter, the empirical application is made to superannuation funds. The hedging ratio is derived in association with the optimal portfolio decision. In this circumstance, currency derivatives are not only adopted as the hedging instrument, but also one asset class of an optimal global investment portfolio. In constructing an efficient portfolio, strategic fund managers, who
primarily aims to maximise the expected return, could also have a secondary constraint, namely the aversion to long term path risk, as was discussed in Chapter 4.
Part V Empirical Long Term Fund Risk Management

Chapter 9 Optimal Portfolio Risk Management for Long Term Fund Investment

The risk management framework for strategic fund managers differs from that applicable to managing corporate cash flow risk in several ways. One important way is that the currency component is not necessarily to be hedged away, but can be treated as an investment asset class in its own right, especially for currency overlay managers. The currency risk management strategy, in the present context, will therefore be developed in conjunction with the optimal portfolio decision. Another difference between the current application and the foregoing empirical discussion is that the risk management in this chapter is mainly focused on long term risk reduction. In a long term context, the paths of different asset classes follow quite different dynamic behaviour. The choice of alternative paths can be an important secondary consideration, if the primary objective is to maximise terminal wealth. Wavelets offer a convenient decomposition of a given time series into a number of components of successively longer time windows. The current discussion shows how to use wavelet analysis to resolve problems of detection, attribution and welfare measurement, including assigning volatility metrics and path risk.

In addition, when the focus of attention shifts to the longer run, one has to incorporate environmental influences, such as macroeconomics of business cycles, interest rates or exchange rates, economic policy and structural change. Considering the difficulty of using one-period return to capture the weak local dependence, which can be quite consistent with strong global dependence, either a value or an accumulated return can be chosen as the underlying variable for portfolio optimisation.

The chapter is structured as follows. Section 9.1 illustrates the background for long term risk management. Section 9.2 provides wavelet decomposition outcomes of some asset classes. Section 9.3 shows how to obtain an optimised portfolio. Section 9.4 contains the description of an optimal outcome. Section 9.5 briefly describes how
the portfolio construction framework can be applied to hedge funds. Section 9.6 concludes.\textsuperscript{24}

\textbf{9.1 Long term risk management}

Although many theoretical models of inter-temporal portfolio allocation have been devised, the long run situation is under-represented in practical portfolio theory and extensions to that theory such as hedging algorithms. The proposed long term risk management which follows does not assume that the future path will exhibit the same long term volatility pattern. Nevertheless, some assets may exhibit characteristic long-term volatility patterns, in the same way that business cycles and longer-term exchange rate variation have not yet vanished from world economies. There may be causality between economic behaviour and economic policy over a long term, even if precise causal models for such behaviour are difficult to quantify.

The first topic to be explored is how to measure these dynamic patterns over a long interval. The way in which they are measured should feed naturally into portfolio selection methodology. Wavelets, as an advanced filtering method, attempt to distinguish two paths with different cyclical patterns. With this approach, the specific feature of one path can be characterized by its multi-scale behavior. The wavelet indicator developed to account for various evolutionary paths enables fund managers to take the aversion to the path risk into account of long term risk management as a secondary objective welfare.

In a long term context, as discussed in Chapter 4, computation and use of one-period returns remain the predominant raw material for portfolio construction. Statistical decomposition of the rates of return typically shows energy increasing with detail – the higher the detail, the greater the energy. Such effect that is shown in the present data can be found in Appendix G. It therefore might be better in this context to resort to investment value or log value for measuring the underlying fund long term performance.

\textbf{9.2 Some wavelet energy decompositions}

The top row of table 9.1 gives the asset classes used to construct the illustrative portfolio, which is chosen as an international equity portfolio for a New Zealand investor wishing to invest in the US and other major stock markets. The total return

\textsuperscript{24} This chapter is largely based on Bowden and Zhu (2009).
Part V Empirical Long Term Fund Risk Management
Chapter 9 Optimal Portfolio Risk Management for Long Term Fund Investment

stock indexes as given refer to the own-currency log value, e.g. the US index is in US dollar terms, the Japanese component in yen.

The remaining asset appearing in table 9.1 is the total return index on a one-month forward contract on the US dollar against the NZ dollar. This corresponds to a portfolio long in zero coupon NZ bonds or bills\(^{25}\), short in USD, embodying the foreign exchange hedging component of the three-fund theorem of international finance (Solnik, 1974). The foreign exchange hedge component assumes particular significance when the stock returns are to be converted back to home currency (here the NZD) as noted above, as indeed they have to be in the present case. In that case the bond portfolio can be referred to as a currency hedge portfolio against the USD, and is in effect a forward contract.

The relevant monthly return on the hedge asset is defined by

\[
r - (1 + r^*) \frac{e_{1} e_{0}}{e_{0}} - r^*,
\]

(9.1)

where:

- \(r\) = NZ one-month bank bill rate as at start of month
- \(r^*\) = one-month US CD rate as at start of month
- \(e\) = exchange rate as 1USD = e NZD, i.e. with the US dollar as commodity currency and NZ dollar as terms: \(e_{1}\) = end of period rate, \(e_{0}\) = beginning of period.

The hedging monthly returns are then accumulated to form a total return index homologous with those used for the equity components of the portfolio. The object variable (or dependent variable) in each case is the log of the total return index.

Time series plots of the wavelet details are given in figures 9.1a, and 9.1b for two of the assets, namely NZ stocks and the forward rate hedge asset. The log value cycles are irregular both as to amplitude and period.

---

\(^{25}\)The one month NZ bank bill rate (BBR) is chosen, while the US short rate is taken as the certificate of deposit (CD) rate. Both are wholesale rates referring to high-grade bank credit.
Figure 9.1a Wavelet decomposition of NZ stock index accumulated return
One can measure the amplitudes of each cyclical component in the form of the wavelet variance on a scale basis, as discussed in Chapter 5. However, the above figures show that the cyclical pattern can be a local phenomenon, differing over time. An alternative is to compute the average variance over the given time horizon and present the results in the form of a table of average wavelet energies (AWE) at the different levels of detail.

Table 9.1 gives AWE decompositions for the asset classes used in the present study using monthly data from Jan 1988 to May 2007. All the data comes from MSCI. The Coif 5 wavelet is used, and computations were done in Matlab (Misiti et al 2005) using Mallat’s algorithm. Very similar results were obtained with the Sym10 filter.
Given 233 observations in total, the maximum detail available for the data run is level 7. The residual is taken to be the trend, though it may remain more complex than the standard log linear trend. Indeed this is one of the strengths of wavelet analysis, in that it makes no pre-judgements about the form of any underlying deterministic or stochastic trend.

Table 9.1 Average wavelet energy decomposition for asset classes

<table>
<thead>
<tr>
<th>Detail Level</th>
<th>Equivalent time period (years)</th>
<th>NZ stocks detail energy (%)</th>
<th>US stocks detail energy (%)</th>
<th>Japanese stocks detail energy (%)</th>
<th>Australian stocks detail energy (%)</th>
<th>USD/NZD forward detail energy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2417</td>
<td>2.29%</td>
<td>1.09%</td>
<td>1.88%</td>
<td>2.50%</td>
<td>0.58%</td>
</tr>
<tr>
<td>2</td>
<td>0.4833</td>
<td>2.75%</td>
<td>1.30%</td>
<td>2.28%</td>
<td>2.91%</td>
<td>0.96%</td>
</tr>
<tr>
<td>3</td>
<td>0.9667</td>
<td>4.51%</td>
<td>2.02%</td>
<td>4.31%</td>
<td>7.06%</td>
<td>1.06%</td>
</tr>
<tr>
<td>4</td>
<td>1.9333</td>
<td>6.48%</td>
<td>2.81%</td>
<td>12.20%</td>
<td>12.14%</td>
<td>2.19%</td>
</tr>
<tr>
<td>5</td>
<td>3.8667</td>
<td>4.94%</td>
<td>6.30%</td>
<td>20.43%</td>
<td>17.54%</td>
<td>8.44%</td>
</tr>
<tr>
<td>6</td>
<td>7.7333</td>
<td>57.55%</td>
<td>57.51%</td>
<td>54.63%</td>
<td>38.53%</td>
<td>43.11%</td>
</tr>
<tr>
<td>7</td>
<td>15.4667</td>
<td>21.49%</td>
<td>28.97%</td>
<td>4.27%</td>
<td>19.32%</td>
<td>43.66%</td>
</tr>
</tbody>
</table>

The power pattern in most cases shows an interior peak at level 6, which corresponds to an average cyclic period of 7.7 years. Note the higher energy of US stocks at longer run energies, especially detail 6. By way of contrast, applying the Coif and Sym filters to the standard geometric Ito model with constant drift reveals an absence of interior local energy peaks. In this case, power accumulates monotonically through different detail levels, effectively becoming a stochastic trend (Appendix H illustrates).

9.3 Portfolio choice

This section develops the band pass and related portfolio choices, implementing the ideas discussed in Chapter 4 and illustrating with the above asset classes. Generalising the familiar mean-variance portfolio construction, wavelet-based portfolios maximise long run reward subject to limits on the energy penalty ($\phi$). The collection of such points is referred to as the ‘reward-energy frontier’, analogous to the efficient frontier of mean variance.

By combining assets into a portfolio, one aims to achieve paths that are less risky for any given level of reward. In the wavelet based approach, it is chosen to carry out the portfolio analysis in terms of (log) values directly, and not one-period returns as such. However, there is a natural relationship between the two. If a set of assets $\{i\}$ have values $V_i$, then the asset proportions are $w_i = V_i / V$ and log
accumulation per period is taken as $\sum_i w_i \Delta \log V_i$ which is a portfolio return. In some formulations, return elements can appear in the objective function (see below). However, in what follows wavelet approximations (A) and details (D) refer to log portfolio value.

**9.3.1 Reward objective and path risk penalty**

In the context of wavelet analysis, the reward objective for long run investment is most naturally taken as some metric assigned to the high level approximations, such that higher values are preferred to lower values at any time point. Let $A^*$ denote the wavelet approximation of maximal order consistent with historical data availability, taken as $T$ observations. It may sometimes be loosely referred to as the trend. In the preceding sections this was taken as $A_7$, as in figures 9.1a, b. A suitable objective might then be of the form

$$\text{Max } \sum_{\tau=1}^T f(\tau) \Delta A^*_\tau,$$

which is a weighted sum of the historical value increments, with $\Delta A^*_\tau = A^*_\tau - A^*_{\tau-1}$.

In equation (9.2), $f(\tau)$ is a non-negative weighting function such that $\sum_{\tau=1}^T f(\tau) = 1$. It can be chosen to be in line with some preference between early or late return accumulation. For instance, a preference for earlier returns could be represented in the form of a discount factor. Some other useful special cases are as follows:

(i) The uniform weighting function $f(\tau) = 1/T$ all $\tau$, corresponds to the usual geometric rate of return over the whole horizon. Multiplying this by $T$ as in the above objective gives $A^*_T - A^*_0$. Hence the objective is simply to maximise the terminal value of the trend. If the wavelet decomposition is carried out on logs to begin with, then the objective is the compounded value growth. If the factor $T$ was missed in the objective (9.2), and logs were to be used, then the objective would be interpreted as the long run trend geometric average rate of return.

(ii) Take $f(\tau) \propto \rho^{T-\tau}$; \ 0 < $\rho$ < 1. The higher the value chosen for $\rho$, the more one weights later values of value growth. The idea behind this is that later value increments of the historical record might have more predictive content for what is to come in the present real time.
As discussed in Chapter 4, the path risk can be defined in an operational version shown in the equation (4.1). The path risk penalty function \( \varphi \) is assumed to be linear in the current application, with nonnegative coefficients \( \{w_k\} \), so the constraint is of the form:

\[
\varphi = \sum_k w_k E_k \leq \nu; \quad w_k \geq 0, \sum_k w_k = 1.
\]  

(9.3)

In equation (9.3), \( E_k \) denotes the average wavelet energy at detail level \( k \), and \( \nu \) is a user-assigned constant, interpreted as a path risk constraint. Thus, by setting some of the weights \( \{w_k\} \) to zero, and assigning heavy penalties to the others, the resulting portfolios will favour variation in the former, but not the latter. One could call these band pass portfolios, motivated by similar usage in electronic system design, where one filters out signals at designated frequencies, allowing others to pass through unhindered.

The empirical illustration below adopts the quasi VaR stance outlined in Chapter 4, where the fund manager is influenced by a need to avoid its investor’s fear of large-scale opportunity losses associated with long swings in value. In this case the longer details are penalised, and the weightings assigned to short run variation are small or even zero. However, the general band pass approach can handle quite different weightings. For example, a fund manager concerned that excessive short run fluctuations in a volatile market might unsettle investors, could elect \( w_k = 0 \) for \( k > 2 \). This would allow lower level details to pass through unhindered while penalising short run fluctuations \( (k = 1,2) \).

9.3.2 Optimisation problem and equivalent utility function

The optimisation problem is to choose the asset weights to maximise the equation (9.2) subject to (9.3). Also incorporated are standard portfolio constraints, such as asset proportions have to add up to unity if they require capital, or be non-negative if fund policy requires this. By varying the allowable energy parameter \( \nu \), and solving the resulting portfolio, one can trace out an efficient frontier in the same way as for classical mean-variance analysis (Markowitz, 1952, 1956, 1959).

An equivalent utility function is

\[
U = T \sum_{\tau=1}^{T} f(\tau) \Delta A_{s, \tau} - \nu \sum_k w_k E_k, \quad (9.4)
\]
where in the programming context, \( \eta \geq 0 \) is interpreted as a Lagrange multiplier. As noted in Chapter 4, there is a parallel with the mean-variance utility function, in this case via equation (9.4). Using the second mean value theorem, the effect of a weighted sum of energies is as though there is a single energy \( E^* \), say, which in turn has the dimension of a variance. So the equivalence relationship with mean-variance (denoted \( \sim \)) can be expressed as:

\[
\phi = \sum_k w_k E_k = E^* \sim \sigma_T^2
\]

\[\sum_{\tau=1}^T f(\tau) \Delta A_{\tau} \sim \mu_T.\]  

The equivalence of the weighted sum of energies with a variance is useful in choosing the allowable path risk constant \( \nu \) in the programming specifications (see below).

Efficient frontiers can be obtained by varying the allowable path risk parameter \( \nu \) and finding the portfolio that maximises the reward function (9.2) subject to the weighted energy constraint (9.3). Plotting the reward against \( \nu \), or loosely just \( \phi \), yields the efficient frontier incorporating the trade-off between reward and path risk. It will be referred to in what follows as the reward-energy efficient frontier.

### 9.4 Application

The portfolios are constructed with the asset classes appearing in table 9.1. These are intended to be operational portfolios, so each of the stock returns have been converted to home currency. The portfolios are constrained to non-negative proportions to the four country stock market portfolios, and there is a single zero capital element, namely the USD/NZD forward contract, which can be shorted, i.e. potentially have a negative portfolio weight. In the objective function, the time weight \( \rho \) is chosen as 0.9 which indicates that the value increment weight is largely assigned to the recent observations. For the energy weights \( \{w_k\} \), equal weights for details 4-7 are assumed but zero weights for energy levels 1-3, where investors are not concerned about short run fluctuations. By way of contrast, in standard mean-variance analysis, investors are assumed to be equally worried by short and long run power elements. In the present band pass application, power bands 1-3, i.e. the shorter run fluctuations, are allowed to pass freely.
Figure 9.2 depicts the resulting reward-energy frontier. The efficient frontier is strikingly similar to that of standard mean variance analysis, with the same parabolic shape extending into the lower inefficient half. As with mean variance, the trade-off (implied $\eta$ value) is higher as the energy bounds diminish.

Table 9.2 gives the optimal asset weights as one moves along the efficient frontier. Note that these do not have to add up to unity because of the presence of a zero capital element, namely the forward contract. Only the stock weights add up to unity. As the energy bounds become more restrictive, the optimal portfolio rebalances to Australian stocks (with lower long scale volatility), while the proportion devoted to US stocks decreases. It is also evident that the use of the USD/NZD forward contract diminishes. It seems to be contradictory to the belief that a forward is useful in diminishing the variance. However, it is consistent with the finding of Thorp (2005). It is also consistent with the conclusions for New Zealand dairy farmer hedging outcomes that the optimal forward hedging ratio declines when the corporate managers become more risk averse. Thus preserving an exposure to unhedged spot USD/NZD can be conservative risk management practice in Australasian fund industries.

It is of interest to see whether the similarity with mean-variance (MV) extends to the path properties of the optimal portfolios. If the two give similar results, one may favour the use of mean-variance analysis given its simplicity. If not, then the
issue of optimality and long-term stability would have to be addressed, with the reward energy efficient (REF) portfolio as a useful starting point.

Comparison of the two approaches can never be rigorous, as they refer to different reward or variation concepts, and standardisation is needed on one or the other. Two alternative comparisons along these lines are as follows.

(a) Normalise on the MV version of the reward i.e. a given mean return over the entire horizon. Select the corresponding portfolio along the REF frontier that generates this mean. Compare the time paths and energy decompositions of the two portfolios, including the path risk, defined for this purpose as the sum of the four longest detail energies as in equations (9.4) or (9.5a). Figure (9.3a) illustrates the two portfolios, while table (9.2a) gives the related energies and the respective portfolio compositions.

(b) Normalise on the path risk $\varphi$. Start with a MV efficient portfolio and calculate its path risk. Find the portfolio along the REF frontier that has the same path risk and compare its mean and reward with that of the original MV portfolio. Figure (9.3b) plots the two time paths of the resulting portfolios, while table (9.2b) contains the energies and portfolio compositions.

A third possible approach (not illustrated here) could be to normalise on the trade-off parameter $\lambda$ between reward and variation, with appropriate interpretation of these dimensions in the respective contexts. Variation would be taken as $\sigma^2$ for MV and as weighted energy for the REF portfolio.

Normalising on the mean as in (a) shows the smoothing effect of the wavelet based approach, reflected in the lower value for the path risk metric $\varphi$ in table 9.2a. The REF portfolio was slower to rise between 1996-2000, but with much less of a subsequent fall. The effect is apparent in the lower detail 6 energy and in the relative path risk (2.73 as against 3.68). In portfolio terms, it is produced by reducing a weighting of the US stocks component in favour of Australian stocks. The latter have virtually the same mean as the US, but materially lower long-term variation. The hedge proportion allocated to the US dollar has also diminished. The Japan weighting disappears altogether.

Normalisation (b), with fixed path risk, results in a higher accumulation path for the REF portfolio. As before, the Australian weight is increased at the expense of US stocks, but the tendency to use USD/NZD forwards remains roughly the same.
Table 9.2a Mean-normalised comparison between MV and REF portfolios

<table>
<thead>
<tr>
<th>Assets</th>
<th>Mean-variance efficient portfolio MV</th>
<th>Reward-energy efficient portfolio REF</th>
</tr>
</thead>
<tbody>
<tr>
<td>NZ</td>
<td>15.96%</td>
<td>18.81%</td>
</tr>
<tr>
<td>US</td>
<td>54.78%</td>
<td>19.62%</td>
</tr>
<tr>
<td>JP</td>
<td>9.90%</td>
<td>0.00%</td>
</tr>
<tr>
<td>AU</td>
<td>19.35%</td>
<td>61.57%</td>
</tr>
<tr>
<td>Forward</td>
<td>57.98%</td>
<td>20.22%</td>
</tr>
<tr>
<td><strong>Reward</strong></td>
<td>1.5966</td>
<td>1.6854</td>
</tr>
<tr>
<td><strong>Path risk ϕ</strong></td>
<td>3.6806</td>
<td>2.7333</td>
</tr>
<tr>
<td>Total detail energy</td>
<td>3.8985</td>
<td>3.0544</td>
</tr>
<tr>
<td><strong>Single Period Mean</strong></td>
<td>0.0098</td>
<td>0.0098</td>
</tr>
<tr>
<td><strong>Single Period Variance</strong></td>
<td>0.001183</td>
<td>0.0015</td>
</tr>
</tbody>
</table>

Figure 9.3a Path comparison, MV and REF portfolios: mean-normalised

Table 9.2b Energy-normalised comparison between MV and REF portfolios

<table>
<thead>
<tr>
<th>Assets</th>
<th>Mean-variance efficient portfolio MV</th>
<th>Reward-energy efficient portfolio REF</th>
</tr>
</thead>
<tbody>
<tr>
<td>NZ</td>
<td>15.96%</td>
<td>0.00%</td>
</tr>
<tr>
<td>US</td>
<td>54.78%</td>
<td>16.76%</td>
</tr>
<tr>
<td>JP</td>
<td>9.90%</td>
<td>0.00%</td>
</tr>
<tr>
<td>AU</td>
<td>19.35%</td>
<td>83.24%</td>
</tr>
<tr>
<td>Forward</td>
<td>57.98%</td>
<td>44.79%</td>
</tr>
<tr>
<td><strong>Reward</strong></td>
<td>1.5966</td>
<td>1.9511</td>
</tr>
<tr>
<td><strong>Path risk ϕ</strong></td>
<td>3.6806</td>
<td>3.6806</td>
</tr>
<tr>
<td><strong>Total detail energy</strong></td>
<td>3.8985</td>
<td>4.0462</td>
</tr>
<tr>
<td><strong>Single Period Mean</strong></td>
<td>0.0098</td>
<td>0.0115</td>
</tr>
<tr>
<td><strong>Single Period Variance</strong></td>
<td>0.001183</td>
<td>0.0017</td>
</tr>
</tbody>
</table>
9.5 Extensions

The preceding development assumes that the manager’s objective is to maximise long-term reward while minimising path risk. A quite different scenario might be that of a hedge fund concerned with identifying portfolios that actually maximise path risk over some designated detail band, perhaps one much shorter than assumed above. This could be accommodated by requiring a minimal reward element and maximising the path risk with an appropriate choice of the energy weights \( w_k \). This seems analogous to a dual formulation from the classic theory of mathematical programming (Dantzig 1963, Rockafellar 1968, Murty 1976). However, the latter would require one to minimise the energy subject to reward constraints, so the ‘hedge fund problem’ is not exactly dual to the long run strategic approach.

9.6 Conclusions

The underlying objective of this chapter has been to develop portfolio technology that allows for non-independent return elements, with dependence that may be weak in the short run, but have a cumulative impact over the long run. There are other ways of attempting the same thing, notably by developing formal macroeconomic or time series models of conditional returns. However, these models require the manager to have superior information in the model prediction which is problematic, given the difficulty in long range forecasting of business cycles. The wavelet based reward-
energy approach is much less demanding in assumptions or informational requirements. It can be viewed as generalising mean variance analysis to the spectral domain, where the latter is interpreted broadly to encompass possible non-stationarity and wavelet technology. The energies correspond collectively to the variance in mean-variance analysis, but refer to path properties as a whole rather than the property of a single stationary distribution. The resulting optimal portfolio choice framework leads to decisions with a higher accumulating path or with a lower long scale risk in the current application.

On the other hand, the wavelet based approach does have some maintained hypotheses of its own, notably that the long-term volatility patterns are characteristic of the data generation process, e.g. an underlying business cycle, and are likely to be repeated in the years to come. There is some comfort in the ability of wavelet analysis to detect structural breaks, which typically appear as sudden energy bursts in the high detail bands (see Chapter 7 and Vuorenmaa, 2005). However, Ramsay (2002) has noted that wavelet decompositions may not be stable outside the sample period. Nevertheless, it could be still possible to endow the wavelet approach with a priori macroeconomic information, with the objective of increasing confidence in the long-term volatility patterns. For instance, if asset returns and value accumulation depend on exchange rates, one might have a fair idea about the causal factors involved for a particular home country. In terms of the current example, the NZ dollar is well known to be driven by world commodity price cycles in conjunction with monetary policy responses, as indicated earlier and by Lance (1996). Likewise, there is some support among NZ economists for a business cycle of about 7-8 years, partly as a result of the commodity-exchange rate cycle (e.g. Kim et al, 1995; Hall & McDermott, 2006). As long as such fundamental characteristics remain alive, the risk management framework developed in the current chapter is applicable.
Concluding Review

This thesis studies the corporate foreign exchange risk management from an empirical perspective, with a particular focus on the development of algorithms and techniques, with which decision theory and risk management practice is combined to develop an empirically implementable financial risk management framework. A series of case studies included in this thesis have illustrated how currency hedging position may expose a firm to financial distress. In corporate finance theory, one rationale for hedging is that it helps to minimise the cost of bankruptcy, or more generally the cost arising from financial distress. The objective welfare criterion for the structured financial risk management framework proposed in this thesis is established on the basis of the generalised value at risk, as it, among the traditional risk management decision theories, links most closely to financial distress costs. It inherits the value at risk point as a natural calibration point, as well as the focus on losses downside of this critical point. The firm’s value at risk can then be seen as an exposure to put options that diminish the value of a firm. This form of option equivalence can be further utilised in deriving the optimal hedging decision in a practical context. The consequent risk management decision therefore improves a firm’s financial value by maximising the value in good times at a minimal generalised value at risk.

Turning to the operational implementation, discussions on the proposed structural hedging on the basis of the generalised value at risk criteria start with developing a measurement tool for the corporate exposures. To account for the most sensitive exposures encountered by the corporate, the present thesis has developed a real income indicator – corporate term of trade. This indicator is a convenient payoff variable for the implementation of the generalised value at risk management approach. The economic basis of this indicator lies in its link with the firm’s net profit margin. In addition, the bounded nature of the resulting time series enables one to draw on a variety of established statistical metrics to measure the effectiveness of hedging.

A detailed decomposition of such corporate exposures assists the corporate managers in detecting the origin and component of the risk. Empirically, the corporate exposure can be investigated with the aid of the wavelet analysis. This can be employed to decompose the natural exposures to detect cycles and trends and analyse influences causing these. Wavelet analysis, which has a wide application in natural
science, differs from classical techniques such as spectral analysis. Firstly, wavelet analysis is not restricted with the data generating process and thus suits both stationary and non-stationary data. Secondly, the wavelet analysis decomposes the data along both time and frequency, which enables users to discover both global as well as local aspects with respect to the data over time. This form of decomposition facilitates the breakdown of time series into long term trends, cyclical patterns, seasonal changes and irregular variations. By decomposing the variations along the frequency and time, wavelet analysis also enables users to detect any structural breaks in the cycle, which can be accompanied by violent short run fluctuations in the low level details. In addition, the underlying orthogonality of the wavelet decomposition means that the composite detail is independent along the frequency. Some statistical metrics, such as variance, covariance as well as correlation in the data, can thus be computed and compared across the frequency.

With the New Zealand dairy industry as the case study, the corporate exposure is expressed in the form of farmer terms of trade, which effectively constructs a NZ dollar price of dairy product relative to the price of framer expenses. An in-depth analysis of the farmer terms of trade then shows industry managers what their corporate’s main exposures are and when these exposures might occur. The wavelet decomposition discloses the negative trend in the farmer terms of trade, especially in earlier years due to the steadily increasing expenses. Fluctuations over scales 5 and 6, which relate respectively to the 4-year and 8-year cycle, largely dominate the time series. The wavelet analysis further reveals cycles as long as 7-8 years, which are mainly produced by the interaction of commodity prices with the exchange rate, but with a strong natural buffering element, especially prior to 1997. The buffering effect implies that, based on historical performance, farmers have little need to implement a long term risk management, and a formal hedging can be constructed in a shorter term context.

In terms of hedging instruments, although commodity prices play an important role in determining the corporate exposure, dairy commodities do not possess well developed forward markets. This implies that currency derivatives will be generally utilised for hedging purposes. The empirical application has been directed to the issue of the forward premium and its usage. It does not exclude the use of options, either on a stand-alone basis or in conjunction with forward covers in some circumstances. If a manager is in the natural position of having to write an adversity put option, then this
could be fixed by buying an offsetting put option in the adverse zone. In effect, one would be using options to hedge a natural option-type position. The empirical results, however, suggest that given the gains to be made from using forwards, the use of options may be an expensive supplement, given the volatility of the NZ dollar in the chosen context. However, the optimal combination of forwards and options could remain a subject for further research.

Turning to the particular context of currency risk management, unconditional hedging results offer qualified support for New Zealand exporters to use currency forwards. The New Zealand dollar has a chronic forward rate discount due to the higher interest rate compared to other OECD countries. In this respect, exporters’ expected income could be better off by transferring foreign currency to home currency with the forward rate rather than the spot rate. However, the use of forward markets is not necessarily a response to risk aversion. The hedge in an aim to reduce the generalised value at risk should be distributed over time, including a spot weighting, rather than relying wholly on the forward rate. Such a strategy is less exposed to welfare losses associated with exceptional times, notwithstanding an ex ante forward discount or premium. Indeed, current hedging results suggest that the component of an unhedged spot, where it appears, arises from increasing risk aversion in some situations. In addition, the empirical hedging outcomes show that the apparent forward rate discount should not be relied on too far forward by hedgers averse to generalised value at risk, despite expected benefit possibly growing in forward maturity time.

The generalised value at risk approach is adapted for use in conditional hedging, where the hedge ratio depends upon available information. Information management and use in real world forecasting and risk management can be active or passive, or somewhere in between. At one extreme, the manager knows enough about the economy and markets to be able to formulate a complete and informative econometric model, one capable of accurate value forecasts. At the other extreme, a manager would use only implicit market forecasts or certainty equivalents, arguing that given market efficiency, no one should even try to do better. This is a passive view of informational use and a simple forward hedging would be the optimal solution to this case. In the chosen exchange rate forecasting context, the theoretical result from Martingale pricing in complete markets is that the forward rate is an optimal predictor. However, there is a large amount of empirical evidence, both from existing
literature and contained in case studies in this thesis, pointing to the inefficiency of the currency market. In this sort of market, managers who have superior information or advanced knowledge may be able to exploit value-improving hedges by actively using information.

One of the findings is that even very simple frameworks, involving only directional rather than complete value assessments, can produce a better result than informational passivity. Over a longer time interval, managers who have useful but limited information may have insufficient data to derive precise value forecasts. A framework of incomplete directional forecasting is adapted for this sort of exigency. In this framework, the categorical prediction model implicit in binomial or trinomial step processes is developed to establish non-homogeneous multinomial directional probabilities. Problems of signal compression and outcome definition can be addressed using methods analogous to neuronal nets and fuzzy membership functions. A further problem is that the true underlying data generation process may not coincide precisely with the estimation model. In this respect, one is replacing maximum likelihood by quasi-maximum likelihood, and convergence to an equivalent set of parameters under the specified model may not always be assured. Likewise, an explicit methodology is also developed to relate hedge parameters to the resulting directional probabilities.

It is difficult to forecast short run currency changes with a structured economic model as economic fundamentals are more important at longer horizons. Given the volatility clustering effects observed in the variation of exchange rate NZD/USD, the mean-GARCH model is employed in a short term hedging context. The findings exhibit significant evidence of auto-correlations in volatilities. The parameter estimators for the interest rate differential are negative, which also leads to the rejection of the unbiased forward hypothesis.

The optimised conditional hedge outcome based on the developed exchange rate forecasting represents an improvement on the unhedged exposure, the simple forward or the passive hedging outcomes. Both in-sample and out-of-sample tests, including the rolling regression tests for hedging results, show the superior performance of the proposed forecasting models. The evidence of value improvement by actively using information, on the other hand, demonstrates that the currency market is not always efficient.
Turning to the context of long term risk management, the focus is on strategic fund management. The long term portfolio optimisation technology in the literature usually emphasises the expected value or expected utility of long run portfolio value. However, objective functions of this type do not take into account what happens along the way. Two alternative portfolios might have the same terminal or long run mean-variance properties, but yet be exposed quite differently to alternative path dynamic, e.g. one dominated with high frequency fluctuations while another consistent with long term macro scale cycles. The latter path could encounter excessive long swings in portfolio value, which is related to the dynamic value at risk. Value histories of this sort could be exposed to investor withdrawal, which is damaging to the reputation of its managers. To account for concerns on these issues, strategic portfolio selection needs to include a secondary objective of managing the path exposure, in addition to the primary objective function regarding the long term expected value or average returns. A path dependent analogue of generalised value at risk is developed to handle this sort of path risk.

Empirically, the path risk management aspects can be handled with a wavelet analysis based reward-energy approach. The proposed wavelet based portfolio technology is much less demanding in assumptions or informational requirements than traditional portfolio theory. However, it does have some maintained hypotheses of its own, notably that the long-term volatility patterns are characteristic of the data generation process, e.g. an underlying business cycle, and are likely to be repeated in the years to come. As the wavelet analysis decomposes the total variation according to the scale, fund managers are able to compare paths dominated with various cyclical patterns. The approach leads to dynamic analogues of mean-variance, such as band pass portfolios that are more sensitive to variability at designated scales. The consequent long term fund investment portfolio in this thesis exhibits less variation over a long cycle at the cost of high frequency fluctuations. The portfolio management framework, as developed in this thesis, can be easily adapted for investors with various preference structures as to multi-scale variation, for example, hedge fund managers could derive a portfolio with apparent long cycle, but relatively lower short run variations.

This thesis represents a series of studies on several essential aspects of the currency exchange risk management, including the defining of decision criteria, measurement and decomposition of corporate exposures, developing the exchange
rate forecasting to incorporate private information into the decision, and some operational techniques in achieving an optimal currency risk management strategy. The thesis studies the risk management from both a short term and long period perspective. The value at risk theory is rationalised in the corporate finance perspective and generalised to a dynamic version, with a particular focus on long run fund management. Some theories and econometric techniques developed in the current research, such as the active currency risk management with incomplete exchange rate forecasting model and the dynamic path risk management using long term band pass portfolio are original to the literature and can be applied to other contexts in future research. The hedging algorithms and operational instruments established in this thesis, including the measurement of corporate exposures, the multi-dimensional wavelet decomposition of corporate exposures and financial investment value, the construction of currency reference rate and the option equivalence fuzzy approach, are new to the empirical research and can also be useful in other applications. In addition, the exchange rate risk management framework developed in this research provides practitioners the guidance on developing a structured hedging strategy to improve financial value of the corporate or the investment in practice.
Appendix A: Wavelets

Wavelet decompositions

The following treatment based on Crowley (2005), though much of the terminology is common in the wavelet literature. Wavelets for a given family are generator functions, indexed by two parameters called the scale \((j)\) and the translation or location \((k)\). Every family may have two sets, the father and mother wavelets, respectively of the form:

\[
\phi_{j,k}(t) = 2^{-j/2} \phi\left(\frac{t - 2^j k}{2^j}\right); \quad \psi_{j,k}(t) = 2^{-j/2} \psi\left(\frac{t - 2^j k}{2^j}\right).
\]

The mother wavelets \((\psi)\)'s integrate to zero, as these are meant to span the cyclical influences. The father wavelets \((\phi)\) are normalised to integrate to one. The scale parameter determines the span of the wavelet, meaning its non-zero support, as each wavelet damps down to zero on either side of its centre. For a given time \(t\), there are contributions from neighbouring wavelets translated to either side of \(t\). Figures A.1 and A.2 depicts the two wavelet generators used in the present study – Coiflet and Symmlet.

![Figure A.1 Coiflet father wavelet (left) and mother wavelet (right)](image1)

![Figure A.2 Symmlet father wavelet (left) and mother wavelet (right)](image2)

The family of functions defined as above are mutually orthogonal. Something analogous to Fourier analysis will therefore hold. Coefficients are formed as
Appendix A: Wavelets

\[ s_{j,k} = \int \phi_{j,k}(t) x(t) dt; \quad d_{j,k} = \int \psi_{j,k}(t) x(t) dt. \]

for \( j = 1,2,\ldots J \), where \( J \) is limited by the number of observations on \( x \) available. As with the inverse transform in Fourier analysis, \( x(t) \) can be recovered in terms of the wavelet functions as:

\[ x(t) = \sum_k s_{j,k} \phi_{j,k}(t) + \sum_k d_{j,k} \psi_{j,k}(t) + \cdots + \sum_k d_{1,k} \psi_{1,k}(t). \]

The detail is written as \( D_j(t) = \sum_k d_{j,k} \psi_{j,k}(t) \). Note that just the one father wavelet has been used in the above with maximal scale.

**Computational procedure**

The quasi Fourier approach illustrated above would be slow computationally. In the present thesis, computations were done in Matlab (Misiti et al 2005) using Mallat’s algorithm, which is considerably more efficient. The algorithm follows through the basic sequence as illustrated in figure 5.1 of the text. The original signal \( x(t) \) is fed through a high pass and low pass filter, one the quadrate of the other, which ensures orthogonality of the two outputs. The low pass filter is adapted to the longer run father wavelets and the higher to the mother wavelets. Output from the high pass filter is downloaded as the level 1 detail \( D_1 \), and the output from the low pass filter becomes the level 1 approximation. Starting afresh with \( A_1 \), the process is successively repeated.

**Power**

The power of the wavelet at each scale is basically set by the coefficients \( s_{j,k} \) and \( d_{j,k} \). Having obtained the details \( D_j \) at each time point \( t \), one can express the average power or energy at each level of detail, relative to the whole:

\[ E_j^D = \frac{1}{E} \sum_t D_j^2, \quad E_j^A = \frac{1}{E} \sum_t A_j^2, \]

\[ E = \sum_t A_{j,t}^2 + \sum_j \sum_t D_{j,t}^2. \quad (A.1) \]

Table 7.1 of the text shows the energies \( E_j^A \) and \( E_j^D \) for the log farmer terms of trade \( ftt \) in the form of percentage contributions relative to \( E \).

**Scale and frequency**

To connect the scale to frequency, a pseudo frequency is calculated. The algorithm works by associating with the wavelet function a purely periodic signal of frequency \( F_c \) that maximises the Fourier transform of the wavelet modulus. When the wavelet is dilated by the scaling factor \( 2^j \), the pseudo frequency corresponding to the scale is expressed as:

\[ F_s = \frac{F_c}{2^j \times \Delta}, \quad \text{where } \Delta \text{ is the sampling period}. \]
Taking the wavelet ‘coif5’ as an example, the centre frequency as seen from figure A.3 is 0.68966 and thus the pseudo frequency corresponding to the scale $2^5$ is 0.02155. As the sampling period is one month, the period corresponding to the pseudo frequency is 3.87 years.

Wavelet coif5 and Center frequency based approximation

### Figure A.3 Scale in terms of equivalent sinusoidal frequency

**Wavelet variance derived by MODWT**

For a time series $X_t$, $t=1, 2, \ldots, N$. The maximal-overlap discrete wavelet coefficients for levels $j=1, \ldots, J$ are:

- **wavelet coefficients**: $W_{j,t} = \sum_{l=0}^{L_j-1} h_{j,l} X_{t-l},$

- **scale coefficients**: $V_{j,t} = \sum_{l=0}^{L_j-1} g_{j,l} X_{t-l},$

where $L_j = (2^j - 1)(L-1) + 1$, $h_{j,l}$ and $g_{j,l}$ are wavelet and scale filters at level $j$ and $J$.

The coefficients satisfy

$$
\frac{1}{N} \sum_{t=0}^{N-1} X_t^2 = \sum_{j=1}^{J} \frac{1}{N} \sum_{j=0}^{N-1} W_{j,t}^2 + \sum_{j=1}^{J} \frac{1}{N} \sum_{j=0}^{N-1} V_{j,t}^2. \quad (A.2)
$$

Percival and Mofjeld (1997) proved that the sample variance can be expressed as the sum of the $J+1$ series.

$$
Var = \frac{1}{N} \sum_{j=0}^{N-1} (X_t - \bar{X})^2 = \frac{1}{N} \sum_{j=1}^{N-1} X_t^2 - \bar{X}^2. \quad (A.3)
$$

Combining equation (A.2) and (A.3), the variance can be decomposed as:

$$
Var = \sum_{j=1}^{J} \frac{1}{N} \sum_{j=0}^{N-1} W_{j,t}^2 + \sum_{j=1}^{J} \frac{1}{N} \sum_{j=0}^{N-1} V_{j,t}^2 - \bar{X}^2. \quad (A.4)
$$
Since \( \tilde{W}_{j,t} \) has a zero mean, the first factor of equation (A.4) reflects the sum of coefficients variation over different scales. The wavelet variance at level \( j \) for time series \( X \) is thus expressed as:

\[
Var(X)_j = \frac{1}{N} \sum_{i=0}^{N-1} (\tilde{W}_{j,i})^2,
\]

(A.5)

\[
Var(X)_{ij} = \frac{1}{N} \sum_{i=0}^{N-1} \tilde{V}_{j,i}^2 - \tilde{X}.
\]

Under same theory, the wavelet covariance and correlation between two time series \( X \) and \( Y \), on a scale by scale basis, can be also constructed.

\[
Cov(X,Y)_j = \sum_{j=1}^{J} \frac{1}{N} \sum_{i=0}^{N-1} W_{j,i} \cdot W_{j,i} \cdot (X) - (Y),
\]

(A.6)

\[
Corr(X,Y)_j = \frac{Cov(X,Y)_j}{\sqrt{Var(X)_j \cdot Var(Y)_j}}.
\]

(A.7)
Appendix B: Reference Exchange Rates

The results and discussion that follow provide a more formal support for the exposition of section 5.3 concerning currency reference rates. The evidence is mostly comprised of straightforward matrix algebra, hence only outlines are given. Some additional remarks describe the treatment of section 5.3.

Nominal exchange rates

Proposition 1 (Nominal currency reference rates)

Let \( S = ((s_{ij})) \); \( i, j = 1, \ldots, n \) be a matrix of bilateral log exchange rates with currency \( i \) as commodity currency and country \( j \) as terms currency, so that 1 country \( i \) unit = \( e^{s_{ij}} R_{ij} \) country \( j \) units. No arbitrage exists across currencies. Then:

(a) A set \( \{ A_i \} \) of CRR’s exists such that \( R_{ij} = \frac{A_i}{A_j} ; s_{ij} = a_i - a_j \) or collectively

\[
S = a1' - 1a'. \tag{B.1}
\]

Conversely, any bilateral set \( \{ S \} \) defined in this way is a no-arbitrage system.

(b) Two equivalent generic representations are:

\[
a = \frac{1}{n} (S - \lambda I) 1, \tag{B.2}
\]

where 1 denotes the unit vector (of ones) and \( \lambda \) is some scalar;

\[
a^w = Sw \tag{B.3}
\]

for some vector \( w \) with \( \sum_j w_j = 1 \).

Representations (B.2,3) are equivalent with \( \lambda = w'S1 ; \sum_j w_j = 1 \). Any CRR vector can be written in the form (B.2) or (B.3).

(c) The choice \( \lambda = 0 \) or \( w = \frac{1}{n} 1 \) gives \( a^0 = \frac{1}{n} S1 \) and \( \sum_i a^0_i = 0 \) (the ‘centred representation’). The centred version \( a^0 \) corresponds to setting the log CRR for any country as the average (log) bilateral rate, with that country taken as commodity currency. For any alternative CRR vector \( a, a^0_i = a_i - a \) and \( a = a^0 - (w'a^0)1 \).

(d) The centred version \( a^0 \) minimises the norm \( a'a = \sum_i a^2_i \).
(e) The choice \( \lambda = \mathbf{e}_n' \mathbf{S} = \sum_i s_{in} \), where \( \mathbf{e}_n \) is the \( n \)th identity vector, corresponds to setting \( \mathbf{a} = \mathbf{s}_n \), in which case \( a_n = 0 \). Suppose currency \( n \) is an arbitrary base or numeraire (e.g. the US dollar) for the system of bilateral rates. The centred CRR’s can be calculated as

\[
a_i^0 = s_{in} - \bar{x}_n; \quad \text{or} \quad a_0 = \mathbf{s}_n - (\frac{1}{n} \mathbf{1}' \mathbf{s}_n) \mathbf{1},
\]

(B.4)

(f) Given any desired weighting system \( \mathbf{w} \) as in representation (B.3), the corresponding CRR’s can be calculated in terms of the centred rates as \( a_i^w = a_i^0 - \sum_j w_j a_j^0 \); or alternatively in terms of a numeraire currency as elements of \( \mathbf{a}^w = \mathbf{s}_n - (\mathbf{w}' \mathbf{s}_n) \mathbf{1} \).
Appendix C: Sufficient Conditions for Quasi Maximum Likelihood Estimation

Even if the true data generating process is unknown, limited information may nevertheless prove consistent with the data, in the sense that their parameter estimates (or quasi-estimates) converge. The following are some general assumptions that will suffice, together with comments relating to the context of Chapter 6.

Assumptions I:

(i) \( \{(Y_t, Z'_t) : t = 1, 2, \cdots \} \) is a set of strictly stationary and ergodic processes;

(ii) the space \( \Theta \) for \( \theta \) is compact in \( \mathbb{R}^p \) \((p \in \mathbb{N})\);

(iii) for a stationary, ergodic, and integrable \( D_t \), \( \sup_{\theta \in \Theta} \left| \ln(\bar{L}_{R,t}(\theta; \sigma)) \right| \leq D_t \);

(iv) there is a unique maximiser \( \theta^* \) of \( E[\bar{L}_R(\theta; \sigma)] \) in the interior part of \( \Theta \).

Comments:

(a) Assumption I (i) describes the conditions for the data generating process of the data. The stationarity and ergodicity condition is crucial in applying the asymptotic theory.

(b) Assumptions I (iii, iv) specify the conditions for the consistence of the QMLE, \( \hat{\theta}_n \). By Assumption I (iii), the strong uniform law of large numbers holds for \( n^{-1} \sum \ln(\bar{L}_{R,t}(\theta^*; \sigma)) \); so that \( \hat{\theta}_n \) converges a.s. to \( \theta^* \) given that \( E[L_R(\cdot; \sigma)] \) is identified.

Assumptions II:

(i) \( \{\nabla_{\theta} \ln(\bar{L}_{R,t}(\theta^*; \sigma)), \mathcal{F}_t \} \) is an adapted mixingale of size \(-1\) (McLeish, 1974), where \( \mathcal{F}_t \) is a smallest \( \sigma \)-algebra generated by \( \{y_1, z_1, \ldots, y_t, z_t\} \);

(ii) \( E[\nabla_{\theta} \ln(\bar{L}_{R,t}(\theta^*; \sigma)) \nabla_{\theta} \ln(\bar{L}_{R,t}(\theta^*; \sigma))] < \infty \);

(iii) \( B \) is positive definite, where \( \text{avar}(\cdot) \) is the asymptotic variance of given argument;

(iv) \( \sup_{\theta \in \Theta} \left\| \nabla_{\theta}^2 \ln(\bar{L}_{R,t}(\theta; \sigma)) \right\|_\infty < D_1 \), where \( \| \cdot \|_\infty \) is the uniform metric;

(v) \( A^* := E \left[ \nabla_{\theta}^2 \bar{L}_R(\theta^*; \sigma) \right] \) is negative definite.

Comments:

(a) The central limit theorem (CLT) can be applied to \( n^{-1/2} \sum \nabla \ln(\bar{L}_{R,t}(\theta^*; \sigma)) \) by Assumptions II (i to iii). Scott (1973) provides sufficient conditions for the central limit theorem, and White (1999, p. 125) proves the CLT given Assumptions II (i to iii).

(b) Assumptions II (iv and v) are used to approximate \( \bar{L}_R(\cdot; \sigma) \) by a quadratic function. The standard second-order Taylor expansion can be applied to \( \bar{L}_R(\cdot; \sigma) \).
(c) By Assumption II (iv), the strong uniform law of large numbers can be applied to the Hessian matrix. The negative definite Hessian matrix is necessary for a non-degenerate asymptotic distribution of the QMLE.
Appendix D: The Generalised Rubinstein Risk Premium for the Segmented Utility Function

The generalised Rubinstein risk premium is defined for an arbitrary risk-averse utility function \( U(R) \) by

\[
\theta = - \frac{E[(R - \mu_R)U'(R)]}{E[U'(R)]}.
\]

It uses the marginal utility weights to adjust the expected outcomes; the weights have a close correspondence with the state price deflators used in martingale pricing (see Bowden 2005a).

The meaning of the risk premium is that if a return or object of certain value \( \mu_R - \theta \) was available, then the investor or manager would be indifferent if the last investment dollar or unit was devoted either to \( R \) or to the certain asset. By adapting the utility function (8.1) to the present context, it can be derived that

\[
\theta = \frac{bF_R(P)}{1 + bF_R(P)} \left[ \mu_R - \mu_R(P) \right], \quad (D.1)
\]

where \( \mu_R = E[R] \) and \( \mu_R(P) \) is defined as the censored mean \( E[R | R \leq P] \). Note that \( \mu_R(P) < \mu_R \), for any \( P \), so the risk premium is always positive and in increasing with \( b \).

To prove formula D.1, it is easiest to use methods from generalised functions (see Bowden 2005a, Appendix C; or Lighthill, 1958), a standard reference. The same result can be obtained – at greater length – by breaking up the domains of integration and using more traditional methods. The utility function is

\[
U(R; P) = R - P + b(R - P)SF(P - R).
\]

Also \( SF(P - R) = 1 - SF(R - P) \) and \( \frac{d}{dR} SF(R - P) = \delta(R - P) \), the Dirac delta function.

In addition, for any smooth function \( \phi(R) \),

\[
\int_{-\infty}^{\infty} \phi(R)\delta(R - P)f(R)dR = \phi(P)f(P).
\]

Hence \( E[\phi(R)\delta(R - P)] = \phi(P)f_R(P) \). Finally, \( E[SF(P - R)] = F_R(P) \) and \( E[RSF(P - R)] = F_R(P)\mu_R(P) \) by definition of the censored mean \( \mu_R(P) \). The desired result follows by substitution.
Appendix E: Using Fuzzy Logic to Smooth Out the Kink

For a continuous random variable, the step function \( SF(x) \) is an indicator function for two distinct sets of positive measures, namely the positive and negative numbers, so that a given number \( x \) belongs to either one or the other. A standard fuzzy indicator function would attach a number between zero and unity to indicate the strength of the classification. Thus one could approximate the step function in equation (8.1) of the text by a Normal distribution function and write \( SF(x) \sim N(x;0,\sigma^2) \) for a suitably chosen value of \( \sigma \). The Logistic distribution function is also useful: \( SF(x) \sim 1/(1+e^{-x/\lambda}) \). By setting the ‘smearing’ parameters \( \sigma \) or \( \lambda \) arbitrarily small, one can approach closer and closer to the ‘all or nothing’ switch given by the unit step function. Figure E.1 depicts a fuzzy GVaR utility function, using the logistic version, which is marginally faster to compute. A value of \( \lambda = 0.01 \) suffices for a fairly close approximation and yields much improved convergence in the empirically based optimisation reported below. For other uses of fuzzy logic in Finance, see Simonelli (2001), Tseng (2001), Zmeskal (2001, 2005).

![Figure E.1 Fuzzy approaches to the exact utility function (P = 0)](image)

Comparing figure E.1 and figure 8.2 of the text for the option equivalent method, it will be evident that the fuzzy method smoothes from above whereas the option equivalent method smoothes from below. One might therefore expect the option equivalent utility function to be marginally more sensitive to behaviour approaching the VaR point \( P \) from above, and hence slightly more defensive, which turns out to be the case. In all cases, the intent is to preserve the sharp curvature at the VaR point and the aggravated penalty slope to the left of this point.
### Appendix F: Exchange rate econometrics

#### Table F.1a Correlogram of standardized residuals for the UIP test outcomes

<table>
<thead>
<tr>
<th>Lag Period</th>
<th>Autocorrelation</th>
<th>Partial Autocorrelation</th>
<th>Q-Stat</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0.021</td>
<td>0.1238</td>
<td>0.725</td>
</tr>
<tr>
<td>2</td>
<td>-0.083</td>
<td>-0.083</td>
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<tr>
<td>3</td>
<td>0.125</td>
<td>0.13</td>
<td>6.217</td>
<td>0.102</td>
</tr>
<tr>
<td>4</td>
<td>-0.112</td>
<td>-0.129</td>
<td>9.6423</td>
<td>0.047</td>
</tr>
<tr>
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<td>10.382</td>
<td>0.065</td>
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<tr>
<td>6</td>
<td>0.131</td>
<td>0.1</td>
<td>15.112</td>
<td>0.019</td>
</tr>
<tr>
<td>7</td>
<td>0.005</td>
<td>0.019</td>
<td>15.119</td>
<td>0.035</td>
</tr>
<tr>
<td>8</td>
<td>0.084</td>
<td>0.103</td>
<td>17.069</td>
<td>0.029</td>
</tr>
<tr>
<td>9</td>
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<td>0.089</td>
<td>21.589</td>
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<td>-0.007</td>
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<tr>
<td>12</td>
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<td>-0.041</td>
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<td>0.063</td>
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<tr>
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<tr>
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<td>0.088</td>
<td>0.033</td>
<td>34.211</td>
<td>0.025</td>
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#### Table F.1b Correlogram of standardized residuals squared for the UIP test outcomes

<table>
<thead>
<tr>
<th>Lag Period</th>
<th>Autocorrelation</th>
<th>Partial Autocorrelation</th>
<th>Q-Stat</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0.119</td>
<td>3.8011</td>
<td>0.051</td>
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<tr>
<td>2</td>
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<td>0.082</td>
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<td>0.04</td>
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<td>0.118</td>
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<td>0.017</td>
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<td>6</td>
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<td>0.054</td>
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<td>7</td>
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### Table F.2a Correlogram of standardized residuals of M-GARCH model

<table>
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<tr>
<th>Lag Period</th>
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<th>Partial Autocorrelation</th>
<th>Q-Stat</th>
<th>Prob</th>
</tr>
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<td>0.064</td>
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<td>0.039</td>
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<td>-0.054</td>
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<td>0.034</td>
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### Table F.2b Correlogram of standardized residuals squared of M-GARCH model

<table>
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<th>Lag Period</th>
<th>Autocorrelation</th>
<th>Partial Autocorrelation</th>
<th>Q-Stat</th>
<th>Prob</th>
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<tbody>
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<td>-0.032</td>
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<td>-0.074</td>
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<td>3</td>
<td>0.129</td>
<td>0.124</td>
<td>6.180</td>
<td>0.103</td>
</tr>
<tr>
<td>4</td>
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<td>0.030</td>
<td>6.381</td>
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<td>-0.019</td>
<td>6.788</td>
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<td>12.168</td>
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<td>-0.039</td>
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<td>0.036</td>
<td>0.039</td>
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<td>-0.069</td>
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<td>0.589</td>
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<td>17.524</td>
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</tbody>
</table>
Appendix G: Energy in Returns versus Levels

Table G.1 is an energy table for returns as distinct from the levels (values) used in the text. As the table indicates, wavelet energy now concentrates in the high detail band, and there is little indication of any interior maximum or other sign of power at lower details.

<table>
<thead>
<tr>
<th></th>
<th>NZ monthly return wavelet transform</th>
<th>US monthly return wavelet transform</th>
<th>JP monthly return wavelet transform</th>
<th>AU monthly return wavelet transform</th>
<th>USD/NZD forward monthly return wavelet transform</th>
</tr>
</thead>
<tbody>
<tr>
<td>E(A7)(value)</td>
<td>0.0054</td>
<td>0.0316</td>
<td>0.0069</td>
<td>0.0242</td>
<td>0.0013</td>
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<tr>
<td>E(D1)</td>
<td>0.3671</td>
<td>0.2267</td>
<td>0.4494</td>
<td>0.2476</td>
<td>0.0869</td>
</tr>
<tr>
<td>E(D2)</td>
<td>0.1456</td>
<td>0.1016</td>
<td>0.219</td>
<td>0.1025</td>
<td>0.0398</td>
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<tr>
<td>E(D3)</td>
<td>0.0885</td>
<td>0.0754</td>
<td>0.1077</td>
<td>0.0615</td>
<td>0.0243</td>
</tr>
<tr>
<td>E(D4)</td>
<td>0.0285</td>
<td>0.0248</td>
<td>0.062</td>
<td>0.0264</td>
<td>0.0077</td>
</tr>
<tr>
<td>E(D5)</td>
<td>0.0108</td>
<td>0.0086</td>
<td>0.0399</td>
<td>0.0126</td>
<td>0.0047</td>
</tr>
<tr>
<td>E(D6)</td>
<td>0.0035</td>
<td>0.0324</td>
<td>0.0229</td>
<td>0.0054</td>
<td>0.0103</td>
</tr>
<tr>
<td>E(D7)</td>
<td>0.0049</td>
<td>0.0037</td>
<td>0.0009</td>
<td>0.0006</td>
<td>0.0018</td>
</tr>
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</table>
Appendix H: Energy Table for a Geometric Ito Process

Table H.1 depicts the wavelet decomposition of a standard geometric Ito process with an annual drift of 10% and annual volatility of 20%. The simulated sample is a year with 365 daily observations for which the maximum scale for wavelet decomposition is 8. As is evident from the table, the energy increases along the scale level.

Table H.1: Energy decomposition for geometric Ito process

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<thead>
<tr>
<th>Detail level</th>
<th>Detail Energy (%)</th>
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<tbody>
<tr>
<td>1</td>
<td>0.90%</td>
</tr>
<tr>
<td>2</td>
<td>1.12%</td>
</tr>
<tr>
<td>3</td>
<td>1.65%</td>
</tr>
<tr>
<td>4</td>
<td>7.29%</td>
</tr>
<tr>
<td>5</td>
<td>2.99%</td>
</tr>
<tr>
<td>6</td>
<td>8.61%</td>
</tr>
<tr>
<td>7</td>
<td>19.26%</td>
</tr>
<tr>
<td>8</td>
<td>58.18%</td>
</tr>
</tbody>
</table>
References


References


References


Crowley, P. (2005). An intuitive guide to wavelets. Bank of Finland/College of Business Texas A&M University, Patrick.Crowley@bof.fi


References


