Analysis of Change in Present Value Measurements

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ANALYSIS OF CHANGE IN PRESENT VALUE MEASUREMENTS

ABSTRACT

In recent years, the leading standard setters for financial reporting have shown an increasing preference for fair value measurement. However, present value is often the only acceptable method of estimating fair value and therefore the actual result of the swing to fair value is likely to be increased use of present value in financial reporting. This paper addresses the issue of interpretation of a change in present value between successive reporting dates and shows that the change can be analyzed by use of the familiar variance analysis framework widely used in management accounting.

Keywords: Fair Value, Present Value, Variance Analysis
ANALYSIS OF CHANGE IN
PRESENT VALUE MEASUREMENTS

1. INTRODUCTION AND MOTIVATION
The International Accounting Standards Board (IASB) and the US Financial Accounting Standards Board (FASB) are showing an increasing preference for fair value measurement (Herrmann et al., 2002). Although fair value is a concept that has a long standing in the accounting literature it is only in recent times that fair value has been widely proposed as a basis for measurement subsequent to initial recognition. It is regarded as the only basis that provides sufficient transparency in, and comparability of, financial statements in comparison to the historical cost basis (Wahlen et al., 2000, 506). In the past, fair value tended to be seen as a substitute for historical cost at initial recognition of transactions such as an exchange in kind, a delayed payment, or as a basis for allocation of cost to individual assets in the purchase of a set of assets. Thus, although accounting standards have increasingly incorporated fair value into financial statements, “… this evolution had resulted in a piecemeal collection of disclosed and recognized fair-value amounts” (Hirst et al., 2004, 456). This is now changing.

In February 2000 the FASB issued CON 7: Using Cash Flow Information and Present Value in Accounting Measurements which provided a framework for using present value as the basis for accounting measurements at initial recognition or fresh-start measurements that “should attempt to capture the elements that taken together would comprise a market price if one existed, that is, fair value” (Paragraph 25). More recently the IASB has issued guidance on the estimation of fair value in IFRS 3 Business Combinations (March, 2004) and in IAS 39 Financial Instruments: Recognition and Measurement (March, 2004). The FASB has issued similar guidance in the Exposure Draft Fair Value Measurements (June, 2004). The
FASB’s recent Working Draft Financial Accounting Series *Fair Value Measurements* (October, 2005) has a new hierarchy of five sources for fair value measurement: firstly, market inputs that reflect quoted price in an active market for identical items in focus, secondly, market inputs that reflect quoted price for identical items in markets that are not active, and quoted prices for similar items in all markets, regardless of level of activity, thirdly, market inputs other than quoted prices that are directly observable for the items, fourthly, market inputs that are not directly observable for the items but that are corroborated by market data through correlation or by other means, thereby incorporating market data that are observable, and fifthly, entity inputs. Therefore, when use is made of valuation techniques, those techniques that maximises the use of market inputs and minimises the use of entity inputs are to be used. Present value enters here and would likely be the most common valuation technique applied. In using present value to estimate fair value, the estimates of the expected cash flows and the risk-adjusted discount rate should reflect market based assessments rather than entity-specific assessments.

Fair value measurements are made on an ongoing or periodic basis. There will be a stream of regular or periodic fair value measures reported over time. Those fair value estimates will vary from year to year as business and economic conditions change. It could be argued that at present although the change in fair value would be measured and reported as income or expense in the income statement, the reasons for the change would not be known to most users, mainly the external users. This paper develops an innovative use of the management accounting technique of variance analysis to analyze these changes. This technique can be used to interpret the change in the fair value of an item between successive reporting dates where the item’s fair value is estimated by present value. The consequences of a drop in fair value of an asset for instance, due to change in estimated future cash flows might well be
different from a drop caused by a rise in market interest rates. The analysis provides a useful explanation of the changes in fair value estimates between successive reporting dates. The quality and degree of conservatism of a firm’s estimates of future cash flows would be reflected in a low or nil quantity variance. On the other hand, a pure price variance might be viewed by the market as generally uncontrollable by the entity. Therefore, a decomposition of changes in fair value is more important than the aggregate change. Variance information in other contexts is widely used by decision makers (Cheatham and Cheatham, 1996; Pierce and O'Dea, 2003). The decomposition exercise would also detect whether a favourable quantity (price) variance is offset by an unfavourable price (quantity) variance. Under such circumstances, no loss or gain due to fair value change would be recognised in the income statement, which might mislead the users. However, the decomposed variances would reveal such offsetting effects. The explanation of changes in the present value estimates of fair value is potentially useful to external users of financial statements, to management as decision makers, and to auditors in examining and testing the representations of management (AICPA, 2003).

2. ANALYSIS
Variance analysis is used here to decompose the change in the total estimate of present value of an asset or liability into a quantity variance and a price variance. The price variance is then further decomposed into a pure price variance and a price-quantity interaction variance. A change in the present value of an item results from changes in either or both of the cash profile of the item and the discount rate. The variance analysis framework can thus be applied to decomposition of the change in the present value of an item between successive reporting dates by viewing the change in the cash flow profile as a quantity change and the change in discount rate as a price change.
(i) Calculation of Present Value
Consider an entity that has an asset or a liability at the opening reporting date, \( t_0 \), that is measured by present value. Let the stream of expected cash flows, estimated at \( t_0 \), be \( \{c_{0j}; j=1,2,\ldots,N_0\} \) and the discount rate be \( R_0 \). If the discount rate reflects the systematic risk associated with the expected cash flow stream, then the present value of the item, \( V_0 \), is given by

\[
V_0 = \frac{c_{01}}{1 + R_0} + \frac{c_{02}}{(1 + R_0)^2} + \frac{c_{03}}{(1 + R_0)^3} + \cdots + \frac{c_{0N_0}}{(1 + R_0)^{N_0}}
\]  

(1)

\[
= \sum_{j}^{N_0} \frac{c_{0j}}{(1 + R_0)^j} .
\]  

(2)

Similarly, at the closing reporting date, \( t_1 \), the present value, \( V_1 \), is given by

\[
V_1 = \sum_{j}^{N_1} \frac{c_{1j}}{(1 + R_1)^j} .
\]  

(3)

The cash flow streams and the comparisons of cash flows made in calculating the change in value are illustrated in Figure 1 below. The cash flow comparisons are for \( \{c_{1j}, c_{0j}\} \), as indicated by the ellipses.

*** Insert Figure 1 here ***

The \( j^{th} \) expected cash flow, as assessed at \( t_0 \), has a present value

\[
v_{0j} = c_{0j} r_{0j} ,
\]  

(4)

which can be regarded as the product of the quantity, \( c_{0j} \) (expected cash flow), and the price (discount factor), \( r_{0j} \), where
\[ r_{0j} = \frac{1}{(1 + R_0)^j}. \]  

The present value of the asset or liability at \( t_0 \) is then given by

\[ V_0 = c_{01}r_{01} + c_{02}r_{02} + c_{03}r_{03} + \cdots c_{0N_0}r_{0N_0} = \sum_{j} v_{0j}, \]  

and, similarly at \( t_1 \)

\[ V_1 = c_{11}r_{11} + c_{12}r_{12} + c_{13}r_{13} + \cdots c_{1N_1}r_{1N_1} = \sum_{j} v_{1j}. \]  

(ii) Variance Analysis

The total change in present value (total present value variance), \( \Delta V \), between the successive reporting dates \( t_0 \) and \( t_1 \) is given by

\[ \Delta V = V_1 - V_0 = \sum_{j} \Delta v_{j}, \]  

where

\[ \Delta v_{j} = v_{1j} - v_{0j} = c_{ij}r_{ij} - c_{0j}r_{0j} \text{ and } j = 1, 2, \ldots, N \quad [N = \max(N_0, N_1)]. \]  

\(^{1}\) For a monetary liability it is likely that \( N_1 = N_0 - 1 \); however in the case of, say, a warranty provision or a plant asset, \( N_1 \) may be greater than, less than, or equal to \( N_0 \). If, for example, \( N_1 = N_0 + 2 \) then \( N = N_1 \) and \( c_{0(N_0+1)} = c_{0(N_0+2)} = 0 \).
$\Delta v_j$ is the present value variance for (any) year $j$ and the cash flows compared in $\Delta v_j$ are illustrated in Figure 1 as those enclosed by the ellipses.

Now $\Delta v_j$ can be restated as

$$\Delta v_j = c_{ij}r_{0j} - c_{0j}r_{ij} + c_{ij}r_{ij} - c_{ij}r_{0j} \quad (11)$$

$$= (c_{ij} - c_{0j})r_{0j} + (r_{ij} - r_{0j})c_{ij}. \quad (12)$$

That is, $\Delta v_j$, can be decomposed into

- a quantity variance, $(c_{ij} - c_{0j})r_{0j}$, and
- a price variance, $(r_{ij} - r_{0j})c_{ij}$. \quad (13) \quad (14)

The quantity variance and the price variance are illustrated in Figure 2 below, assuming that both the price (discount factor), $r_{ij}$, and the quantity (expected cash flow), $\{c_{ij}\}$, at $t_i$ are greater than corresponding amounts at $t_0$.

*** Insert Figure 2 here ***

Furthermore, the price variance, $(r_{ij} - r_{0j})c_{ij}$, can be restated as

$$(r_{ij} - r_{0j})c_{ij} = (r_{ij} - r_{0j})c_{ij} = (r_{ij} - r_{0j})c_{0j} + (r_{ij} - r_{0j})(c_{ij} - c_{0j}). \quad (15)$$

That is, the price variance can itself be decomposed into

- a pure price variance, $(r_{ij} - r_{0j})c_{0j}$, and
- a price-quantity interaction variance, $(r_{ij} - r_{0j})(c_{ij} - c_{0j})$. \quad (16) \quad (17)

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2 The decomposition of a price variance into a pure price variance and a price-quantity interaction variance traces to Kwang and Slavin (1962).
Collecting these results, the present value variance for a given year \( j \), \( \Delta v_j \), can be expressed as

\[
\Delta v_j = (c_{ij} - c_{0j})r_{0j} + (r_{ij} - r_{0j})c_{0j} + (r_{ij} - r_{0j})(c_{ij} - c_{0j}).
\]  

(18)

The quantity variance is \((c_{ij} - c_{0j})r_{0j}\), the pure price variance is \((r_{ij} - r_{0j})c_{0j}\), and the price-quantity interaction variance is \((r_{ij} - r_{0j})(c_{ij} - c_{0j})\). This decomposition into three variances is illustrated in Figure 3 below, assuming that at time \( t_1 \) the price (discount factor) and quantity (expected cash flow) are greater than at \( t_0 \).

*** Insert Figure 3 here ***

From Equations (12) and (15) above it follows that the total present value variance, \( \Delta V \), between reporting dates \( t_0 \) and \( t_1 \) is given by

\[
\Delta V = \sum_{j=1}^{N} (c_{ij} - c_{0j})r_{0j} + \sum_{j=1}^{N} (r_{ij} - r_{0j})c_{ij}.
\]  

(19)

The first term in Equation (19) for \( \Delta V \), viz,

\[
\sum_{j=1}^{N} (c_{ij} - c_{0j})r_{0j},
\]  

(20)

is the sum of the quantity variances for the individual cash flows and is thus the total quantity variance arising from the difference between the expected cash profiles at \( t_0 \) and \( t_1 \), both evaluated at the \( t_0 \) discount rate.

The second term, viz,
\[ \sum_{i=1}^{N} (r_{ij} - r_{0j})c_{i,j}, \]  

is the sum of the price variances for the individual cash flows at \( t_1 \) and is thus the total price variance arising from the change in the discount rate between \( t_0 \) and \( t_1 \).

It is shown in the Appendix Section of the paper that, for a given value of the item at the initial reporting date, the price variance is determined by the duration of the item and the change in the discount rate.

The total price variance can also be restated as

\[ \sum_{i=1}^{N} (r_{ij} - r_{0j})c_{i,j} = \sum_{i=1}^{N} (r_{ij} - r_{0j})c_{0j} + \sum_{i=1}^{N} (r_{ij} - r_{0j})(c_{ij} - c_{0j}). \]  

(22)

The first term on the right hand side of Equation (22), viz,

\[ \sum_{i=1}^{N} (r_{ij} - r_{0j})c_{0j}, \]  

(23)

is the sum of the pure price variances for the individual cash flows at \( t_0 \) and is thus the total pure price variance arising from the change in the discount rate between \( t_0 \) and \( t_1 \).

Similarly, the remaining term, viz,

\[ \sum_{j=1}^{N} (r_{ij} - r_{0j})(c_{ij} - c_{0j}), \]  

(24)

is the price-quantity interaction variance, which is partly caused by the difference in price (discount rate) and partly caused by the difference in quantity (expected cash flows).

3. NUMERICAL EXAMPLE OF PRESENT VALUE VARIANCE ANALYSIS

Consider a manufacturing entity that warrants the quality of its product for an initial three-year period following sale. The resulting warranty provision is likely to be measured by
present value. Assume that the market based assessment of cash flows and discount rates at times \( t_0 \) and \( t_1 \) are as shown in Table 1 and that the present value estimate of the warranty liability at \( t_1 \) is therefore greater than at \( t_0 \).

*** Insert Table 1 here ***

(i) Total Variance
Applying Equations (6) and (7), the present value, \( V_0 \), of the liability at \( t_0 \) is $571.71 and the present value, \( V_1 \), at \( t_1 \) is $631.56. As a result, the total change in present value, \( \Delta V \), applying Equation (19), is $59.84, which is the total present value variance between \( t_0 \) and \( t_1 \).

This $59.84 variance has a positive sign and similar to cost or expense variances will be considered unfavorable as the present value is in regard to a liability. Conversely, if the present value had arisen in relation to an asset then the positive sign would be considered favorable, similar to the situation with sales or revenue variances.

The change in present value (fair value) between the successive reporting dates, \( t_0 \) and \( t_1 \), would be recognized in the income statement for the reporting period ended \( t_1 \), and as an increase in the warranty provision at \( t_1 \) – summarized by the following journal entry.
This total present value variance, $\Delta V$, can be analyzed by decomposing it as follows.

**(ii) Quantity Variance**

The present value quantity variance as shown in Expression (20) is

$$\sum_j (c_{1j} - c_{0j}) r_{0j}.$$  \hspace{1cm} (25)

Multiplying terms, this gives the quantity variance as

$$\sum_j (c_{1j} - c_{0j}) r_{0j} = \sum_j c_{1j} r_{0j} - \sum_j c_{0j} r_{0j}.$$ \hspace{1cm} (26)

The quantity variance as shown in Equation (26) is the present value of the stream of expected cash flows at time, $t_1$, discounted at the time $t_0$ discount rate, minus the present value of the stream of expected cash flows at time, $t_0$, discounted at the time $t_0$ discount rate.

Thus, the quantity variance is

$$= PV_{11%}(360, 250, 130) - PV_{11%}(330, 230, 120)$$

$$= \$50.57 .$$

This would be considered an unfavorable variance as it is in regard to a liability. Conversely, the variance would have been regarded as favorable if it were in regard to an asset.

**(iii) Price Variance**

The present value price variance as shown in Expression (21) is

$$\sum_j (r_{1j} - r_{0j}) c_{1j}.$$ \hspace{1cm} (27)
Multiplying terms gives the price variance as
\[
\sum_{j} (r_{1j} - r_{0j})c_{0j} = \sum_{j} r_{1j}c_{1j} - \sum_{j} r_{0j}c_{1j} .
\] (28)

The price variance as shown in Equation (28) is the present value of the stream of expected cash flows at time \( t_1 \) discounted at the \( t_1 \) discount rate, minus the present value of the stream of expected cash flows at \( t_1 \) discounted at the \( t_0 \) discount rate.

\[
= \text{PV}_{10\%} \{360, 250, 130\} - \text{PV}_{11\%} \{360, 250, 130\}
\]

\[
= \$9.27 .
\]

The variance is positive in sign and would be considered an unfavorable price variance as it is in regard to a liability.

The price variance decomposes into a pure price variance and the price-quantity interaction variance.

(iv) Pure Price Variance

The present value pure price (pure discount factor) variance as shown in Expression (21) is
\[
\sum_{j} (r_{1j} - r_{0j})c_{0j} .
\] (29)

Multiplying terms gives the pure price variance as
\[
\sum_{j} (r_{1j} - r_{0j})c_{0j} = \sum_{j} r_{1j}c_{0j} - \sum_{j} r_{0j}c_{0j} .
\] (30)

The pure price variance as shown in Equation (30) is the present value of the stream of expected cash flows at time \( t_0 \) discounted at the \( t_1 \) discount rate, minus the present value of the stream of expected cash flows at \( t_0 \) discounted at the \( t_0 \) discount rate.

\[
= \text{PV}_{10\%} \{330, 230, 120\} - \text{PV}_{11\%} \{330, 230, 120\}
\]

\[
= \$8.53 .
\]
The variance is positive in sign and would be considered an unfavorable pure price variance as it is in regard to a liability.

(v) Price-Quantity Interaction Variance

The present value price-quantity interaction variance as shown in Equation (22) is

$$\sum_{j} (r_{ij} - r_{0j})(c_{1j} - c_{0j}).$$

(31)

Multiplying terms gives the price-quantity interaction variance as

$$\sum_{j} (r_{ij} - r_{0j})(c_{1j} - c_{0j}) = \sum_{j} r_{ij}c_{1j} - \sum_{j} r_{0j}c_{1j} - \sum_{j} r_{ij}c_{0j} + \sum_{j} r_{0j}c_{0j}$$

(32)

$$= PV_{10\%}\{360, 250, 130\} - PV_{11\%}\{360, 250, 130\}$$

$$- PV_{10\%}\{330, 230, 120\} + PV_{11\%}\{330, 230, 120\}$$

$$= 0.74.$$

This price-quantity interaction variance would be considered an unfavorable variance as it is in regard to a liability.

Thus the unfavorable price variance of $9.27 between the successive reporting dates, $t_0$ and $t_1$, arises from an unfavorable pure price variance, and an unfavorable price-quantity interaction variance as follows:

$$9.27 = 8.53 + 0.74.$$

(vi) Interpretation of the Results

Thus the unfavorable total present value variance of $59.84 between the successive reporting dates, $t_0$ and $t_1$, arises from an unfavorable quantity (expected cash flow) variance, an unfavorable pure price (discount rate) variance and an unfavorable price-quantity interaction variance as follows:
$59.84 = $50.57 + $8.53 + $0.74.

This decomposition of the change in present value is summarized by the following journal entries.

| 1. | Present value (PV) quantity variance | $50.57 |
|    | PV pure price variance | $8.53 |
|    | PV price quantity interaction variance | $0.74 |
|    | Warranty provision | $59.84 |

(Recording the decomposition of the unfavorable increase in the warranty provision at \( t_1 \) by charging it to the decomposition variance accounts.)

| 2. | Warranty expense | $59.84 |
|    | PV quantity variance | $50.57 |
|    | PV pure price variance | $8.53 |
|    | PV price quantity interaction variance | $0.74 |

(Recording the unfavorable increase in the warranty provision at \( t_1 \) by expensing the decomposition variance accounts to the warranty expense account.)

Decomposition of the total change in the present value of the warranty provision is potentially useful to internal and external users of the financial report as well as to auditors.

Plausible explanations for a change in a warranty provision, either individually or in combination, include:

(a) Entity related:
   (i) Change in the entity’s credit risk rating
   (ii) Change in sales levels
   (iii) Change in the proportion of defective products
   (iv) Change in costs
   (v) Change in warranty claims criteria
(b) Market related:

A change in market interest rates.

Management’s awareness of the areas in which change occurred would have motivated the variation in assessment of cash flows and the discount rate underlying the change in present value. While this knowledge would also indicate the direction of change in present value, the decomposition would demonstrate the relative impacts of the operative factors and thus provide useful information to management for planning and control. Users of the entity’s external financial report could benefit, as the decomposition would obviously provide a sound basis for management to provide the disclosures required in financial reporting standards, for example, paragraph 116 (and supporting explanatory paragraphs) of IAS 1 Presentation of Financial Statements. The decomposition would be useful to auditors when auditing fair value measurements and disclosures, as it would highlight the sources of change in a fair value estimate and thus provide a relevant basis for testing that management can adequately support any significant market based assessments (AICPA, 2003).

4. CONCLUSION

This analysis developed in this paper should benefit internal users of financial reporting information. By use of this technique, managers can readily see the relative impacts of factors within and outside their control. This paper shows that the change in the present value of an asset or liability between successive reporting dates can be decomposed into component variances that are more amenable for analysis than is the aggregated amount. The technique

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3 Paragraph 116 states that “An entity shall disclose in the notes information about the key assumptions concerning the future, and other sources of estimation uncertainty at the balance sheet date, that have a significant risk of causing material adjustments to the carrying amounts of assets and liabilities within the next financial year. In respect of those assets and liabilities, the notes shall include details of: (a) their nature, and (b) their carrying amounts at the balance sheet date.”
developed for this purpose is an adaptation of the familiar variance analysis framework used in management accounting.

The decomposition of the change in present value could be used to facilitate improved quality of disclosure of information to users of the entity’s external financial report. Auditors would have an improved basis for testing the representations of management with respect to present value measurements. The analysis presented here should assist auditors in meeting their responsibilities regarding the requirement to express an opinion on financial statements that contain an increasing proportion of fair value disclosures, much of which will be based on the present value measurement method.
APPENDIX

Determinants of the price variance

The present value, \( V \), of an item can be stated as

\[
V = f(c_1, c_2, c_3, \ldots c_N, R),
\]

(33)

where \( \{c_j\} \) are the cash flows expected over \( N \) periods and \( R \) is the discount rate.

Then, \( dV \), the total change in value, is given by

\[
dV = \frac{\partial f}{\partial c_1} dc_1 + \frac{\partial f}{\partial c_2} dc_2 + \cdots + \frac{\partial f}{\partial c_N} dc_N + \frac{\partial f}{\partial R} dR,
\]

(34)

where \( \frac{\partial f}{\partial c_j} \) and \( \frac{\partial f}{\partial R} \) are the partial derivatives of \( V \) with respect to \( \{c_j\} \) and \( R \) respectively, and \( \{dc_j\} \) and \( dR \) are the differentials of \( \{c_j\} \) and \( R \).

Thus

\[
dV = \frac{1}{1 + R} dc_1 + \frac{1}{(1 + R)^2} dc_2 + \cdots + \frac{1}{(1 + R)^N} dc_N
\]

\[
+ [- \frac{c_1}{(1 + R)^2} - \frac{2c_2}{(1 + R)^3} - \cdots - \frac{Nc_N}{(1 + R)^{N+1}}]dR
\]

(35)

\[
= pv\{dc_j\} - \frac{1}{1 + R} \left[ \frac{c_1}{1 + R} + \frac{2c_2}{(1 + R)^2} + \cdots + \frac{Nc_N}{(1 + R)^N} \right]dR,
\]

(36)

where \( pv\{dc_j\} \) is the present value of the incremental cash flows and is thus the quantity variance.

Thus

\[
dV = pv\{dc_j\} - \frac{V}{1 + R} (D)(dR),
\]

(37)

where \( D \) is the duration of the cash flows, and is defined as

\[
D = \frac{\sum_{j=1}^{N} N_j c_j}{\sum_{j=1}^{N} c_j}.
\]

(38)
Comparison of Equation (37) with Expression (12) - (14) shows that the price variance is given by

\[ \text{price variance} = \frac{V}{1 + R} (D)(dR). \quad (39) \]

That is, for a given value of the item at the initial reporting date, the magnitude of the price variance is determined by, \( D \), the duration of the item, and, \( dR \), the magnitude of the change in the discount rate.

It should be noted that for any example, the result in Equation (39) will not be exact as duration is a point concept and varies over the interval. Thus the issue arises as to which value to assign to \( D \) – the value for the initial cash flows and discount rate, the final values, or an average. Using the initial values for the example discussed in the paper, Equation (39) gives the value of the pure price variance as

\[ \frac{V}{1 + R} (D)(dR) = \frac{(571.713)}{1.11} (1.633)(0.01) = 8.41 \text{ (U)}. \]

However, using Expression (23) the value of the pure price variance is

\[ \sum_{i=1}^{N} (r_{ij} - r_{0j})c_{ij} = 8.53 \text{ (U)}, \]

which is the correct value. The approximation deteriorates as the size of the changes in \( \{c_{ij}\} \) and \( R \) increases.
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Figure 1

Cash Flow Stream at $t_0$ and $t_1$

At $t_0$:

$C_{01}$ $C_{02}$ $C_{03}$ $\ldots$ $C_{0N_0}$

At $t_1$:

$C_{11}$ $C_{12}$ $C_{13}$ $\ldots$ $C_{1N_1}$

0 1 2 3 4 time
Figure 2

Variance Analysis

![Diagram showing price variance and quantity variance](image-url)

- Price Variance
- Quantity Variance

Symbols:
- $r_{ij}$
- $r_{0j}$
- $c_{0j}$
- $c_{1j}$

Price

Quantity
Figure 3

Decomposition of Price Variance

![Diagram showing decomposition of price variance with areas for pure price variance, interaction variance, and quantity variance.]
Table 1
Warranty example

Panel A: time $t_0$

<table>
<thead>
<tr>
<th>Year of estimation</th>
<th>Expected cash flows over next three years</th>
<th>Discount rate per annum</th>
<th>Present Value as per Equation (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$t_0$</td>
<td>$330$</td>
<td>$230$</td>
<td>$120$</td>
</tr>
</tbody>
</table>

Panel B: time $t_1$

<table>
<thead>
<tr>
<th>Year of estimation</th>
<th>Expected cash flows over next three years</th>
<th>Discount rate per annum</th>
<th>Present Value as per Equation (7)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$t_1$</td>
<td>$360$</td>
<td>$250$</td>
<td>$130$</td>
</tr>
</tbody>
</table>