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INNOVATION AND GROWTH WITH FINANCIAL, AND OTHER, FRICTIONS*

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The generation and implementation of ideas are crucial for economic performance. We study this in a model of endogenous growth, where productivity increases with innovation, and where the exchange of ideas (technology transfer) allows those with comparative advantage implement them. Search, bargaining, and commitment frictions impede the idea market, however, reducing efficiency and growth. We characterize optimal policies involving subsidies to innovative and entrepreneurial activity, given both knowledge and search externalities. The role of liquidity is discussed. We show intermediation helps by financing more transactions with fewer assets, and, more subtly, by ameliorating holdup problems. We also discuss some evidence.

Keywords: innovation, growth, liquidity, intermediation, search, bargaining

JEL Classification nos.: E5, G2, O3

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1 Introduction

It is commonly argued that the generation and implementation of new ideas are major factors underlying economic performance and growth, and that financial development plays a role in this process.\footnote{An early proponent of the view that financial factors are crucial for growth is Goldsmith (1967), but there is by now a large literature. See Aghion and Howitt (1997), Levine (2005) and Acemoglu (2009) for comprehensive treatments and bibliographies. See Greenwood et al. (2010, 2013), Opp (2010) and Cole et al. (2011) for recent papers with more references.} This project is our attempt to better understand the issues in a model where decisions to innovate and implement new ideas are endogenous. Based on the premise that some people are better at research and others at development, the model incorporates a market for ideas in order to study technology transfer. This market helps get ideas into the hands of those better able to implement them, but is hindered by search, bargaining and commitment frictions that slow reallocation and hence the advance of knowledge. Realistically, our idea market is thin, agents are not price takers, and there are fixed costs that are hard to recoup due to holdup problems. Also, commitment problems impede credit, generating a role for liquidity. We show how financial intermediaries (e.g., banks) contribute to growth in two ways: they reallocate liquidity to those that need it most; and, perhaps more surprisingly, they ameliorate holdup problems.

By way of preview, in our setup, individual producers have access to the frontier technology $Z$, but may also come up with ideas for innovation that increase their own productivity to $z > Z$. This raises individual profit in the short run, but later knowledge enters the public domain. In the simplest case, an innovator with an idea tries to develop it on his own, and succeeds with probability $\sigma$, indexing the quality of the match between the idea and his expertise. Innovations advance individual productivity, and collectively determine the evolution of the technology frontier. The model has a balanced-growth equilibrium, where the growth rate depends on the number of innovators, their probabilities of success, the distance by
which innovations increase individual productivity, and the way they aggregate to move the frontier. Naturally, since knowledge is (partially) a public good, equilibrium is inefficient absent intervention, and we characterize the optimal corrective subsidy. This benchmark model is, however, only a stepping stone toward studying economies where individuals do not necessarily develop their own ideas.

As discussed in the literature, the question is this: When people come up with new ideas, should they try to implement them on their own, or trade them to others, say entrepreneurs who may be better at development, marketing and related activities? Given heterogeneity in abilities, it is beneficial if some specialize in research and others in development. As Katz and Shapiro (1986) say: “Inventor-founded startups are often second-best, as innovators do not have the entrepreneurial skills to commercialize new ideas or products.” As The Economist (2005) puts it: “as the patent system has evolved, it ... leads to a degree of specialization that makes business more efficient. Patents are transferable assets, and by the early 20th century they had made it possible to separate the person who makes an invention from the one who commercializes it. This recognized the fact that someone who is good at coming up with ideas is not necessarily the best person to bring these ideas to market.” And as Lamoreaux and Sokoloff (1999) say: “The growth of the U.S. economy over the nineteenth century was characterized by a sharp acceleration of the rate of inventive activity and a dramatic rise in the relative importance of highly specialized inventors as generators of new technological knowledge. Relying on evidence compiled from patent records, we argue that the evolution of a market for technology played a central role in these developments” (emphasis added).

Our idea market has a liquidity problem, motivated by limited commitment, which impedes credit. This is especially important because knowledge is difficult to collateralize – if you sell someone an idea on credit, and they renege, can you repossess the information? Of course, that depends on intellectual property rights,
patent protection, etc. Perhaps less obviously, it also depends on search frictions that mean entrepreneurs do not know in advance who they will meet, and hence do not know how much liquidity they may need. This leads to a role for intermediation, which reallocates liquidity, and hence redirects resources to more productive users. Here the resources in question are ideas. In fact, the theory applies to any factor of production, but we frame the discussion in terms of ideas, consistent with these factors expanding knowledge, and the notion that knowledge is a (partially) nonrival good. Again, equilibrium is inefficient, and we characterize optimal subsidies to innovative and entrepreneurial activities. These results are novel, we think, because of the interaction between knowledge and search externalities. We also show how it is easier to achieve efficiency with than without intermediation.

Although we formally model direct technology transfer, we understand that this is but one mechanism by which innovators and entrepreneurs interact – e.g., they can also enter into longer-term partnerships, as with venture capital. We are pretty sure that many of the same insights would emerge in a model with partnerships, but concentrate on situations where innovators want to sell their ideas outright, for several reasons. One very important advantage of direct transfers is that they avoid strategic problems with joint implementation. Another is that they allow innovators to go “back to the drawing board” to come up with more new ideas, which is their forte, rather than getting tied up in development. Moreover, direct technology transfers have been somewhat neglected in theory, and we think they are worth studying, even if they are just one of many contributors to economic growth. And we think it is important to model this market as one with frictions, although, to focus on other issues, this paper abstracts from private information.

The focus instead is on how search, bargaining and liquidity problems interact with innovation, and how this generates a role for financial intermediation that has not been previously recognized. As usual, one reason to study markets with
these frictions is that we can think of perfect competitive markets as a limiting case. However, people who study this market claim the frictions are realistic and relevant. In his study of patent licensing contracts, e.g., Sakakibara (2010) says: “since there is no public market for patents, the price of patents is determined by a private negotiation between a licensor and a licensee ...[and] once the matching process is completed, the terms of the contract are negotiated between a licensor and a licensee” (emphasis added). Hence, we think this market is best described as one with search and bargaining, and although in theory one can shut down these frictions, as well as the liquidity problem, in practice it seems interesting to keep them.

Our goal is to build a framework that is tractable, at least to those who know search, yet allows us to analyze the creation, transfer and diffusion of knowledge in a growth context. By incorporating key elements from other work we integrate various insights from the literature. Thus, having knowledge entering the public domain in the longer run follows much research on Schumpeterian growth (see the references in footnote 2); the way productivity evolves is similar to the knowledge production function in Jones (1999); and the role of banks is related to King and Levine (1993). Yet our framework differs in important ways from previous papers – e.g., they do not have search and bargaining frictions. Specifically, our framework identifies a strategic problem that arises from the interaction between bargaining and financial frictions: because investment in liquidity assets is sunk at the time of negotiations, there is a holdup problem that implies that entrepreneurs under-

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3 Gans and Stern (2010) argue that one important source of inefficiency in the market for ideas is the lack of thickness. Hagiu and Yoffie (2013) discuss the high search costs on both sides of the patent market.

4 Also, using patent data from the nineteenth century, Lamoreaux and Sokoloff (1999) say “it was evident patent agents and lawyers often perform the functions of intermediaries in the market for technology, matching inventors seeking to sell new technological ideas with buyers eager to develop, commercialize, or invest in them.” In this paper, patent agents and lawyers are not modeled explicitly, but these observations clearly speak to the importance of search and matching in the market.
invest in liquid assets. We then argue that financial intermediaries can alleviate this problem. Also, search hinders innovative and entrepreneurial activity, and this affects the corrective tax-subsidy policies. In addition, we examine general equilibrium linkages between markets for ideas, labor markets and asset markets to uncover some findings not in the existing literature. Finally, we provide some evidence on the relevance of the frictions. Using the World Bank Enterprise Surveys, we show that technology transfer is positively correlated with financial development across countries, especially for small firms.

In the rest of the paper, we present a sequence of increasingly involved models, as a way to build up to the general case. Section 2 characterizes equilibrium when there is no technology transfer. Section 3 adds an idea market with perfect credit. In these relatively simple versions, the key endogenous decisions concern participation by innovators and entrepreneurs, and efficiency generally requires intervention. Section 4 introduces credit frictions, shows how innovation is hindered by liquidity shortages, and discusses inefficiencies due to holdup problems. Section 5 adds intermediation, shows how this allows the economy to finance more transactions with the same supply of liquid assets, and helps get around holdup problems by allowing entrepreneurs to undo otherwise sunk investments. Section 6 discusses the evidence. Section 7 concludes.

\(^5\)Silveira and Wright (2010) use a related setup to also study markets for ideas, but that paper does not have growth. Moreover, it does not analyze entry/participation decisions, corrective policies, banking, or implications for labor markets.

\(^6\)As regards other work, Holmes and Schmitz (1990,1995) also have individuals differing in innovation and implementation ability, but only study perfect markets. Many people study credit frictions and entrepreneurship; for recent papers and references, Cole et al. (2011), Chatterjee and Rossi-Hansberg (2012), Greenwood et al. (2013). Other related work includes Chaney (2008), Berentsen et al. (2012) and Chu et al. (2012). Alvarez et al. (2008), Lucas (2009) and Lucas and Moll (2011) also study similar issues using different approaches. In Lucas and Moll (2011), e.g., there is bilateral matching, but agents get ideas for free from anyone they meet, while here they have to pay for them. Also, knowledge in our model is a rival good in the short run but a public good in the long run; we have ex ante investments; we model liquidity and intermediation; and we include a nontrivial labor market to interact growth and employment. There are also papers that highlight creative destruction, e.g., Aghion and Howitt (1992) or Klette and Kortum (2004); while obviously important, creative destruction is not the focus of this paper.
2 A Simple Model

Here we present a most rudimentary version of the environment to lay the foundation for a general framework.

2.1 Basic Assumptions

A \([0, 1]\) continuum of agents live forever in discrete time. Each period there is a frictionless centralized market where agents trade a numeraire consumption good \(c\), labor \(h\), and an asset \(a\). Think of \(a\) as a Lucas (1978) tree in fixed supply \(A\). There is no reproducible capital in the benchmark model, but Appendix 1 shows how to add it. To generate balanced growth, assume that each unit of the asset bears a dividend \(\delta\) of an intermediate good that is transformed into \(Z\delta\) units of final consumption \(c\), where \(Z\) is the aggregate state of knowledge, or productivity.\(^7\) Thus, \(Z\) is the price of intermediate goods in terms of numeraire. Let \(w\) be the wage and \(\phi\) the asset price. Then the value function for agents entering this market is

\[
W(a, z; Z) = \max_{c, h, a'} \{u(c) - \chi h + \beta V(a', Z')\} \quad (1)
\]

\[
\text{st } c = (\phi + Z\delta) a + wh - \phi a' + \pi(z),
\]

where \(u(c)\) satisfies the usual assumptions, \(\chi\) is the disutility of labor, \(V(a', Z')\) is the continuation value, and \(\pi(z)\) is profit as a function of individual productivity \(z\).

We interpret each individual as operating his own firm, although it is equivalent to have him engage a manager. In either case,

\[
\pi(z) = \max_H \{zf(H) - wH\}, \quad (2)
\]

where \(f(H)\) satisfies the usual assumptions and \(H\) is labor demand (individuals may work for themselves, or for others, given a frictionless labor market). Output \(f(H)\) is in units of the intermediate good, which is transformed into \(zf(H)\) units.
of $c$. Individual $z$ can differ from $Z$ if an agent innovates. A fraction $\bar{n}_i$ of the agents have opportunities to innovate, which means they come up with new ideas, but not all of them pan out: the success probability is $\sigma$, where $\sigma$ is a random draw from $F_i(\sigma)$, and subscript $i$ indicates this is the CDF associated with innovators, to be distinguished from the entrepreneurs introduced later. One can think of $\sigma$ as capturing the quality of the idea vis-a-vis the skill of the individual.

A successful innovation increases individual productivity by a factor $\eta$, so that for those who try to innovate:

$$z = \begin{cases} Z(1 + \eta) & \text{with prob } \sigma \\ Z & \text{with prob } 1 - \sigma \end{cases}$$

(3)

The number of successful innovations is $N = \bar{n}_i \int \sigma dF_i(\sigma) = \bar{n}_i \mathbb{E} \sigma$, and the aggregate state evolves according to $Z' = G(N)Z$. Knowledge is a public goods in the long run, in the sense that it enters the public domain, and yields an advance in aggregate productivity after one period (one can extend this to several periods). Thus, aggregate knowledge is higher next period if more ideas are implemented successfully today, $G'(N) \geq 0$.

As an example, consider aggregating across individuals by

$$Z' = \rho \left( \int_0^1 z_j^+ dj \right)^{1/\varepsilon} = \rho \left[ 1 - N + N(1 + \eta)^\varepsilon \right]^{1/\varepsilon} Z,$$

(4)

where the second equality uses (3), and $\rho$ is an exogenous component while $\varepsilon$ affects the substitutability of individual innovations.$^8$ As special cases, before adjusting for $\rho$, $\varepsilon = 1$ implies productivity next period is given by the average this period (we all contribute equally to the frontier); $\varepsilon = +\infty$ implies it is given by the maximum (we stand on the shoulders of the best); and $\varepsilon = -\infty$ implies it is given by the minimum (we are dragged down by the worst). However, except for constructing examples,$^8$

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$^8$The simplicity of (4) is due to the fact that, although $\sigma$ is random, each success advances $z$ by a fixed amount $\eta$. We also solved the model with $\eta$ random, but it added little other than notation. Also note that, in this example, agents that fail to innovate use the frontier $Z'$ next period, and $Z' < z$ is possible, although one can raise $\rho$ if one wants to avoid this.
we do not need functional forms, and the growth rate in general is similar to what one gets with a standard knowledge production functions,

\[ 1 + g = Z'/Z = G(N). \] (5)

### 2.2 Equilibrium

We seek a balanced growth equilibrium, where \( c, w \) and \( \phi \) grow at the same rate as \( Z \), while \( h \) is constant. To pursue this, first eliminate \( h \) and \( \pi \) to rewrite (1) as

\[
W(a, z; Z) = \frac{\chi}{w}(\phi + \delta Z)a + \max_c \left\{ u(c) - \frac{\chi}{w}c \right\} + \frac{\chi}{w} \max_H \left\{ zf(H) - wH \right\} + \max_{a'} \left\{ \beta V(a', Z') - \frac{\chi}{w}a' \right\},
\] (6)

where it is understood that \( Z' = G(N)Z \) with \( N = \bar{n}_i\mathbb{E}\sigma \). Notice \( W \) is linear in wealth, and \( W_a = \chi(\phi + \delta Z)/w \). The FOC's are

\[
u'(c) = \chi/w, \ zf'(H) = w \text{ and } \phi \chi/w = \beta V_a(a', Z'). \] (7)

The continuation value depends on whether an agent has an opportunity to innovate: for those that do not \( V(a, Z) = W(a, Z; Z) \); for those that do

\[
V(a, Z) = W(a, Z; Z) + \mathbb{E}\sigma \left\{ W \left[ a, Z(1 + \eta); Z \right] - W(a, Z; Z) \right\}, \] (8)

which adds the expected surplus. Inserting \( V_a = W_a \) into the FOC for \( a' \), we get the Euler equation

\[
\frac{\chi}{w}\phi = \beta \frac{\chi}{w'}(\phi' + \delta Z'). \] (9)

Since the stationary solution \( \phi = Z\delta\beta/(1 - \beta) \) is the unique bounded and non-negative solution to (9), the asset must be priced fundamentally, by the present value of its dividend stream; this will not necessarily be the case when we introduce liquidity concerns. We also need the wage \( w \), which we get from goods-market clearing. In terms of supply,

\[
S(w) = N(1 + \eta)Zf(H_1) + (1 - N)Zf(H_0) + A\delta Z,
\]
where $H_1$ solves (2) for successful innovators and $H_0$ solves it for the rest. Notice from the FOC’s $Z(1+\eta)f'(H_1) = w$ and $Zf'(H_0) = w$ that $H_0$ and $H_1$ depend only on $w/Z$, and that $S'(w) < 0$. In terms of demand, the FOC $u'(c) = \chi/w$ implies $D'(w) > 0$. Balanced growth in this model requires $u(c) = \log(c)$ (Waller 2011), which means $D = w/\chi$ does not depend on $Z$. Setting $S(w) = D(w)$, we get

$$\frac{w}{Z} = \chi \left[N(1+\eta)f(H_1) + (1-N)f(H_0) + A\delta\right].$$

(10)

Since $H_1$ and $H_0$ depend only on the normalized wage $\bar{w} = w/Z$, so does this condition, and there is a unique equilibrium $\bar{w}$ that clears the market. From $\bar{w}$, all the other endogenous variables follow easily.

As an example, suppose $f(H) = 1 - \exp(-H)$. Then profit maximization implies $f(H_1) = 1 - w/Z(1+\eta)$ and $f(H_0) = 1 - w/Z$. This makes supply linear, $S(w) = Z(1+N\eta + \delta A) - w$, so we can solve explicitly for $\bar{w} = (1+N\eta + A\delta)\chi/(1+\chi)$, $c = w/\chi$ and so on. Although this example is simple, for any increasing and concave $f(H)$ the results are basically the same. In general, the growth rate $g$ is given by (5), which depends on the number of ideas implemented, $N = \bar{n}_i E\sigma$. It is easy to see how the equilibrium is affected by changes in parameters. Thus, as the average match between ideas and skills, parameterized by $F_i$, improves, $g$ increases, along with $w$ and $c$. An improvement in the overall quality of ideas, captured by $\eta$, has similar effects. An increase in $A\delta$ raises $c$ and $w$, through a wealth effect, but cannot affect the growth rate $g$ in this version of the model; it can below, however, due to liquidity considerations.

### 2.3 Entry

The effects just discussed are fairly mechanical. We now endogenize the choice by individuals to participate in the innovative process. Thus, suppose that the $\bar{n}_i$ potential innovators choose whether to engage in research at cost $\kappa_i$. Let the number of active innovators be $n_i \in [0, \bar{n}_i]$. Now, to calculate the expected individual gain
from trying to innovate, recall that the expected probability of success is $\mathbb{E}\sigma$, and the gain normalized by $Z$ is $\Delta = (\pi_1 - \pi_0)/Z$, with $\pi_1 = Z(1 + \eta)f(H_1) - wH_1$ and $\pi_0 = Zf(H_0) - wH_0$. Then, since $W$ is linear in wealth with slope $\chi/w$, the expected gain from attempting to innovate is $\bar{\kappa}_i = \Delta \chi/\bar{w}\mathbb{E}\sigma$. This means the number of active innovators in equilibrium is:

$$ n_i = \begin{cases} 
0 & \text{if } \kappa_i > \bar{\kappa}_i \\
[0, \bar{n}_i] & \text{if } \kappa_i = \bar{\kappa}_i \\
\bar{n}_i & \text{if } \kappa_i < \bar{\kappa}_i 
\end{cases} \quad (11) $$

As shown in Figure 1, the balanced growth path can now be characterized by two curves in $(n_i, \bar{w})$ space, one representing entry (11), and the other market clearing (10), except now $N = n_i\mathbb{E}\sigma$, rather than $N = \bar{n}_i\mathbb{E}\sigma$, since there are only $n_i$ active innovators.

<table>
<thead>
<tr>
<th>$H_0$</th>
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<th>$\bar{w}$</th>
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Table A: Effects of Parameters in Basic Model

Notes: Parameters: $\kappa_i$: research cost, $A$: asset supply, $\delta$: dividend, $\eta$: technology improvement, $\chi$: disutility of labor, $\beta$: discount factor; Endogenous variables: $H_0, H_1$: employment, $\bar{w}$: normalized wage, $n_i$: active innovators, $c$: consumption, $\phi$: asset price, $g$: growth rate, $N$: no. of successful innovations

In this version of the model, the entry decision gives a horizontal line at $\bar{w} = \Delta \chi\mathbb{E}\sigma/\kappa_i$, while the market clearing curve is strictly increasing, so they intersect uniquely. The solution is interior, $n_i \in (0, \bar{n}_i)$, as long as $\kappa_i$ is not too high or too low. An increase in $\kappa_i$ shifts the entry curve down, reducing $n_i$ and growth. So does an increase in $A\delta$, this time through a shift in the market clearing condition. In terms of employment, which is one of the variables that interest us most, an increase in $\kappa_i$ raises both $H_0$ and $H_1$, but not necessarily $H = NH_1 + (1 - N)H_0$, because $N$ falls. Several other results are summarized in Table A.
2.4 Efficiency

Having described the equilibrium growth path, we now turn to efficiency. Consider the planner’s problem:

\[
J(Z) = \max_{c,H_0,H_1,n_i} \left\{ u(c) - \chi [NH_1 + (1 - N)H_0] - \kappa_i n_i + \beta J(G(N)Z) \right\}
\]

subject to

\[
c = NZ(1 + \eta)f(H_1) + (1 - N)ZF(H_0) + A\delta Z,
\]

where the constraint \( n_i \in [0, \bar{n}_i] \) is implicit, as is \( N = n_i \mathbb{E}\sigma \). The FOC for an interior \( n_i \) implies

\[
\kappa_i = \{ u'(c) [Z(1 + \eta)f(H_1) -ZF(H_0)] - \chi(H_1 - H_0) + \beta V'(Z')G'(N)Z \} \mathbb{E}\sigma. \quad (12)
\]

The RHS of (12) is the marginal social benefit of innovative activity: the first term in braces is the utility change due to output increasing; the second is the change in the disutility of working; the third is the discounted benefit of knowledge in the future \( \beta V'(Z')G'(N)Z \). All this is multiplied by the average success rate, \( \mathbb{E}\sigma \). The envelope condition is

\[
J'(Z) = 1 + \beta J'(Z')G(N) = \frac{1}{Z} + \frac{\beta G(N)}{Z(1 + g)} + \frac{\beta^2 G(N)^2}{Z(1 + g)^2} + \cdots = \frac{1}{Z(1 - \beta)}. \quad (13)
\]

Notice how this takes account of knowledge lasting forever.

Combining (13) and (12), we have

\[
\kappa_i = \left[ u'(c)Z\Delta + \frac{G'(N)}{rG(N)} \right] \mathbb{E}\sigma \quad (14)
\]

where \( r \equiv 1/\beta - 1 \). The analogous equilibrium condition is \( \kappa_i = u'(c)Z\Delta\mathbb{E}\sigma \), which clearly entails too few innovators, because they ignore the external long-run impact of the knowledge they create. This is of course easily corrected by a subsidy \( \tau_i \) that reduces the cost of innovative activity to \( \kappa_i - \tau_i \) financed by lump-sum taxes, and, with our utility function, these taxes affect leisure but nothing else. We summarize all of this as follows:
**Proposition 1** In the model without a market for ideas where $n_i$ is determined by entry, there is a unique interior equilibrium as long as $\kappa_i$ is not too high or low. Absent intervention, it is inefficient. The optimal policy, which yields efficiency, is a subsidy $\tau_i = G'(N)\bar{E}\sigma/rG(N) > 0$.

3 Technology Transfer with Perfect Credit

Results related to Proposition 1 can be found in textbooks on growth (see footnote 1), but we wanted to lay out these implications as a benchmark. The next step is to consider the case where there is a frictional idea market, combining search externalities with the knowledge externalities captured above.

3.1 A Market for Ideas

We now introduce entrepreneurs who, for ease of presentation, do not come up with their own ideas, but may have comparative advantage at implementation. The measure of potential entrepreneurs is $\bar{n}_e$, while $\bar{n}_i$ is again the measure of potential innovators, and the rest of the population, with measure $1 - \bar{n}_i - \bar{n}_e$, work and consume but get involved in neither innovative or entrepreneurial activity. Each period, before the centralized market convenes, there is a decentralized market where an entrepreneur $e$ and innovator $i$ might meet bilaterally according to a standard matching function $\mu(n_i, n_e)$, where $n_j \leq \bar{n}_j$ is the measure of active type $j = i, e$. Thus, the meeting probability in this market is $\alpha_j = \mu(n_i, n_e)/n_j$ for type $j$, and constant returns to scale in $\mu$ implies that $\alpha_j$ depends only on market tightness, $n_e/n_i$. Later we endogenize $n_j$ by assuming type $j$ can participate at cost $\kappa_j$, but we begin with $n_j = \bar{n}_j$, say because $\kappa_j = 0$.

Each period, an active innovator draws $\sigma_i$ from $F_i(\sigma_i)$, then matching begins. Given $\sigma_i$, if $i$ meets $e$, the latter draws $\sigma_e$ from $F_e(\sigma_e|\sigma_i)$. By assumption, $i$ and

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9This way of integrating models of frictional decentralized trade and frictionless centralized trade, by having alternating markets, follows Lagos and Wright (2005).
e both observe \((\sigma_i, \sigma_e)\). Although private information is of course relevant in these kinds of markets, we abstract from that to concentrate on other frictions.\(^\text{10}\) There are gains from trade in the event that \(\sigma_e > \sigma_i\). In this case, \(i\) and \(e\) bargain over a payment \(p\) that the latter will make in the next centralized market, where for now \(e\) can commit to any \(p\) in the relevant range. Given there is no private information, we use generalized Nash bargaining (one can get the same results using various strategic bargaining solutions). Let \(\theta\) be the bargaining power of \(e\). The outside option for \(e\) is using the public technology \(Z\), and the outside option for \(i\) is trying to implement the idea on his own. Recalling that \(\Delta = (\pi_1 - \pi_0)/Z\) is the gain from successfully implementing, one can easily show generalized Nash bargaining delivers

\[
p = p(\sigma_e, \sigma_i) = [\theta \sigma_i + (1 - \theta) \sigma_e] \Delta.
\]

(15)

Whoever takes the idea out of a meeting tries to implement it and improve his productivity from \(z = Z\) to \(z = Z(1 + \eta)\). To reduce notation, ideas are rival goods in the short run, so if \(i\) sells his idea he cannot also try to implement it (this can be relaxed, as in Silveira and Wright 2010, where ideas can be rival, partly public, or pure public goods). After the idea market, agents enter the centralized market, as in Section 2. Exiting the centralized market, the continuation values, now indexed by type \(i\) or \(e\), are

\[
V^i(a, Z) = W^i(a, Z; Z) + \frac{\chi}{\bar{w}} \Delta \mathbb{E} \sigma_i + \alpha_i (1 - \theta) \frac{\chi}{\bar{w}} \Delta \bar{\mathbb{E}} (\sigma_e - \sigma_i)
\]

(16)

\[
V^e(a, Z) = W^e(a, Z; Z) + \alpha_e \theta \frac{\chi}{\bar{w}} \Delta \bar{\mathbb{E}} (\sigma_e - \sigma_i),
\]

(17)

where the expected increase in the success rate due to trade is given by

\[
\hat{\mathbb{E}}(\sigma_e - \sigma_i) = \mathbb{E}(\sigma_e - \sigma_i | \sigma_e > \sigma_i) \Pr(\sigma_e > \sigma_i).
\]

\(^\text{10}\) We also mention that at least some parts of the theory work fine in the special case where \((\sigma_i, \sigma_e)\) is nonrandom, in which case there is no issue about information in a particular bilateral meeting. One can also reinterpret the probability \(1 - \alpha_e\) that \(e\) meets no one in terms of information frictions – e.g., \(e\) meets someone with an idea outside his area of expertise, and hence chooses to not trade lest he gets a lemon. See Lester et al. (2012) or Li et al. (2012) for recent, and less trivial, analyses of information frictions in related search-and-bargaining models.
In particular, compared with (8), the second term in (16) reflects the fact that $i$ can still try to implement his idea, while the last term reflects the fact that he might get an opportunity to sell it. The number of successful innovations is now

$$N = n_i \mathbb{E} \sigma_i + n_e \alpha_e \mathbb{E} (\sigma_e - \sigma_i).$$

The first term is the baseline success rate when ideas are implemented by $i$, while the second captures additional successes gained by technology transfer in matches where $\sigma_e > \sigma_i$. The growth rate is still $1 + g = G(N)$, but $N$ and $g$ now additionally depend on the distribution $F_e$ and the matching function $\mu$. Given $n_j = \bar{n}_j$ is fixed, $g$ is independent of $\theta$, which divides the gains from trade but does not determine which trades get made. Also, $g$ is independent of $\delta A$ (although again that changes below).

### 3.2 Entry

We now endogenize $n_i$ and $n_e$ by considering two-sided entry, which works in our model, because $f(H)$ is concave (it does not work in typical search models, e.g. Pissarides 2000, where the technology is linear, and one can only consider one-sided entry). Thus, $i$ and $e$ choose whether to enter the idea market, at costs $\kappa_i$ and $\kappa_e$. The measure of active innovators $n_i$ still satisfies (11), except now the cost threshold increases to $\tilde{\kappa}_i = u'(c) Z \Delta [\mathbb{E} \sigma_i + \alpha_i (1 - \theta) \mathbb{E} (\sigma_e - \sigma_i)]$, since $i$ is willing to pay more to participate when there are potential options to sell ideas. Similarly, the measure of active entrepreneurs satisfies

$$n_e = \begin{cases} 0 & \text{if } \kappa_e > \tilde{\kappa}_e \\ [0, \bar{n}_e] & \text{if } \kappa_e = \tilde{\kappa}_e \\ \bar{n}_e & \text{if } \kappa_e < \tilde{\kappa}_e \end{cases}$$

(18)

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$^{11}$It is easy to work out examples with $G(N) = \rho [1 - N + N(1 + \eta)^\varepsilon]^{1/\varepsilon}$ and $f(H) = 1 - \exp(-H)$, as in Section 2. Suppose, e.g., that $\sigma_e = 1$ with probability 1 while $\sigma_i$ is uniform on $[0, 1]$. Then $\varepsilon = 1$ implies $g = \rho [n_i + \mu(n_i, n_e)]/2 - (1 - \rho)$; $\varepsilon = \infty$ implies $g = \rho (1 + \eta) - 2$; and $\varepsilon = -\infty$ implies $g = \rho - 1$. 

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15
with $\bar{e} = u'(c)Z\Delta\alpha e\theta\hat{E}(\sigma_e - \sigma_i)$. Equilibrium solves (18) and (11), plus market clearing (10). Appendix 2 shows there is a unique interior equilibrium, $n_j \in (0, \bar{n}_j)$ for $j = i, e$, as long as $\kappa_i$ and $\kappa_e$ are not too high or too low.

### 3.3 Efficiency

The planner's problem for this version of the model is given by

$$J(Z) = \max_{c, H_0, H_1, n_i, n_e} \{u(c) - \chi[NH_1 + (1 - N)H_0] - \kappa_i n_i - \kappa_e n_e + \beta J[G(N)Z]\}$$

subject to

$$c = NZ(1 + \eta)f(H_1) + (1 - N)zf(H_0) + \delta ZA,$$

with the implicit constraints $n_j \in [0, \bar{n}_j]$ and $N = n_i \hat{E}\sigma_i + \mu(n_i, n_e)\hat{E}(\sigma_e - \sigma_i)$. Here we take as given the matching process, and that payment $p$ is determined by bargaining with parameter $\theta$, but we choose entry on both sides of the idea market.

Assuming an interior solution, we get the FOC's for $(n_i, n_e)$,

$$\kappa_i = \frac{u'(c)Z\Delta + \frac{G'(N)}{rG(N)}}{\frac{G'(N)}{rG(N)}} \left[\hat{E}\sigma_i + \mu_i\hat{E}(\sigma_e - \sigma_i)\right],$$

$$\kappa_e = \frac{u'(c)Z\Delta + \frac{G'(N)}{rG(N)}}{\frac{G'(N)}{rG(N)}} \mu_e\hat{E}(\sigma_e - \sigma_i),$$

where it is understood that $\mu$ is evaluated at $(n_i, n_e)$. Comparing this with equilibrium, we get the optimal subsidies, as summarized by:

**Proposition 2** With an idea market and two-sided entry, as long as $\kappa_i$ and $\kappa_e$ are not too high or low, there is a unique interior equilibrium. Absent intervention, it is inefficient. The optimal policy, which yields efficiency, involves subsidies

$$\tau_i = \frac{G'(N)[\hat{E}\sigma_i + \mu_i\hat{E}(\sigma_e - \sigma_i)]}{rG(N)} - u'(c)Z\Delta\hat{E}(\sigma_e - \sigma_i) \left[(1 - \theta)\frac{\mu}{n_i} - \mu_i\right],$$

$$\tau_e = \frac{G'(N)\mu_e\hat{E}(\sigma_e - \sigma_i)}{rG(N)} - u'(c)Z\Delta\hat{E}(\sigma_e - \sigma_i) \left(\theta\frac{\mu}{n_e} - \mu_e\right).$$

These results are somewhat novel relative to growth theory without search frictions. To explain them, note that in addition to the inefficiencies due to knowledge
externalities discussed above, there are now search externalities. The former are corrected by the first terms in $\tau_i$ and $\tau_e$, while the latter are corrected by the second terms. The corrections for search externalities are of course related to Hosios’ (1990) general conditions for efficiency, saying that agents’ bargaining powers should be commensurate with their contributions to the matching process. For entrepreneurs this means $\theta = \mu_e n_e / \mu$, and for innovators $1 - \theta = \mu_i n_i / \mu$. Constant returns in $\mu$ implies that one holds iff the other holds, so the Hosios conditions yield efficient participation by both $i$ and $e$. Even if $\theta$ satisfies the Hosios condition, however, we naturally still want to subsidize participation due to knowledge externalities. When the Hosios condition fails, the optimal policy balances search and knowledge externalities. While it is not technically difficult to derive these results, we submit that they contain a lot of economic content.

As a special case, we can fix $n_e = \bar{n}_e$ but determine $n_i$ through entry, to better compare the results with Section 2. In Figure 1, when the idea market was closed, the entry condition (11) gave a horizontal line at $\bar{w}$. Now it gives a curve sloping downward due to congestion effects: bigger $n_i$ reduces the arrival rate $\alpha_i$ and hence the return to innovation. Market clearing (10) still generates an increasing curve, so we still have a unique equilibrium, and the qualitative effects of parameter changes are the same as in Table A. There are also new effects related to search and bargaining. Increasing $\theta$ shifts down the entry curve, reducing $n_i$, $N$, $g$, $\bar{w}$ and $c$. Increasing the matching rate $\alpha_i$, either because $\mu$ improves or $\bar{n}_e$ increases, shifts up both curves, increasing $\bar{w}$ and $N$ but lowering $H_0$ and $H_1$. Notice higher $\alpha_i$ means higher growth, even though $n_i$ might go up or down. Also notice that for any $\theta > 0$ there is a holdup problem: at the time of bargaining, $\kappa_i$ is sunk, and so cannot affect $p$. In the extreme case $\theta = 1$ the entry curve is again horizontal, and an increase in $\alpha_i$ implies $n_i$ falls, but $\bar{w}$ and $N$ do not change. This is a complete crowding-out effect, with the fall in $n_i$ exactly offsetting the improvement in matching. For $\theta < 1$,
the holdup problem is still there but less drastic.

We summarize this to facilitate comparison with Proposition 1 as follows:

**Proposition 3** With an idea market and one-sided entry by \( i \), as long as \( \kappa_i \) is not too high or low, there is a unique interior equilibrium. Absent intervention, it is generally inefficient. The optimal policy, which yields efficiency, is the \( \tau_i \) in Proposition 2.

4 Technology Transfer with Imperfect Credit

We begin with the \( n \)'s fixed and an exogenous credit constraint. Then we endogenize both.

4.1 Exogenous Liquidity

Suppose that when \( e \) meets \( i \) in the idea market his payment must satisfy \( p \leq x \). There are two standard interpretations. One is that \( i \) insists on *quid pro quo*, \( e \) is holding transferable assets worth \( x \), and he cannot hand over more than he has (as in monetary models like those surveyed in Williamson and Wright 2010). Another is that \( e \) can promise to pay \( p \) to \( i \) in the next centralized market, but then \( e \) can renege, so \( i \) will only accept promises collateralized by the value \( x \) of \( e \)'s pledgeable assets (as in credit models like those surveyed in Gertler and Kiyotaki 2010). On the first interpretation there is finalization when the idea changes hands; on the second there is deferred settlement; but other than this irrelevant timing difference, nothing depends on the interpretation. In any case, for an idea to be traded to happen, two conditions have to be met: as always, \( e \) must have a higher probability of success, \( \sigma_i \leq \sigma_e \); and now, \( x \) must be big enough to cover \( i \)'s reservation price, \( \sigma_i \Delta \). Thus, we need \( \sigma_i \leq \min\{\sigma_e, x/\Delta\} \).

If the bargaining solution derived in Section 3, without the liquidity constraint, satisfies \( p \leq x \), then \( p = \Delta [\theta \sigma_i + (1 - \theta)\sigma_e] \) as before. It is easy to check that the
liquidity constraint is not violated iff
\[ \sigma_e \leq B \left( \sigma_i, \frac{x}{\Delta} \right) \equiv \frac{1}{1 - \theta} \left( \frac{x}{\Delta} - \theta \sigma_i \right). \] (19)

When \( \sigma_e > B(\sigma_i, x/\Delta) \), the unconstrained bargaining outcome \( p \) is infeasible, which leads to the following: if \( x/\Delta \geq \sigma_i \) the agents close the deal with \( e \) paying \( \tilde{p} = x < p \); and if \( x/\Delta < \sigma_i \) there is no trade because \( x \) cannot cover \( i \)'s reservation price. As shown in Figure 2, there is no trade in the region labeled \( A_0 \) because there are no gains from trade; there is no trade in \( A_3 \) because \( e \) cannot meet \( i \)'s reservation price; there is unconstrained trade in \( A_1 \) where \( e \) pays \( p_i \) and there is constrained trade in \( A_2 \) where \( e \) pays \( \tilde{p} = x \).

The number of successful innovations is given by
\[ N = \tilde{n}_i E \sigma_i + \tilde{n}_i \alpha_i E(\sigma_e - \sigma_i; x), \] (20)

where the expected increase in the success rate due to trade is now
\[ E(\sigma_e - \sigma_i; x) = E(\sigma_e - \sigma_i \mid \min\{\sigma_e, x/\Delta\} > \sigma_i) \Pr(\min\{\sigma_e, x/\Delta\} > \sigma_i), \]

less than it was with perfect credit. We can still write goods-market supply and demand as above, although there is now an additional effect on supply coming through \( N \), since \( \Delta \) depends on \( x \) and \( \bar{w} \). Appendix 3 shows by example that this can lead to multiplicity, but we can still guarantee uniqueness if \( \eta \) is not too big, as we assume from now on.\(^{13}\)

\(^{12}\)Again one can write down strategic bargaining models that generate this outcome. Also, note that the results partly depend on ideas being indivisible, so \( i \) cannot give \( e \) part of it. Still, they could trade using a lottery – \( e \) pays \( p \), then only gets the idea with probability \( \xi \). As Silveira and Wright (2010) show, if one allows lotteries, the main results go through, in the sense that there are still some meetings where \( i \) inefficiently keeps his idea even though \( \sigma_e > \sigma_i \).

\(^{13}\)To see how supply can be nonmonotone, derive
\[ S'(w) = \frac{N f'(H_1)}{f''(H_1)} + (1 - N) \frac{f'(H_0)}{f''(H_0)} + Z[(1 + \eta)f(H_1) - f(H_0)] \frac{dN}{dw}. \] (21)

The first two terms capture the result that, holding \( N \) fixed, higher \( w \) lowers hours and output. The final term is positive, however, because higher \( w \) relaxes the liquidity constraint, spurring
4.2 Endogenous Liquidity

The next important step is to endogenize $x$. First, from the total supply $A$, assume that a fraction $A_1 = \gamma A$ of the assets are liquid – i.e., they are transferable or pledgeable – while the remaining $A_0 = (1 - \gamma)A$ are not. Hence, only $A_1$ facilitate trade in the idea market, although $A_0$ can always be traded in the centralized market. While the stock $A_1$ is exogenous, the price and hence the value of liquid assets is endogenous, and this is what matters, since we now constrain $p$ by $x = (\phi + Z\delta) a'_1 / Z$, where $a'_1$ is the individual holdings of liquid assets. While it is certainly interesting to ask why certain assets can or cannot be used to facilitate trade in certain markets, we follow much good work (e.g., Kiyotaki and Moore 1997 or Holmstrom and Tirole 2011) and simply impose this – but as a special case we can set $\gamma = 1$, so that all assets are liquid.\(^{14}\) In any case, in general, agents now hold a portfolio $a = (a_0, a_1)$.

The dividend on both assets is still $\delta$, while the price of $a_j$ is now $\phi_j$. The gross return on $A_j$ is

$$1 + r_j = \frac{\phi_j' + Z'\delta}{\phi_j}. \tag{22}$$

As before, the illiquid asset $A_0$ must trade at the fundamental price, $\phi_0 = \beta\delta Z / (1 - \beta)$, which means $1 + r_0 = (1 + g) / \beta$. This is not necessarily true for the liquid asset $A_1$, however. Define the spread by

$$s \equiv \frac{r_0 - r_1}{1 + r_1} = \frac{(1 + g)\phi_1}{\beta(\phi_1' + Z'\delta)} - 1. \tag{23}$$

This is the marginal cost of liquidity – i.e., the return one sacrifices by holding $A_1$.

Figure 2 still applies, with $x = (\phi + Z\delta) a'_1 / Z$ endogenous but predetermined in the meeting. In addition to the equilibrium conditions described above, we now

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\(^{14}\)One interesting extension is to endogenize the quantity of liquid assets by modelling how individuals, and the economy as a whole, accumulate liquid assets over time to improve the allocation of ideas.
have to clear the market for $A_1$, which occurs when $s$ equates demand and supply. In terms of demand, consider first agents who have no possibility of buying ideas (everybody except active type $e$ agents). Since they will not sacrifice return for liquidity, we have the following: they are happy to hold any amount of $A_1$ if the spread is $s = 0$; they demand $0$ if $s > 0$; and they demand arbitrarily large positions if $s < 0$. In other words, demand for $A_1$ by these agents is horizontal at $s = 0$. Now consider active type $e$ agents. Integrating across the $A_j$ regions in Figure 2, their payoff in the idea market is:

$$V^e(a, Z) = W^e(a, Z; Z) + \alpha_e \theta \frac{X}{w} \int_{A_1} (\sigma_e - \sigma_i) Z \Delta + \alpha_e \frac{X}{w} \int_{A_2} \left[ \sigma_e Z \Delta - a_1(\phi_1 + Z \delta) \right]$$

In Appendix 4, we show that the FOC for $e$’s choice of $x = (\phi + Z \delta) a_1 / Z$ is given by $s = \ell(x)$, where $s$ is the spread and $\ell(x)$ is the marginal benefit of liquidity,

$$\ell(x) \equiv \alpha_e \frac{X}{\Delta} \int_{\sigma_e}^{1} \left( \sigma - \frac{x}{\Delta} \right) dF_e \left( \sigma \; \frac{x}{\Delta} \right)$$

$$- \alpha_e \int_{0}^{\frac{x}{\Delta}} \left\{ 1 - F_e \left( \sigma_i, \frac{x}{\Delta} \right) \right\} dF_i(\sigma_i).$$

The first term on the RHS of (24) gives the increase in $e$’s expected payoff from not losing deals because he cannot meet $i$’s reservation price, while the second gives the decrease from paying more when he could have closed the deal at $x$. The FOC $\ell(x) = s$ equates the marginal benefit and cost of liquidity. We also have to consider the SOC, which is not trivial in this kind of model, in general, but with that in mind, as in Wright (2010), we can describe market demand for liquidity $L(x)$ as follows. First, if $s < 0$ then demand is unbounded. Second, if $s = 0$ then type $e$ agents satiate in liquidity at $x(0)$. In this case type $e$ in aggregate demand $n_e x(0)$, and, if there is any left, others hold the rest, which they are happy to do at $s = 0$. Third, if $s > 0$ is not too big then type $e$ agents demand the $x = x(s)$ that solves $s = \ell(x)$, and everyone else demands $0$. Finally, $s > 0$ is too big then type $e$ as well as everyone else demand $x = 0$. One can also show that market demand is
decreasing.\textsuperscript{15}

\textbf{4.3 Asset Market Equilibrium}

To complete the analysis of the market for $A_1$, consider supply. Again, $A_1$ is fixed but the real value of liquid assets depends on the price $\phi_1$, or equivalently the spread $s$. Using the definitions of $x$, $r_0$ and $r_1$, and setting $\bar{n}_e a_e = A_1$ for all $s > 0$, we can write

$$s = s(x) = \frac{\phi_1 a_e}{\beta x} - 1 = \frac{x - \delta A_1/\bar{n}_e}{\beta x} - 1.$$  \hspace{1cm} (25)

This relation gives the spread required to make the real value of $A_1$ equal to $x$. Since $s(0) = -\infty$, $s'(x) > 0$ and $s(\infty) = r$, supply and demand intersect uniquely. Write asset market equilibrium as $AM(x, \bar{w}) = 0$, where

$$AM(x, \bar{w}) \equiv s(x) - L \left[ x/\Delta(\bar{w}) \right].$$  \hspace{1cm} (26)

From this we get a unique market-clearing $x$ for any $\bar{w}$, with $\partial x/\partial \bar{w} < 0$.

Asset-market equilibrium is shown in Figure 3, for different values of $\theta$ that translate into different demand, and different values of $A\delta$ that translate into different supply. Note that $\ell(x)$ can become negative, but market demand $L(x)$ is truncated below by the axis, since $s < 0$ always implies excess demand. Clearly, $e$ can be satiated at an $x(0)$ that is below the value of $x$ that allows him to close the deal in every idea market meeting. This can only occur if $\theta < 1$, however, as one can check $\theta = 1$ and $s = 0$ implies $e$ chooses $x$ so that he can close deals with probability 1. Heuristically, when $e$ buys liquid assets he is making an investment at cost $s$ (the forgone return). If $\theta < 1$, he has to share the surplus generated by this investment with $i$ – another holdup problem – and so $e$ underinvests in liquidity unless $\theta = 1$. Notice that $\theta = 1$ does not generally satisfy the Hosios condition,

\textsuperscript{15}For details, again, see Wright (2010). The result that $e$ satiates in liquidity at $x(0)$ can best be understood by noting there is always some $\hat{x}$ such that $x > \hat{x}$ implies the second term in (24) dominates the first, and so the marginal value of liquidity is negative, because any additional $x$ only increases $p$. This is standard in models with liquidity constraints and Nash bargaining.
however, so with endogenous entry efficiency requires $\theta = 1$ to avoid this holdup problem, plus subsidies to promote optimal participation (see below).

In any case, if $A_1$ is above some threshold $A_1^*$, defined below, then $s = 0$; and if $A_1 < A_1^*$, then $s > 0$. This much is standard (Geromichalos et al. 2007; Lagos and Rocheteau 2008; Lester et al. 2012). The novelty here concerns deriving the implications for innovation and growth. To pursue this, recall the goods market clearing condition (10), which we reproduce as $GM(x, \bar{w}) = 0$ with

$$GM(x, \bar{w}) \equiv \bar{w} - N(1 + \eta)f[H_1(\bar{w})] - (1 - N)f[H_0(\bar{w})] - A\delta. \quad (27)$$

As long as $\eta$ is not too big, this delivers $x$ as a function of $\bar{w}$, with $\partial x / \partial \bar{w} \geq 0$. Equilibrium is characterized by $(x, \bar{w})$ satisfying asset- and goods-market clearing, (26)-(27), from which we can easily find the rest of the variables. As shown in Figure 4, existence and uniqueness are immediate, given that $\eta$ is not too big.

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Table B: Effects of Parameters with Imperfect Credit

Notes: Parameters: $\gamma$: fraction of liquid assets, $A_0$: illiquid assets, $\eta$: technology improvement, $\chi$: disutility of labor, $\beta$: discount factor, $\theta$: bargaining power; Endogenous variables: $H_0, H_1$: employment, $\bar{w}$: normalized wage, $x$: liquidity, $s$: spread, $c$: consumption, $g$: growth rate, $N$: no. of successful innovations.

Table B reports the effects of parameters when $A_1 < A_1^*$. An increase in $\theta$, e.g., shifts the $AM$ curve up while $GM$ is unaffected, increasing $x, \bar{w}$ and $g$. Intuitively, low $\theta$ makes $e$ try to economize on liquidity, since he gets less of the surplus when buying ideas, and so he more frequently cannot meet the reservation price, which reduces idea trade and innovation. One can also show growth increases if matching frictions are reduced or $n_i$ increases, but not necessarily if $n_e$ increases. Consider a
rise in \( n_i \). This shifts \( GM \) and \( AM \) up, promoting growth via two effects. First, there are more meetings in the idea market. Second, since the arrival rate \( \alpha_e \) increases, \( e \) holds more liquidity. An increase in \( n_e \), however, while still increasing meetings, reduces rather than increases \( \alpha_e \), which has a negative effect on liquidity.

### 4.4 Entry and Efficiency

As in the previous versions, we now consider entry by \( i \). Appendix 5 shows equilibrium exists, although may not be unique. It also makes the point that liquidity does not necessarily promote growth – i.e., we can get \( \partial n_i/\partial A_1 < 0 \). And again we derive the optimal subsidy \( \tau_i \) simply by comparing the equilibrium and planner’s solution. We summarize the main results as follows:

**Proposition 4** With an idea market and imperfect credit, for fixed \( n_j \) there is a unique equilibrium if \( \eta \) is not too big. With entry by \( i \), equilibrium exists and \( n_i \in (0, \bar{n}_i) \) if \( \kappa_i \) is not too big or too small. It is inefficient unless three conditions are satisfied: entrepreneurs have all the bargaining power, \( \theta = 1 \); the supply of liquid assets is abundant, \( A_1 \geq A_1^* \equiv (\pi_1 - \pi_0) (1 - \beta)\bar{n}_e/\delta \); and the subsidy is set to

\[
\tau_i = \frac{G'(N)[\mathbb{E}\sigma_i + \mu_i \mathbb{E}(\sigma_e - \sigma_i)]}{rG(N)} + u'(c)Z \Delta_\mathbb{E}(\sigma_e - \sigma_i)\mu_i.
\]

### 5 Technology Transfer with Intermediation

It is now time to consider financial intermediation, and how that may contribute to innovation and growth.\(^{16}\)

#### 5.1 Banking

As is now becoming standard in this type of model, banks are introduced as in Berentsen et al. (2007) (see, e.g., Chiu and Meh 2011 or He et al. 2014 for other

\(^{16}\)In spirit, the following discussion is somewhat related to King and Levine (1993), although we think it is fair to say that the microfoundations are very different.
applications). These banks accept deposits at interest rate $r_d$ and make loans at $r_l$, although in equilibrium competition yields $r_l = r_d$ (this is not true in Chiu and Meh 2011, which has a transactions cost). For simplicity here borrowers can commit to repay bank loans, and bankers can commit to honor deposits, in the next centralized market, although one can endogenize these decisions. After meeting and observing the realization $(\sigma_i, \sigma_e)$ in the idea market, $e$ can choose to deposit his assets in, or borrow assets from, banks. Lack of commitment between $e$ and $i$ means that claims on liquid assets are still needed to trade in the idea market, even with commitment between $e$ and his bank.\footnote{By banks we mean any institution that can reallocate liquidity, in the spirit of Diamond and Dybvig (1983). Chiu and Meh (2010) allow a fixed cost $\xi$ to banking, and can capture financial development as a reduction in $\xi$. Similarly, Silveira and Wright (2010) assume that when $e$ is short of liquidity he can try to raise additional funds, but only succeeds with probability $\zeta$, so financial development is an increase in $\zeta$. Here we set $\xi = \zeta = 0$, and financial development is captured by comparing the outcomes with and without banking. One can also in principle introduce a parameter governing the efficiency of the banking sector by assuming that only a fraction of agents can access banks.}

For $e$ in the centralized market, we now have

$$W(a_1, d, z; Z) = \frac{X}{w}(\phi_1 + \delta Z)a_1 + \max_c \left\{ u(c) - \frac{X}{w}c \right\} + \frac{X}{w} \max H \{ z f(H) - wH \} + \max_{a_1'} \left\{ \beta V^k(a_1', Z') - \frac{X}{w} \phi_1 a_1' \right\} - \frac{X}{w} Zd(1 + r_d),$$

which is the same as (6) in the baseline model except for the last term, which gives the real value of debt obligations to a bank $d$ (if one has deposits in the bank then $d < 0$). Without loss in generality, bank loans are settled every period in the centralized market, and for this discussion we set $A_0 = 0$, since as demonstrated above illiquid assets do not affect growth. Then in the idea market, after observing $(\sigma_i, \sigma_e), i$ and $e$ bargain knowing that $e$ can always get a loan at interest rate $r_d$. The generalized Nash solution then delivers

$$p = p(\sigma_e, \sigma_i, r_d) = \Delta \left[ \theta \sigma_i + (1 - \theta) \frac{\sigma_e}{1 + r_d} \right].$$

It is easy to see the following: $\sigma_e < \sigma_i(1 + r_d)$ implies $e$ will deposit $x$ and not trade, because the gain does not cover the interest cost; and if $\sigma_e \geq \sigma_i(1 + r_d)$ then...
trades, depositing \( x - p(\sigma_e, \sigma_i) \) when \( \sigma_e < \bar{B}(\sigma_i, x) \) and borrowing \( p(\sigma_e, \sigma_i) - x \) when \( \sigma_e > \bar{B}(\sigma_i, x) \), with

\[
\bar{B}(\sigma_i, x) = \frac{1 + r_d}{1 - \theta} \left( \frac{x}{\Delta} - \theta \sigma_i \right).
\]

See Figure 5. Now asset-market clearing requires \( r_d = s \), where the spread is the same as in Section 4. Goods market clearing is also the same as before, with

\[
N = n_i \mathbb{E}(\sigma_i) + n_i \alpha_i \mathbb{E}[\sigma_e - \sigma_i | \sigma_e > \sigma_i(1 + r_d)] \Pr[\sigma_e > \sigma_i(1 + r_d)],
\]

since trade happens iff \( \sigma_e > \sigma_i(1 + r_d) \). Finally, deposits and loans have to net out, which requires

\[
\alpha_e \int_{A_1 \cup A_2 \cup A_3} p(\sigma_e, \sigma_i) \leq x, \text{ with } = \text{ when } r_d > 0.
\]

Summarizing, equilibrium now consists of \((x, r_d, \bar{w})\) clearing the asset and goods market, plus the netting of deposits and loans, which after simplification yield:

\[
(x - \gamma \delta A/\bar{n}_e) / \beta x = 1 + r_d \tag{28}
\]

\[
\chi \left[ N(1 + \eta)f(H_1) + (1 - N)f(H_0) + A\delta \right] = \bar{w} \tag{29}
\]

\[
\Delta \alpha_e \int_0^{1/r_d} \int_{\sigma_i(1+r_d)}^1 \left[ \theta \sigma_i + \frac{(1-\theta)\sigma_e}{1+r_d} \right] dF_e dF_i = x \text{ for } r_d > 0 \tag{30}
\]

We can write (29) as \( GM(r_d, \bar{w}) = 0 \) in \((r_d, \bar{w})\) space, with \( \partial \bar{w} / \partial r_d < 0 \). Similarly, we can write (28) and (30) as \( BM(r_d, \bar{w}) = 0 \) with

\[
BM(r_d, \bar{w}) = -\frac{\gamma \delta A}{\bar{n}_e [1 - \beta (1 + r_d)]} - \Delta \alpha_e \int_0^{1/r_d} \int_{\sigma_i(1+r_d)}^1 \left[ \theta \sigma_i + (1-\theta)\frac{\sigma_e}{1+r_d} \right] dF_e dF_i
\]

defining another negative relationship between \( r \) and \( \bar{w} \). Given these two downward sloping curves, we can show existence, but not uniqueness. There are two types of equilibria: one with \( r_d = 0 \) arises when \( A_1 \) is big, in which case ideas are traded whenever \( \sigma_e > \sigma_i \); and one with \( r_d > 0 \) arises when liquid assets are scarce. Importantly, the relevant threshold for sufficient liquidity is now \( A_1^* \), which is below the threshold \( A_1^* \) required for efficiency in the economy without banking (see Proposition 5 below).
5.2 The Roles of Intermediation

Banking here has two distinct roles. The first concerns reallocating liquidity: a given quantity $A_t$ can be channeled to those who need it most, which is not generally efficient without banks, because $e$ do not know how much liquidity he will need before he leaves the centralized market. This function is especially important when the arrival rate $\alpha_e$ is low, because that makes $e$ want to economize on liquidity. This illustrates clearly how search interacts with liquidity considerations and intermediation. This is relevant to the extent that, as some people argue, there is a shortage of liquid assets in reality (e.g., Caballero 2006). The second and more novel function of banking is that it helps get around the holdup problem associated with investments in liquidity by allowing entrepreneurs to undo these investments and hence increasing their reservation payoffs. Intuitively, without banks, when $i$ asks for a high $p$, $e$ would like to claim that he shouldn’t have to pay so much because he needs to cover his cost, the spread $s$. But $i$ counters that this is a sunk cost, which leads to a high $p$ and hence underinvestment in liquidity. When banks are open, however, $e$ has the option of depositing his assets, which in equilibrium earns $r_d = s$, and therefore the cost is not completely sunk.

Of course, not everyone can do this, since deposits can exceed loans only if $r_d = 0$. But since each individual behaves competitively with respect to banking, the threat by $e$ of putting his money in the bank and earning the going rate $r_d$ is credible in bilateral negotiations. This is especially important when $\theta$ is low, because then the holdup problem is severe. This illustrates how bargaining interacts with liquidity considerations and intermediation. The effect has not been noticed, we think, because the related papers on intermediation and liquidity assume competitive pricing, which means they ignore holdup problems. Bargaining is especially pertinent in the idea market, which is sufficiently thin that the competitive price-taking hypothesis is not compelling, and where there is often one-off trade so that reputation may
not overcome the problem. Therefore, financial intermediation may be particularly significant in the context of technology transfer, innovation and growth. Table C reports the effects of parameters, assuming an equilibrium with \( r_d > 0 \) exists uniquely. An increase in \( \gamma \), e.g., shifts the \( BM \) curve down while \( GM \) is unaffected, reducing \( r_d \), increasing \( \bar{w} \) and raising \( N \) and \( g \).\(^{18}\)

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Table C: Effects of Parameters with Intermediation

Notes: Parameters: \( \gamma \): fraction of liquid assets, \( A_0 \): illiquid assets, \( \chi \): disutility of labor, \( \beta \): discount factor, \( \theta \): bargaining power; Endogenous variables: \( H_0, H_1 \): employment, \( \bar{w} \): normalized wage, \( r_d \): deposit rate, \( s \): spread, \( c \): consumption, \( g \): growth rate, \( N \): no. of successful innovations

### 5.3 Entry and Efficiency

The last step in this version of the model is to again endogenize entry by \( i \) and solve for the optimal subsidy. The results, verified in Appendix 6, are summarized in Proposition 5 below. What we emphasize is that, compared to Proposition 4, one can see explicitly the two functions of banks: efficiency now requires a smaller quantity of liquid assets; and we do not need \( \theta = 1 \), because banking eliminates the holdup problem associated with investments in liquidity.

**Proposition 5** With an idea market and intermediation, for fixed \( n_j \) there exists an equilibrium. With entry by \( i \), an equilibrium with \( r_d = 0 \) exists if \( \alpha_e = \mu(n_i, n_e)/n_e \) is not too big. It is generally inefficient unless two conditions are satisfied: \( A_1 \geq A_{1}^{**} = \Omega A_1^* \), where

\[
\Omega = \alpha_e \int_0^1 \int_{\sigma_i}^1 [\theta \sigma_i + (1 - \theta)\sigma_e]dF_e(\sigma_e|\sigma_i)dF_i(\sigma_i) \leq 1;
\]

\(^{18}\)The effects of \( \eta \) is not reported in Table C because its change will shift both \( GM \) and \( BM \), and always leads to ambiguous results.
and \( \tau_i \) is set as in Proposition 4. We do not need the third condition in Proposition 4 for the economy without banking, \( \theta = 1 \).

6 Some Evidence

Here we report some evidence to support the case that technology transfer can be an important part of the innovation process, and that credit imperfections hinder this process. Our empirical analysis makes use of the firm level data obtained from the World Bank Enterprise Surveys conducted between 2002 and 2005. The whole sample includes 4059 firms across 33 countries. We follow closely the statistical analysis in Carluccio and Fally (2009), but appropriately modify the sample and choice of variables to address our own research questions. Before going to detail, we highlight two findings: (i) in some countries (e.g., Germany), direct technology transfers from outside parties are an important way for firms to acquire new technology; (ii) firms’ use of technology transfer is positively correlated with the financial development in a country, particularly for small firms.

Using survey responses, we can determine whether a firm has acquired a new technology in the period 2002-2005. Given our interest in direct technology transfer, we restrict attention to arm’s length transfers from outside parties. In particular, firms in our sample are asked to report the most important way that they acquired new technology in the last 36 months. We focus on transfers through new licensing or turnkey operations obtained from international sources, domestic sources, universities and public institutions. We do not include transfers resulting from hiring, transfers from parent companies, internal development, and development in cooperation with other partners. In Table 1 (all data tables are at the end of the paper), we report cross-country summary statistics regarding the fraction of firms using direct technology transfers, and its relationship to financial development and firm size. Direct transfers are an important source of technology acquisition in some coun-
tries. In Germany, 12.6% of firms in the survey reported that the most important way they acquire technology is through new licensing or turnkey operations from international sources, domestic sources, universities and public institutions.

To study the effects of intermediation on technology transfer, we follow the literature and proxy financial development of a country by the ratio of private credit to GDP, taken from Beck et al. (1999). Table 2 indicates that, overall, a higher level of financial development is associated with higher rates of technology transfer. The positive correlation is more significant for smaller firms, and tends to become smaller or even reversed as firm size increases. Tables 3-5 report results from three regressions to uncover the effects of financial development. Other control variables in the regression include market size, price of investment, openness, investment level, firm size, presence of foreign capital and industry dummies.\textsuperscript{19} Table 3 reports results from a simple OLS regression. This yields a positive relationship between private credit to GDP and technology transfer, significant at the 10\% level. This positive relation is strongly strengthened when the square of private credit to GDP is introduced, significant at the 1\% level, when we control for firm and country specific variables.

To deal with endogeneity issues, in Table 4, we follow Carluccio and Fally (2009) and Djankov et al. (2007) to instrument for private credit over GDP by legal origin and use 2SLS. This leads to considerably larger coefficients than the OLS regressions. Technology transfer is positively affected by private credit to GDP, with significant results at the 1\% level in all six specifications. The strong positive effects still exists when controls for country and firm characteristics are excluded. Table 5 shows results from a probit regression, which are similar in terms of economic conclusions. The general pattern over the different specifications is that the level of financial development has positive but diminishing effects on technology transfer, and the

\textsuperscript{19} Variable definitions accompany the Tables; See Carluccio and Fally (2009) for a more detailed discussion of the statistical approach.
effect is greater for smaller firms. This is all broadly consistent with our theory.

While the above analysis focuses on how technology transfer depends on the level of financial development in a country, there is also an empirical literature that studies how the decision to acquire technology depends on a firm’s own liquidity and financial constraints. Montalvo and Yafeh (1994), e.g., examine investment in foreign technology by Japanese firms in the form of licensing agreements. They conclude that “liquidity is an important consideration in the firm’s decision to invest in foreign technology.” In particular, they find that “Cash flow has a positive impact, and $REALCF$ (cash flow of firms with limited access to main bank loans) is always positive and significant. Furthermore, the coefficient of $REALCF$ is much higher than that of cash flow, implying that non-keiretsu firms are more liquidity constrained than group-affiliated firms”. Also, Gorodnichenko and Schnizter (2013) study Business Environment and Enterprise Performances Surveys from 2002 to 2005, covering a broad array of sectors and countries, and containing direct measures of innovation and financial constraints. They find evidence that innovative activity is strongly influenced by financial frictions.

Finally, our theory suggests banking enhances efficiency because entrepreneurs with access to banks are in a better position when negotiating with innovators, and therefore acquire technology at better terms. Ideally, one would test this by investigating the correlation between buyers’ access to financing and the prices they pay for technology transfers. Unfortunately, owing to the lack of reliable data, few papers have examined this relationship.20 One exception is Sakakibara (2010), who examined the determinants of patent prices using a unique dataset of 661 Japanese patent licensing contracts. He found that, after controlling for the attributes of licensors, licensees, contracts and patents, the size of a licensee has negative and highly significant effect on the price of licensing, and concluded “large licensees

20Researchers studying patent pricing told us that reliable public data was hard to find because the price of transactions and buyers characteristics are highly confidential.
appear to exercise greater bargaining power.” To the extent that large firms tend to have better access to financing, as is often assumed, one can argue that the model predictions are consistent with this evidence.

This discussion of evidence is brief, and in the future more empirical work could be done to better uncover the importance of technology transfer, how it depends on liquidity and financial considerations, and the implications for growth. The paper’s main goal has been to lay out a theoretical framework within which one can organize such empirical work, and the discussion in this section is mainly an illustration of how some simple observations support the general approach. We also believe that a current shortage of definitive data should not stand in the way of considering new models – there is no reason to think that there will never in the future emerge more information to better test these kinds of theories.

7 Conclusion

We conclude as we began by suggesting that the generation and implementation of new ideas are major factors underlying economic performance and growth, and that financial considerations play a role in this process. By studying a series of environments, increasing in complexity, the paper has constructed an endogenous growth model with participation decisions by innovators and entrepreneurs, where productivity increases with research and development. Progress is aided by the exchange of ideas, since those who come up with them are not necessarily the best at implementing them. In case it is not obvious, a well-functioning market for ideas contributes to innovation in two ways: (i) it gets ideas into the hands of those who are better able to develop them; and (ii) it encourages entry into innovative activity in the first place, since innovators not only can try to implement on their own, they may have opportunities to trade their ideas. However, the market incorporated search, bargaining and credit frictions that hinder trade. We did not model all of
the institutional details, but tried to capture these frictions at an abstract level. Other frictions, including private information, were downplayed, but it would surely be interesting to bring them into the mix in future work.

A main goal was to see how intermediation affects the market for ideas, and thus affects technology transfer, innovation and growth. One result is that banks allow the economy to get by with fewer liquid assets, by reallocating them from those that need them less to those that need them more. This helps get around a basic search/matching problem that implies entrepreneurs do not always have sufficient liquidity when they contact an innovator, since they did not know how much liquidity they might need before contacting him. A perhaps more subtle result is that intermediaries also mitigate holdup problems in bargaining by allowing entrepreneurs to undo otherwise sunk investments in liquidity. With or without intermediation, the framework provides several useful insights, e.g., how to optimally subsidize innovative and/or entrepreneurial activity in the presence of search and knowledge externalities. We studied existence, uniqueness, efficiency and comparative statics for a series of increasing intricate models. There is much left to do in terms of theory, and empirical work, clearly, but we think we learned a lot from this exercise.
Appendix 1: Capital

Consider a CRS technology \( f(K, ZH, ZT) \), where \( K \) is capital and \( T \) is a fixed input, say the talent of the owner. We subsume depreciation in the notation \( f \). Here we present the case where \( n_i \) is endogenous while \( n_e \) is fixed. Consider the planner’s problem (equilibrium is similar):

\[
V(Z, K) = \max \left\{ u(c) - \chi[(1 - N)H_0 + NH_1] - \kappa_i n_i + \beta V[G(N)Z, K'] \right\}
\]

\[
st \quad c = Nf[K_1, Z(1 + \eta)H_1, Z(1 + \eta)T] + (1 - N)f(K_0, ZH_0, ZT) + \delta ZA - K'
\]

\[
K = NK_1 + (1 - N)K_0,
\]

plus \( n_i \in [0, \bar{n}_i] \) and \( N = n_i E_s \), where the choice is \( (c, H_0, H_1, K_0, K_1, K', n_i) \). After eliminating constraints, FOC’s are:

\[
\begin{align*}
H_0 & : \quad u'(c)Zf^0_H = \chi \\
H_1 & : \quad u'(c)(1 + \eta)Zf^1_H = \chi \\
K_1 & : \quad f^1_K = f^0_K \\
K' & : \quad u'(c) = \beta V_K(Z', K') \\
n_i & : \quad \kappa_i / E_s = (f^1 - f^0 - f^1_KK_1 + f^0_KK_1)u'(c) - \chi(H_1 - H_0) + \beta V_G[NZ, K']G'(N)Z
\end{align*}
\]

where \( f^0_H = f_H(K_0, ZH_0, ZT) \), etc. The envelope conditions are

\[
\begin{align*}
V_Z(Z, K) & = \Phi u'(c) + \beta V_Z(Z', K')G(N) \\
V_K(Z, K) & = (f^0 + 1 - \delta)u'(c)
\end{align*}
\]

where \( \Phi \equiv N(1 + \eta)\left[(f^1_HH_1 + f^1_TT) + (1 - N)\left(f^0_HH_0 + f^0_TT\right) + \delta A\right] \).

A balanced growth equilibrium has \( Z, c, K, f^1 \) and \( f^0 \) growing at rate \( Z' / z = G(N) \) while \( H_0, H_1 \) and \( n_i \) are constant. By CRS, \( \Phi \) is also constant, implying \( V_Z = \Phi / c(1 - \beta) \). Then equilibrium is given by \( (H_0, H_1, K_0, K_1, n_i, c, K, N) \) solving

\[
\begin{align*}
N & = n_i E_s \\
G(N) & = \beta(f^0_K + 1 - \delta) \\
\kappa_i / E_s & = u'(c)(f^1 - f^0 - f^1_KK_1 + f^0_KK_1) - \chi(H_1 - H_0) + \beta \frac{\Phi G'(N)Z}{c(1 - \beta)} \\
c & = f^0_H/Z \\
c & = Nf^1 + (1 - N)f^0 + (1 - \delta)K - G(N)K + \delta ZA \\
f^0_H & = (1 + \eta)f^1_H \\
f^0_K & = f^1_K \\
K & = NK_1 + (1 - N)K_0
\end{align*}
\]

34
It is now straightforward to study this model following the analysis in the text, with similar results.

Appendix 2: Two-Sided Entry

We show there is a unique equilibrium in the two-sided entry model of Section 3, with \( n_i \in (0, \tilde{n}_i) \) and \( n_e \in (0, \tilde{n}_e) \) if \( \kappa_i \) and \( \kappa_e \) are not too high or low. The equilibrium conditions are

\[
\hat{w} / f_H^0 = N(1 + \eta) f(H_1) + (1 - N)f(H_0) + A\delta \tag{31}
\]

\[
\kappa_i = \Delta(\hat{w})[E\sigma_i + (1 - \theta)\frac{m(n_i,n_e)}{n_i} \hat{E}(\sigma_e - \sigma_i)]\chi/\hat{w} \tag{32}
\]

\[
\kappa_e = \Delta(\hat{w})[\theta\frac{m(n_i,n_e)}{n_e} \hat{E}(\sigma_e - \sigma_i)]\chi/\hat{w} \tag{33}
\]

where \( N = n_i E\sigma_i + \mu(n_i,n_e) \hat{E}(\sigma_e - \sigma_i) \). Define \( \zeta = n_e / n_i \), and write (31)-(33) as

\[
\kappa_i = \Delta(\hat{w})[E\sigma_i + (1 - \theta) \mu(1,\zeta) \hat{E}(\sigma_e - \sigma_i)]\chi/\hat{w} \tag{34}
\]

\[
\kappa_e = \Delta(\hat{w})[\theta \mu(1/\zeta,1) \hat{E}(\sigma_e - \sigma_i)]\chi/\hat{w}. \tag{35}
\]

In \((\hat{w}, \zeta)\) space, the former gives a strictly increasing curve and the latter a strictly decreasing curve. The unique intersection determines equilibrium \((\hat{w}, \zeta)\). Denote this wage by \( \hat{w}(\kappa_i, \kappa_e) \), where \( \partial \hat{w} / \partial \kappa_i < 0 \) and \( \partial \hat{w} / \partial \kappa_e < 0 \). Also, \( \hat{w}(\kappa_i, \kappa_e) \) gets arbitrarily large for entry costs sufficiently small.

The \((\hat{w}, \zeta)\) pair still needs to satisfy goods market clearing

\[
\hat{w} / \chi = n_i [E\sigma_i + \mu(1,\zeta) \hat{E}(\sigma_e - \sigma_i)][(1 + \eta)f(H_1) - f(H_0)] + f(H_0) + A\delta,
\]

and we need to check that \((n_i, n_e)\) is interior,

\[
n_i = \frac{\hat{w} / \chi - f(H_0) - A\delta}{[E\sigma_i + \mu(1,\zeta) \hat{E}(\sigma_e - \sigma_i)](1 + \eta)f(H_1) - f(H_0)} \in (0, \tilde{n}_i) \tag{36}
\]

\[
n_e = \zeta n_i \in (0, \tilde{n}_e). \tag{37}
\]

The numerator in (36) is a strictly increasing function of \( \hat{w} \) and is 0 for an unique \( \hat{w} \). So we can find \( \hat{\kappa}_i \) and \( \hat{\kappa}_e \) such that \( \hat{w}(\hat{\kappa}_i, \hat{\kappa}_e) / \chi - f(H_0) - A\delta = 0 \), implying \( n_i = n_e = 0 \). By continuity, we can then find \( \kappa_i \) and \( \kappa_e \) close to but bigger than \( \hat{\kappa}_i \) and \( \hat{\kappa}_e \) such that (36)-(37) are satisfied. This establishes that \((\hat{w}, \zeta)\) is unique. To see that \((n_i, n_e)\) is unique, note that equilibrium is given by an intersection of two curves in the \((n_i, n_e)\) space. One is the strictly decreasing relationship between \( n_i \) and \( n_e \).
implicitly defined by (31) given \( \bar{w} \); the other is the strictly increasing relationship defined by (37) given \( \zeta \). Then \((n_i, n_e)\) is determined by the unique intersection. ■

Appendix 3: Multiple Equilibria

Here we provide an example to show supply can be non-monotonic, and hence we can get multiplicity, in the model of Section 4 without the assumption made in the text that \( \eta \) is not too big. Set \( A\delta = 0 \). Letting \( f(H) = 1 - \exp(-H) \), it is easy to solve for:

\[
\begin{align*}
  f [H_0(\bar{w})] &= \begin{cases} 
    1 - \bar{w} & \text{if } \bar{w} \leq 1 \\
    0 & \text{if } \bar{w} > 1
  \end{cases} \\
  f [H_1(\bar{w})] &= \begin{cases} 
    1 - \bar{w}/(1 + \eta) & \text{if } \bar{w} \leq 1 + \eta \\
    0 & \text{if } \bar{w} > 1 + \eta
  \end{cases}
\end{align*}
\]

Given \( N \), supply is

\[
S = \begin{cases} 
  Z [N(1 + \eta - \bar{w}) + (1 - N)(1 - \bar{w})] & \text{if } \bar{w} \leq 1 \\
  Z [N(1 + \eta - \bar{w})] & \text{if } \bar{w} \in (1, 1 + \eta) \\
  0 & \text{if } \bar{w} \geq 1 + \eta
\end{cases}
\]

To describe \( N(\bar{w}) \), first compute:

\[
\Delta(\bar{w}) = \begin{cases} 
  \eta - \bar{w} \log(1 + \eta) & \text{if } \bar{w} \leq 1 \\
  1 + \eta - \bar{w} \left[ 1 - \log \left( \frac{\bar{w}}{1 + \eta} \right) \right] & \text{if } \bar{w} \in (1, 1 + \eta) \\
  0 & \text{if } \bar{w} \geq 1 + \eta
\end{cases}
\]

Since \( \Delta'(\bar{w}) < 0 \) for \( \bar{w} < 1 + \eta \) and \( \Delta(1 + \eta) = 0 \), \( x/\Delta(\bar{w}) \) is strictly increasing and approaches \( \infty \) as \( \bar{w} \to 1 + \eta \). So \( \eta > x \) implies there is a \( \bar{w}' \in (0, 1 + \eta) \) such that

\[
\min \left\{ \frac{x}{\Delta(\bar{w})}, 1 \right\} = \begin{cases} 
  \frac{x}{\Delta(\bar{w})} & \text{if } \bar{w} \leq \bar{w}' \\
  1 & \text{if } \bar{w} > \bar{w}'
\end{cases}
\]

Moreover, we have

\[
\bar{w}' = \begin{cases} 
  \in (0, 1] & \text{if } x > \eta - \log(1 + \eta) \\
  (1, 1 + \eta) & \text{if } x < \eta - \log(1 + \eta)
\end{cases}
\]

and \( N = n_i E(\sigma_i) + n_i \alpha_i \int_0^{\bar{x}} \int_{\sigma_i}^{1} (\sigma_e - \sigma_i) dF_e(\sigma_e|\sigma_i) dF_i(\sigma_i) \). Then, after simplification,

\[
S'(\bar{w}) = Z \left( -1 - \Delta'(\bar{w}) \eta n_1 \alpha_i \frac{x}{\Delta(\bar{w})^2} \int_{\bar{x}(\bar{w})}^{1} \left[ \sigma_e - \frac{x}{\Delta(\bar{w})} \right] dF_e(\sigma_e|\sigma_i) \frac{x}{\Delta(\bar{w})} f_i \left( \frac{x}{\Delta(\bar{w})} \right) \right),
\]

where \( \Delta'(\bar{w}) < 0 \) for \( \bar{w} < 1 + \eta \).

Therefore supply can have a positive slope when the distribution is sufficiently concentrated over the relevant region, as shown in Figure 6. Then it is easy to
specify demand so that we get multiplicity. Note that the above construction uses \( \bar{w} < 1 + \eta \) as well as \( \eta > x \). The restriction made in the text that \( \eta \) is not too big rules this out and allows us to prove uniqueness.

Appendix 4: Entrepreneurs

Here we formulate \( e' \)'s maximization problem as in Section 4. Start with the intuitive expression

\[
V^e(a_0, a_1, Z) = (1 - \alpha_e) W^e(a_0, a_1, Z; Z) + \alpha_e \int_{A_0} W^e(a_0, a_1, Z; Z) \\
+ \alpha_e \int_{A_1} \left\{ \sigma_e W^e \left[ a_0, a_1 - \frac{pZ}{\phi_1 + Z\delta}, Z; Z \right] + (1 - \sigma_e) W^e \left[ a_0, a_1 - \frac{pZ}{\phi_1 + Z\delta}, Z; Z \right] \right\} \\
+ \alpha_e \int_{A_2} \{ \sigma_e W^e [0, 0, Z(1 + \eta); Z] + (1 - \sigma_e) W^e(0, 0, Z; Z) \} \\
+ \alpha_e \int_{A_3} W^e(a_0, a_1, Z; Z).
\]

The first term is \( e' \)'s payoff when he does not meet anyone. The second is his payoff when he meets \( i \) but there are no gains from trade. The third is his payoff from (unconstrained) trade at \( p \). The fourth is his payoff from (constrained) trade at \( \bar{p} \). The final term is his payoff to not trading because he cannot meet \( i \)'s reservation price. Now algebra leads to

\[
V^e(a_0, a_1, Z) = W^e(a_0, a_1, Z; Z) + \frac{\alpha_e \theta \chi}{w} \int_{A_1} (\sigma_e - \sigma_i) Z \Delta + \frac{\alpha_e \chi}{w} \int_{A_2} [\sigma_e Z \Delta - a_1(\phi_1 + Z\delta)]
\]

Notice \( a_1 \) affects the area of the different \( A_j \) regions, and hence the probability of trade, as well as the terms of trade when the constraint binds, as seen in the integrand of the last term. Consider now the portfolio choice \( (a'_0, a'_1) \) in the centralized market. Since the choice of illiquid asset \( a'_0 \) is actually irrelevant for \( e' \)'s payoff, given illiquid assets are priced fundamentally, we can ignore it. For the liquid asset, it is convenient to redefine \( e' \)'s choice as \( x = a'_1(\phi_1 + Z\delta)/Z \), rather than \( a'_1 \), analogous to using real rather than nominal balances in monetary theory. Given this, write \( \tilde{V}^e(0, x, Z) = V^e_t(0, a_1, Z) \) where

\[
\tilde{V}^e(0, x, Z) = \text{const} + \frac{\chi}{\bar{w}x} + \frac{\alpha_e \theta \chi}{\bar{w}} \int_0^{\sigma_e} \int_{\sigma_i} (\sigma_e - \sigma_i) \Delta dF_e(\sigma_e|\sigma_i)dF_i(\sigma_i)
\\
+ \frac{\alpha_e \chi}{\bar{w}} \int_0^{\sigma_e} \int_{B(\sigma_i, x)} (\sigma_e \Delta - x) dF_e(\sigma_e|\sigma_i)dF_i(\sigma_i).
\]
Then
\[
s x' = \left[ \frac{(1 + g) \phi_1}{\beta(\phi'_1 + Z' \delta)} - 1 \right] x' = \frac{(1 + g) \phi_1 a'}{\beta Z'} - x',
\]
implying \( \phi_1 a' = (\beta s x' Z' + \beta x' Z') / (1 + g) \). Then we can rewrite \( e \)'s objective function in the centralized market maximization problem as
\[
-s x + \alpha_e \theta \int_0^{\tilde{x}} \int_{\sigma_i}^{B(\sigma_i, \frac{\tilde{x}}{Z})} (\sigma_e - \sigma_i) \Delta F_e(\sigma_e | \sigma_i) dF_i(\sigma_i)
\]
\[
+ \alpha_e \int_0^{\tilde{x}} \int_{\sigma_i}^{1} (\sigma_e \Delta - x) dF_e(\sigma_e | \sigma_i) dF_i(\sigma_i).
\]
Maximizing wrt \( x \), using Leibniz Rule and a little algebra, we get the FOC \( s = \ell(x) \) where \( \ell(x) \) is defined in (24).  

**Appendix 5: Entry & Credit Frictions**

Here we substantiate some claims made in Section 4. Equilibrium \( (n_i, x, \bar{w}, N) \) satisfies
\[
(1 - \beta) x \bar{n}_e - \delta A_1 - \beta x \bar{n}_e L \left[ \frac{x}{\Delta(\bar{w})} ; n_i \right] = 0 \quad (38)
\]
\[
\bar{w} / \chi - N(1 + \eta) f [H_1(\bar{w})] - (1 - N) f [H_0(\bar{w})] - A \delta = 0 \quad (39)
\]
\[
N - n_i \mathbb{E} \sigma_i - n_i \alpha_i \int_0^{\tilde{x}} \int_{\sigma_i} \Delta F_e(\sigma_e | \sigma_i) dF_i(\sigma_i) = 0 \quad (40)
\]
plus the entry condition
\[
\kappa_i > \bar{\kappa}_i(0) \text{ if } n_i = 0; \quad \kappa_i = \bar{\kappa}_i(n_i) \text{ if } n_i \in (0, \bar{n}_i) \text{ and } \kappa_i < \bar{\kappa}_i(\bar{n}_i) \text{ if } n_i = \bar{n}_i, \quad (41)
\]
where
\[
\bar{\kappa}_i(n_i) = \frac{\chi}{\bar{w}} \Delta \mathbb{E} \sigma_i + \frac{\chi}{\bar{w}} \Delta (1 - \theta) \frac{\mu(n_i, \bar{n}_e)}{n_i} \int_0^{\tilde{x}} \int_{\sigma_i} \Delta F_e(\sigma_e | \sigma_i) dF_i(\sigma_i).
\]

As \( n_i \) increases, both the upward sloping \( GM \) curve and the downward sloping \( AM \) curve shift up in \( (x, \bar{w}) \) space. Therefore, (38)-(40) define an increasing and continuous function \( \bar{w} = \zeta_w(n_i) \) from \( [0, \bar{n}_i] \) onto \([\bar{w}(0), \bar{w}(\bar{n}_i)]\). Moreover, \( (x, \bar{w}) \) pairs that satisfy (38)-(40) define a function \( x = \zeta_x(\bar{w}) \) with range \([\bar{w}(0), \bar{w}(\bar{n}_i)]\). We now need to check the entry condition. First, since \( \bar{\kappa}_i(n_i) \) is strictly decreasing in \( n_i \), for any \( \bar{w} \in [\bar{w}(0), \bar{w}(\bar{n}_i)] \) and \( x = \zeta_x(\bar{w}) \), there is a unique \( n_i \in [0, \bar{n}_i] \) satisfying (41). So we can construct a continuous mapping from \( \bar{w} \in [\bar{w}(0), \bar{w}(\bar{n}_i)] \) to \([0, \bar{n}_i]\. 

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Therefore, given the continuous increasing function \( \kappa_w(n_i) \), this ensures that an equilibrium exists.

Next we show that \( n_i \) can decrease with \( A_1 \). Given \( n_i \in (0, \bar{n}_i) \), we derive

\[
\left( \frac{\delta A_1}{x} - \beta x \bar{n}_e \frac{L'}{\Delta} \right) dx + \beta \bar{n}_e x L' \frac{x}{\Delta^2} \Delta' \bar{d}w - \beta x \bar{n}_e \frac{dL}{dn_i} \bar{d}n_i = \delta dA_1
\]

\[
\bar{G} \bar{d}w - [(1 + \eta) f(H_1) - f(H_0)] dN = 0
\]

\[-\mu \Omega \frac{1}{\Delta} dx + \mu \Omega \frac{x}{\Delta^2} \Delta' \bar{d}w + dN - (\mathbb{E} \sigma_i + \mu_i \Phi) n_i dN = 0
\]

\[-\left[ \frac{\chi}{\bar{w}} \Delta(1 - \theta) \frac{\mu \Omega}{n_i \Delta} \right] dx + \bar{E} \bar{d}w - \left[ \frac{\chi}{\bar{w}} \Delta(1 - \theta) \frac{d(\frac{\mu}{n_i})}{dn_i} \Phi \right] dn_i = 0
\]

where

\[
\Phi = \int_0^1 \int_{\sigma_i}^{\sigma_i} (\sigma_e - \sigma_i) dF_e(\sigma_e | \sigma_i) dF_i(\sigma_i)
\]

\[
\Omega = \int_0^1 (\sigma_e - \frac{x}{\Delta}) f_i \left( \frac{x}{\Delta} \right) dF_e(\sigma_e | \sigma_i)
\]

\[
\bar{G} = \frac{1}{\Delta} - N(1 + \eta)f'(H_1)H_1'(\bar{w}) - (1 - N)f'(H_0)H_0'(\bar{w})
\]

\[
\bar{E}(\theta) = \frac{\chi}{\bar{w}^2} \Delta \mathbb{E} \sigma_i - \frac{\chi}{\bar{w}} \mathbb{E} \sigma_i \Delta' + \frac{\chi}{\bar{w}^2} \Delta(1 - \theta) \frac{\mu}{n_i} \Phi - \frac{\chi}{\bar{w}} \Delta'(1 - \theta) \frac{\mu}{n_i} \Phi + \frac{\chi}{\bar{w}} \Delta(1 - \theta) \frac{\mu}{n_i} \Omega \frac{x}{\Delta^2} \Delta'
\]

Note that \( \bar{E}(\theta) > 0 \) at least for \( \theta \approx 1 \). Then we have

\[
\Upsilon = \left[ \begin{array}{c}
dx \\
\bar{d}w \\
\bar{d}N \\
\bar{d}n_i \\
\end{array} \right] = \left[ \begin{array}{c}
\delta dA_1 \\
0 \\
0 \\
0 \\
\end{array} \right]
\]

where

\[
\Upsilon = \left[ \begin{array}{cccc}
\frac{\delta A_1}{x} - \beta x \bar{n}_e \frac{L'}{\Delta} & \beta \bar{n}_e x L' \frac{x}{\Delta^2} \Delta' & 0 & \frac{-\beta x \bar{n}_e \frac{dL}{dn_i}}{\Delta} \\
0 & \bar{G} & -(1 + \eta)f(H_1) + f(H_0) & 0 \\
-\mu \Omega \frac{1}{\Delta} & \mu \Omega \frac{x}{\Delta^2} \Delta' & 1 & -(\mathbb{E} \sigma_i + \mu_i \Phi) \\
-\frac{\chi}{\bar{w}} \Delta(1 - \theta) \frac{\mu \Omega}{n_i \Delta} & \bar{E} & 0 & \frac{-\chi}{\bar{w}} \Delta(1 - \theta) \frac{\partial(\frac{\mu}{n_i})}{\partial n_i} \Phi \\
\end{array} \right]
\]

One can show \( \det(\Upsilon) > 0 \) at least for \( \theta \approx 1 \). Then

\[
\det(\Upsilon) \frac{\partial \bar{w}}{\partial A_1} = \delta [(1 + \eta)f(H_1) - f(H_0)] (1 - \theta) \Omega \frac{\chi}{\bar{w}} n_i \left( \mathbb{E} \sigma_i + \mu_i \Phi \right) - \mu \left( \frac{\partial(\frac{\mu}{n_i})}{\partial n_i} \Phi \right)
\]

So \( \partial \bar{w} / \partial A_1 = 0 \) when \( \theta = 1 \) and \( \partial \bar{w} / \partial A_1 > 0 \) when \( \theta \in (\theta_0, 1) \) for some \( \theta_0 < 1 \). Since \( \partial N / \partial \bar{w} = \bar{G} / [(1 + \eta)f(H_1) - f(H_0)] > 0 \), we have \( \partial N / \partial A_1 = 0 \) when \( \theta = 1 \).
and $\partial N/\partial A_1 > 0$ when $\theta \in (\theta_0, 1)$. Then we have

$$
\text{det}(\mathbf{Y}) \frac{\partial n_i}{\partial A_1} = \delta \left\{ \frac{\chi}{\bar{w}} \Delta (1 - \theta) \frac{\mu}{n_i} \Omega \Delta \bar{G} - [(1 + \eta) f(H_1) - f(H_0)] \mu \Omega \frac{1}{\Delta} \bar{E} \\
+ [(1 + \eta) f(H_1) - f(H_0)] \mu \Omega \frac{x}{\bar{w}} \Delta (1 - \theta) \frac{\mu}{n_i} \Delta \right\}
$$

Therefore, $\partial n_i/\partial A_1 < 0$ when $\theta \in (\theta_0, 1)$. ■

**Appendix 6: Equilibrium with Intermediation**

We prove existence in the model of Section 5. First consider fixed participation.

Then $GM(r_d, \bar{w}) = 0$ defines $\bar{w}$ as a decreasing function of $r_d$ in $(r_d, \bar{w})$ space, with intercept $\bar{w}_0$ given by the solution to (10) with

$$
N = \bar{n}_i \mathbb{E}(\sigma_i) + \bar{n}_i \alpha_i \int_0^1 \int_{\sigma_i}^1 (\sigma_e - \sigma_i) dF_e(\sigma_e | \sigma_i) dF_i(\sigma_i).
$$

As $r_d \to \infty$, $\bar{w}$ converges monotonically to $\bar{w}_1 > 0$, defined as the solution to (10) with $N = \bar{n}_i \mathbb{E}(\sigma_i)$. As regards the $BM(r_d, \bar{w})$ curve, first, $r_d = 0$ when $\bar{w} \geq \bar{w}_2$, with $\bar{w}_2$ solving

$$
\frac{\gamma \delta A}{\bar{n}_e (1 - \beta)} = \Delta (\bar{w}_2) \alpha_e \int_0^1 \int_{\sigma_i}^1 [\theta \sigma_i + (1 - \theta) \sigma_e] dF_e(\sigma_e | \sigma_i) dF_i(\sigma_i).
$$

Second, the $BM(r_d, \bar{w})$ curve hits $r_d = r$ as $\bar{w} \to 0$, and it is strictly decreasing for $r_d \in [0, r)$. These observations ensure an intersection (interior or not), so equilibrium exists. There are two types of equilibria: (i) $r_d = 0$ and $\bar{w} = \bar{w}_0$; and (ii) $r_d \in (0, r)$ and $\bar{w} \in (\bar{w}_1, \bar{w}_0)$. When equilibrium with $r_d > 0$ exists uniquely, $\bar{w}_2 > \bar{w}_0$ and the $BM$ curve crosses the $GM$ curve from above. We conclude that when $r_d = 0$, $A\delta$ has no effect; and when $r_d > 0$ a rise in $A\delta$ or $\theta$ lowers $r_d$ and increases $N$ and $g$.

This completes the case without entry. The case with entry is similar.■
References


Empirical Variable Definitions

**Dependent**

Technology Transfer: *Firm-specific variable*. Binary variable equal to one if the firm’s (self reported) most important source of technology is any of: “new licensing or turnkey operations from international sources,” “new licensing or turnkey operations from domestic sources,” “new licensing or turnkey operations from domestic sources,” “obtained from universities or public institutions.” [2005:Q61b]

**Independent - Explanatory**

Private credit/GDP: *Country-specific variable*. The ratio of private credit by deposit money banks to GDP, used as a proxy for a country’s level of financial development. Taken from Beck et al (1999).

Private credit/GDP: *Country-specific variable*. The previous term squared.

**Independent - Instruments**

Legal origin: *Country-specific variable*. A set of three dummy variables, French-civil, German-civil, and common law, indicating the origin of a country’s legal system. A country’s legal code can have multiple influences. Taken from Djankov et al (2007), and the CIA World Factbook.

**Independent - Controls**

Market size: *Country-specific variable*. The population of the country in which a firm operates. Taken from Penn World Tables 6.3.

Price level of investment: *Country-specific variable*. PPP over investment level, divided by exchange rate with US$, multiplied by 100. Taken from Penn World Tables 6.3.

Openness: *Country-specific variable*. Exports plus imports, divided by GDP. Taken from Penn World Tables 6.3.

Investment level: *Country-specific variable*. Investment as a share of GDP. Taken from Penn World Tables 6.4.

Firm size: *Firm-specific variable*. Number of permanent, full-time employees employed at a firm, self reported. [2005:Q66a]

Presence of foreign capital: *Firm-specific variable*. Dummy variable equal to one if a positive percentage of a firm is owned by foreign individuals or businesses, self reported. [2005:S5b]

Industry dummies: *Firm-specific variable*. A set of seven dummy variables designating a firm’s industry. A firm belongs to a certain industry if the majority of its operations are in the specified field. Industries are: mining, construction, manufacturing, transport, wholesale, real estate, hotel and restaurant services, and “other” if none of these are applicable. [2005:Q2a-g; 2002:q2a-g]
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<th>Above Mean Private Credit to GDP (%)</th>
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Table 3: OLS Regression of Technology Transfer on Private Credit, Uninstrumented

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<th>(4)</th>
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<td>0.0287*</td>
<td>0.0276*</td>
<td>0.1308***</td>
<td>0.1607***</td>
<td>0.1649***</td>
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<td></td>
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<td>(0.0159)</td>
<td>(0.0381)</td>
<td>(0.0464)</td>
<td>(0.0468)</td>
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<td>-0.0839***</td>
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<td></td>
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Note: * = Significant at 10% level, ** = Significant at 5% level, and *** = Significant at 1% level. Standard deviations are in parentheses.
Table 4: Two-Stage Least Squares Regression of Technology Transfer on Private Credit

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<tr>
<th>Independent Variables</th>
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<th>(4)</th>
<th>(5)</th>
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</table>

**Note:** Private credit is instrumented by legal origin, * ≡ Significant at 10% level, ** ≡ Significant at 5% level, and *** ≡ Significant at 1% level. Standard deviations are in parentheses.
Table 5: Probit Regression of Technology Transfer on Private Credit

<table>
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<td>Log market size</td>
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<td>Price level of investment</td>
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</table>

Note: Private credit is instrumented by legal origin. * ≡ Significant at 10% level, ** ≡ Significant at 5% level, and *** ≡ Significant at 1% level. Standard deviations are in parentheses.
Figure 1: Equilibrium of Basic Model with Endogenous Innovation

Figure 2: Bargaining Outcome with Credit Frictions
Figure 3: $S(x)$ and $\ell(s)$

\[ \beta = .96 \quad A = 1 \quad \gamma = n = .5 \quad F_I \sim U[0,1] \quad F_e \sim U[3,1] \]

Figure 4: Effects of Increasing Liquidity or Bargaining Power
\[ \sigma_e = \sigma_i (1 + r) \]

**Figure 5:** Bargaining Outcome with Intermediation

\[ A_0 : \text{No trade & Save} \]
\[ A_1 : \text{Trade & Save} \]
\[ A_2, A_3 : \text{Trade & Borrow} \]

**Figure 6:** Example: \( \sigma_e = 1, \sigma_i \sim \text{beta}(a, b) \)