Employment participation, unemployment and non market work: Composition models of the United States labour force

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Abstract

Data on the proportions in each of the labour force status categories sum to one and form a composition. Using the log ratios of the parts and a transform to orthogonal coordinates in a lower dimension provides a consistent way of handling this constraint. This paper emphasizes that a composition is a vector of values not a single number and shows a coordinate for employment participation is directly associated with a coordinate defining the level of search by those without jobs. The model is extended to deal with heterogeneity due to gender and current status and to generate a new analysis of the gross flows data. The associations observed provide evidence that micro decisions can be strongly modified by the macro situation and a Keynesian demand driven framework fits very well.

JEL: E24, J64

Keywords: Composition models, unemployment, gross flows data, labor force status, US labor force, log ratio models, rationing.

1 Introduction

This paper is concerned with exploring the simple basic question, “What happens in the labour market when the number of jobs changes with a given population”. The short answer is that the number of unemployed and those doing non market work, must also change even if there is no change in their values or preferences and current prices. As Aitchison(1986) emphasized, partition of a population into the three parts requires consideration of the shares as a multivariate variable.

The interdependence of the parts has been known to bias relationships and make correlations between them meaningless since Pearson(1897) and was explored further by Chayes(1965) but is still usually ignored. One objective of this paper is

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to avoid those problems by applying the composition analysis tools developed by Aitchison (1986) and Egozcue et al (2003) to analyse the relationships between the categories using a consistent set of orthogonal coordinates. Fry (2011) gives examples of other applications of Aitchison’s framework in economics. Bergman (2008) used compositional analysis tools and a VAR for a time series analysis of Swedish labour force data, but used it as an example and did not emphasize or explore the economic interpretation of the transforms he used.

In the case of labour, the population is typically divided into those employed (E) and those not. Some of the latter search for jobs (U), while others do not and engage in non market work (N). We refer to the three categories as the parts. Participation in the parts is commonly defined by internationally used definitions. We do not view these categories as a definitive guide to long term attitudes of the population to the labour market but regard them as a current response to a continuously changing job and social environment. Those for whom current wages are above their reservation wage may not seek work in the observation period but may still be available if offered a job. In this paper the term “labour force” is used with its conventional reference to the data but it should not be interpreted as implying that those in it are the total available labour supply.

The issue of whether the primary classification should be based on employment or participation in a notional work force (W = E + U) or labour supply is not new. Clark and Summers (1979) argued N and U were not different states at least for some groups Flinn and Heckman (1983) showed that there are differences in behaviour and outcome for those in the two groups so they are useful categories. Goldsmith et al (1995) broadened the debate by considering psychological differences and their effects on decision pathways. There has been a large literature exploring these issues. Krussell et al (2016) explores a number of related ideas and emphasizes the interaction of all three parts but still uses a concept of the labour force as an important component. We explore alternative emphases for discussing the interaction of the three parts and differences between subgroups within the population.

In the CPS data for the United States, the employment status of about 5 percent of persons changes each month, and a further 2 percent without jobs change their status. The gross changes are always large relative to the net changes. The JOLTS data provide further evidence of large adjustments with each month and substantial change in both hires and job openings. Some part of this change is due to short term jobs and the changing season but both are merely parts of the overall level of change. Together the data gives a picture of continuous change in the labour market, and changing odds of continuing in employment or obtaining work for its participants. Among those who lose a job, the many with limited assets have neither time nor concern to base their decisions on detailed long term calculations of the present worth of uncertain and often unknown characteristics of transient alternatives. In this changing environment they have to do something. Both sides of the market are affected. Employers faced with changing cash flows can offer or terminate jobs. Individuals can decide to retain a job, and if without one accept an offer, continue searching or focus on non-market work. Many will face new non-market conditions. Persons may make other adjustments in household structure and relationships as a part of their response.

Individuals searching for a job may not obtain one. The observed outcome is
not an equilibrium but a rationed outcome. The rationing occurs in the job search process. Employers work within socially accepted norms and hire persons from those whose search satisfies conditions conventionally described as ‘unemployed’, or from others with or without a job who are available and become known to them. Many who would like work do not obtain jobs. Solow(1990) argued that we need a different framework for thinking about the labour market. Rationing governed by social conventions can play such a role.

In this paper we show that focusing on those with and without jobs, leads to simple descriptive tools, associated with a very high proportion of the variance in the data. It provides an alternative to the emphasis on transition and hazard probabilities in studies of Shimer(2012) and others, and gives the number of jobs a key role. In our model, job search is an option for all without a job. That group is truly heterogenous and we take a first step in dealing with that by distinguishing different patterns by gender.

Short term changes based on BLS data as published are at the center of our analysis. Many including Abowd and Zellner(1985), Blanchard and Diamond(1990), Shimer(2012) and Elsby, Hobijn and Sabin(2015), have made careful adjustments to the data to better measure their concept of change. Their adjustments may not enhance and improve the data for analysis from the perspective used here but are a rich field for further study using composition tools.

In any inquiry, the number and detail of the parts will depend on the purpose of the investigation, but it needs to be sufficient to reflect the main facets of behaviour being studied or factors modifying them. At the broadest level we can use just the proportions \((E,U,N)\). Aitchison showed that using ratios of the parts maintained coherence of conclusions about subcompositions. In this paper we explore the ratio structure of 6, 9 and 18 part systems to include gender differences and differences depending on current status. Each step expands the understanding of the data structure and shows differences in subpopulation behaviour. Other classifications are necessary to discuss industry and skill specific issues.

It is a truism that the framework of ideas used in data analysis influences the conclusions. Part of our objective is to illustrate how a single overarching composition analysis framework enables articulation of alternative ways of exploring the data with focus on employment, searching for work (conventionally referred to as unemployment) and non market work(those not in the market economy “labour force” only in the simple sense that they are not currently searching for paid work).

Some features of the multivariate structure can be emphasized and explored using ternary diagrams to portray the time sequence of changes. Ternary diagrams are widely used in other areas of study, but do not appear to have been used for this aspect of the labour force data. They demonstrate distinct patterns associated with each business cycle, and changes in the structure of response to changes in employment, and are explored in Jackson and Khaled(2017) which looked at 40 years of US experience and provides background for this paper. At a six part level, that paper shows the population totals can give a picture not representative of either of the two important subpopulations. Evidence from there suggests structural change between business cycles. Using dummy variables to describe such change gives a simple general model for a large share of the variance in the data. Structural change creates problems in time series analysis, and the treatment here should only be
regarded as a very simple first approximation.

Our results are a challenge to the conventional approaches to this data. First, we show that using log ratios of the components gives an alternative clear picture of the relationships between the parts, and enables consistent use of standard multivariate tools. Second, we show that there are strong patterns in the relationship of the three components within business cycle periods, and they differ between business cycles. These patterns can be described by differences in search behaviour. Third, within a cycle changes in employment are associated with about 95 to 98 percent of the variation in the remaining features of the data at an aggregate level. The model of employment as an exogenous variable and rapid short term changes in response fits the observations very well. Fourth, consistent with the work of Elsby et al, there are important changes in the proportions of the $U$ and $N$ parts. We find the log ratio for the labour force generally moves counter cyclically, and its inverse, the movements in the log ratio for $N$ relative to the others is procyclic. This highlights the difference arising from using ratios rather than shares of the parts. Fifth, it is not necessary to construct elaborate models of the gross flows. It is simpler to describe the changes as a set of consistent changes in contemporaneous choices for different categories of individuals in a changing macro environment. Sixth, we show that the differences between male and female participants in the market are important in describing the aggregate pattern and this provides an example of treating the heterogeneity which has been emphasized in many of the recent papers. Seventh, we explore the relation with the vacancy data, the extent to which it is a surrogate for the level of employment, and ways in which widespread use of the Beveridge curve is ill conceived. Our results provide an alternative critique to Diamond and Seygul(2014). Eighth, we interpret these results as being consistent with the Keynesian view that the level of employment is not the result of conventional market equilibrium processes and is determined outside this market.

Section 2 outlines the composition analysis tools and three simple alternative ways of looking at the data. Section 3 uses the CPS data and explores six part models using the coordinates introduced in Section 2. Section 4 considers the gross flows data and uses the employment focus to provide a model for all the flows. Section 5 reviews ways in which the approaches described here interface other recent work and challenge many of the conventional views.

## 2 Some elements of Composition Analysis

The main tool in the rest of this paper is a coordinate transformation prior to analysis of the data. The data are in $\mathbb{R}_D$ but the points lie in a non negative region of dimension at most $\mathbb{R}_{D-1}$. The work of Aitchison and Egozcue et al provides the framework of orthogonal log ratio coordinates in $\mathbb{R}_{D-1}$. For a detailed introduction to the transforms readers should look to texts in the compositions literature like Pawlowsky-Glahn, Egozcue and Tolesano-Delgado(2015).
2.1 Orthogonal Coordinates for a Composition

For analysis of elements in a simplex a standard form is to have

\[ a = (a_1, a_2, ..., a_D) \]

and \( a_i > 0 \) and

\[ \sum_{i=1}^{D} a_i = 1 \]

Conversion to this form is referred to as closure with

\[ C(a) = \left( \frac{a_1}{s}, \frac{a_2}{s}, ..., \frac{a_D}{s} \right) \]

with \( s = a_1 + a_2 + ... + a_D \). We write the closure operation applied to \((E,U,N)\) as \((E,U,N)\).

Aitchison realised that for many problems with data in a simplex the ratios of the elements are the natural elements of analysis. For three elements the ratios between any pair can be calculated from \(E/N\) and \(U/N\). Working with ratios is more convenient using logarithms, so he defined an additive log ratio (alr) as the transform

\[ \text{alr}(a) = \left( \ln \frac{a_1}{a_D}, \ln \frac{a_2}{a_D}, ..., \ln \frac{a_{D-1}}{a_D} \right) \]

Any element can be selected as divisor. This transforms the data from the non-negative quadrant of \(\mathbb{R}_D\) to elements in \(\mathbb{R}_{D-1}\). With three elements there are three distinct sets of two ratios, each of which provide all the information about the ratio structure of the data. However they are in a non-orthogonal coordinate system, so very difficult to interpret. Aitchison also defined a more symmetric transform the centralised log ratio given by

\[ \text{clr}(a) = \left( \ln \frac{a_1}{g}, \ln \frac{a_2}{g}, ..., \ln \frac{a_D}{g} \right) \]

with \( g = \left( \prod_{i=1}^{D} a_i \right)^{\frac{1}{D}} \). The clr vectors are a transform into \(\mathbb{R}_D\), but now have the property that the values sum to zero so they are in a space \(\mathbb{R}_{D-1}\) which contains all the information about the ratio structure of the data.

Aitchison showed that the appropriate measure of distance between points \((x, y)\) in the simplex is constructed from the distance between all pairs of clr values

\[ d_a(x, y) = \sqrt{\frac{1}{2D} \sum_{i=1}^{D} \sum_{j=1}^{D} (\log \frac{x_i}{x_j} - \log \frac{y_i}{y_j})^2} \]

Egozcue et al defined a set of D-1 orthogonal contrasts which transform the clr data by an isometric log ratio transform (ilr). This preserves all the ratio information in the data and the Aitchison distance measure between the points. The data can then take any value in \(\mathbb{R}_{D-1}\), and can be analysed with standard procedures. Finding the ilr values requires a transform matrix \(H\) of shape \((D - 1 \times D)\) such that

\[ HH' = I_{D-1} \]
and

$$H'H = I_D - \frac{1}{D}1_D$$

where $1_D$ is a matrix of shape $(D,D)$ with all elements 1.

Egozcue et al use sequential binary partitions (SBP) to construct an orthogonal representation of the composition data. Their first step is to classify all elements into two groups. Assign a 1 for those in the numerator and a -1 for those in the denominator. Repeat the process within each subgroup until there are no further groups with two elements.

For the three categories and the clr transform to $(E, U, N)$ we can choose those with jobs, versus those without as the first contrast with SBP terms $(1,-1,-1)$, and a second between those looking for a job and those not doing so among all without jobs, with terms $(0,1,-1)$. This choice of contrasts gives a focus on employment in the first row, and on job search in the second.

$$B = \begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & -1 \end{pmatrix}$$

(1)

Each row is referred to as a balance since it includes all the information in the data about the ratio of the parts involved. To preserve isometry, separate weights are needed for those in each group. If the SBP process has been followed and there are $r$ in the first group and $s$ in the second group the coefficients for orthogonal coordinates are

$$a_+ = \frac{1}{r} \sqrt{\frac{rs}{r+s}}$$

and

$$a_- = -\frac{1}{s} \sqrt{\frac{rs}{r+s}}$$

(2)

These coefficients give the transform matrix

$$H = \begin{pmatrix} 0.816497 & -0.408248 & -0.408248 \\ 0 & 0.707107 & -0.707107 \end{pmatrix}$$

(3)

A data set $X$ of observations of $n$ compositions of $D$ components, is converted to an $n$ by $D$ matrix $X^c$ of clr values and to ilr coordinates by $Z = X^cH'$. We refer to the coordinates in the columns of $Z$ as $z_1, z_2, ..., z_{D-1}$ where the application to items within a row vector or the whole column depends on the context. After an SBP process each ilr coordinate can be expressed as

$$\sqrt{\frac{rs}{r+s}} \left( \frac{ln g_r}{g_s} \right)$$

where $g_r$ and $g_s$ are the geometric means of the parts in numerator and denominator of a log ratio contrast. The vector of $D-1$ ilr coordinates has an orthogonal basis enabling simple analysis of the variance and the relationships between them. For
some purposes a center of the data is needed. The mean of the columns of $Z$ is an \textit{ilr} transform of the mean of $X^c$.

It is easy to reverse the operations. $X^c = ZH$ gives the \textit{clr} values. For each row $x^c$ of $X^c$ take $Exp(x^c_i)$ for each element of $x^c$, then apply the Closure operation of dividing each of the elements of the row $x^c$ by their sum to give $x = C(Exp(x^c))$.

2.2 Alternative Coordinate Sets

With the SBP process there are alternative ways of constructing a set of orthogonal coordinates. In general there is an infinite set of orthogonal coordinate systems but the finite set of those generated with the SBP are often simpler to interpret. It is important to note that using a different set of coordinates does not change the data. It merely changes the way it is represented. The points are still the same, and their relationship is still the same, but a different conceptual framework is being used to talk about them and describe what is happening.

There are alternatives to using the SBP process to find a set of orthogonal coordinates. We use such alternatives in Section 4. The simple coefficients $a_+$ and $a_-$ are often inadequate for such models of the data.

It is essential to recognise that a change in any one of the coordinates changes the composition. The composition is a multivariate result. It has a unique representation in the coordinates, but a change in any coordinate implies a change in at least two of the parts of a composition. Treating the proportions of the parts as independent variables is simply invalid.

2.2.1 The employment focus

The employment focus was used above. It uses the partition $(1, -1, -1)$ to identify the ratio of those employed to those in other categories. To model the data write $ln(E, U, N)$ as $(e, u, n)$ and take employment as the reference category. The first \textit{ilr} coordinate becomes $z_1 = \frac{1}{\sqrt{6}}((e - u) + (e - n))$. We define $emp = z_1$ to refer to this contrast and also use that name for generalisations of this concept. This measure of the employment level in the economy is a scaled sum of the log ratio of those with employment, and the two groups who do not have a job. If you are searching for a job, the proportion of the population who can obtain one of the available jobs matters. The turnover in jobs, makes the ratio of the proportion with jobs to those searching one factor in their decision. The ratio of those with work to those in non market work is also important as they are potential searchers. $z_1$ also gives information about the market income flow available to households and to each person relative to the average earnings of those with work.

The second coordinate looks at all without jobs by focusing on the outcome of their decision problem. It uses the log ratio of those searching for a job with the conventional search definitions, and those who do not with $z_2 = \frac{1}{\sqrt{2}}(u - n)$. We define $jos = z_2$ as the contrast generated by the level of searching for a job. If the number in jobs is determined by decisions of employers the second coordinate reflects the response of those without jobs to the situation they face. The institutional structures in the market act to ration the jobs among applicants whose reservation wage is below the current market rates. The fact that some do not get a job does not mean they are in a search equilibrium. They are available for work at the current
terms. They may keep on looking until continuing turnover in the jobs on offer lead
to them being selected for a job or they may avoid the search costs and engage in
non market work until they decide to recommence search.

These coordinates can be obtained from the negation of the \( alr \) coordinates with
\( E \) as the divisor. The first coordinate is just a multiple of their sum and the second
a multiple of their difference.

### 2.2.2 A labour force focus

The conventional concept of a ‘labour force’ is just the set of parts to which indi-
viduals must belong if not in \( N \). Those in \( E \) and \( U \) are commonly referred to as
in the labour force so the coefficients \( (1,1,-1) \) identify the groups. We obtain this
from the employment contrast by interchanging \( E \) and \( N \) and reversing the signs.
The first \( ilr \) coordinate is a scaled sum of the balance between having a job and
doing non market work, and balance between searching for work versus doing non
market work with
\[
z_1 = \frac{1}{\sqrt{6}}((e-n) + (u-n))
\]
The second coordinate is the difference
\[
z_2 = \frac{1}{\sqrt{2}}(e-u).
\]
and gives the balance between employment and unemployment of
those in the labour force. Since the second coordinate involves \( e \), both coordinates
respond to changes in \( e \).

If the focus is on non market work rather than the labour force, the sign of \( z_1 \)
is changed. Changing the sign changes a counter cyclical variable to a pro cyclical
one or vice versa but the orthogonal properties are maintained.

### 2.2.3 An unemployment focus

Categories \( E \) and \( N \) are now the alternative set so coefficients \( (-1,1,-1) \) generate
the first \( ilr \) coordinate, and \( (1,0,-1) \) the second. \( z_1 = \frac{1}{\sqrt{6}}((e-u) + (n-u)) \) and
\[
z_2 = \frac{1}{\sqrt{2}}(e-n).
\]
For most groups in the population, \( u \) is much smaller than \( e \) or \( n \) so
there are large changes in the first coordinate and there is relatively little change in
the second coordinate. With the unemployment focus, we have coordinates to show
the balance between jobs and non market work and its relationship with the level
of unemployment.

### 2.2.4 A data based components approach

A singular value decomposition of the \( clr \) data yields a set of orthogonal components
in \( \mathbb{R}_{D-1} \). It is normally quite difficult to interpret compared with the alternatives
above however it may be helpful in identifying characteristics of the other transforms.
The svd for each of the \( ilr \) coordinate sets give relationships which identify the same
line within the data space. They are merely alternative representations of that line.
Using forty years of the monthly seasonally adjusted data and ignoring structural
change the \( H \) matrix is
\[
H = \begin{pmatrix}
-0.5367 & 0.8012 & -0.2646 \\
-0.6153 & -0.1571 & -0.7724
\end{pmatrix}
\]
This value of \( H \) is not too different from the matrix for the unemployment focus
above. It corresponds to a model where unemployment is changing almost orthogon-
ally to factors affecting the desired balance between market and non market work.
2.2.5 Assumptions and Terminology

Various terms are used in the compositions literature for the ilr coordinates. They are often referred to as coordinates or balances. For a composition of only two categories with count data the log ratio is in fact a log odds for the parts. For larger numbers of categories the weighted sums of log odds of component parts are a way of forming a higher level comparison but they are not odds ratios and are referred to as balances for the higher level comparison. The term is not to be interpreted as log odds of the amalgamation of the categories but as a means of obtaining consistent ratio comparisons of the shares of the parts and the ways in which they change.

2.3 Extending the models to include Gender differences

The ilr transforms from Section 2.2 are easily expanded to explore the data from additional perspectives. For gender subgroups, and data in the sequence \((E_m, U_m, N_m, E_f, U_f, N_f)\) each ilr transform is defined by a transform matrix \(H\). There are alternative ways of exploring the patterns associated with gender. One is to consider the log ratios of the the aggregate focus groups then gender groups for each of the status categories. For the employment focus the matrix \(H_1\) is

\[
H_1 = \begin{pmatrix}
0.577 & -0.289 & -0.289 & 0.577 & -0.289 & -0.289 \\
0.000 & 0.500 & -0.500 & 0.000 & 0.500 & -0.500 \\
0.707 & 0.000 & 0.000 & -0.707 & 0.000 & 0.000 \\
0.000 & 0.707 & 0.000 & 0.000 & -0.707 & 0.000 \\
0.000 & 0.000 & 0.707 & 0.000 & 0.000 & -0.707 \\
\end{pmatrix}
\]

For the labour force and unemployment focus, the first two rows are replaced by the appropriate balances. This treatment of gender differences is used for \(H_1, H_2\) and \(H_3\) in Section 3.

A second method is to follow the two main contrast groups with a contrast for gender balance, followed by a difference in the behaviour of the two main contrasts for each gender. This is used for \(H_4, H_5\) and \(H_6\). A further group of data models, \(H_7\) to \(H_9\) have a gender balance and look at the focus within each gender group. \(H_4\) and \(H_7\) are given by

\[
H_4 = \begin{pmatrix}
0.577 & -0.289 & -0.289 & 0.577 & -0.289 & -0.289 \\
0.000 & 0.500 & -0.500 & 0.000 & 0.500 & -0.500 \\
0.408 & 0.408 & 0.408 & -0.408 & -0.408 & 0.408 \\
0.577 & -0.289 & -0.289 & -0.577 & 0.289 & 0.289 \\
0.000 & 0.500 & -0.500 & 0.000 & -0.500 & 0.500 \\
\end{pmatrix}
\]

\[
H_7 = \begin{pmatrix}
0.408 & 0.408 & 0.408 & -0.408 & -0.408 & -0.408 \\
0.816 & -0.408 & -0.408 & 0.000 & 0.000 & 0.000 \\
0.000 & 0.000 & 0.000 & 0.816 & -0.408 & -0.408 \\
0.000 & 0.707 & -0.707 & 0.000 & 0.000 & 0.000 \\
0.000 & 0.000 & 0.000 & 0.000 & 0.707 & -0.707 \\
\end{pmatrix}
\]

The matrices \(H_5\) and \(H_6\) use the labour force and unemployment focus. For \(H_4, H_5\) and \(H_6\) the third contrast is common, but the remaining four coordinates differ for each set. In the final group \(H_7, H_8\) and \(H_9\) the first contrast is common, but the others all differ.
3 The American Experience

We use the official BLS seasonally adjusted monthly data on labour force status by gender from 1975(1) to 2017(8). Ternary plots of \((E, U, N)\) for the monthly data suggest different behaviour of the composition in different business cycles. Large gender differences were also observed so it is important to examine the extended vector \((E_m, U_m, N_m, E_f, U_f, N_f)\) including the status of males and females separately.

For \(ilr\) coordinates generated with the matrix \(H_i\) we use the name \(ilri\). The matrices \(H_i\) are referred to as data models because they provide alternative ways of thinking about the measurements. Our main focus is on employment so the model \(H_1\) and the data \(ilr1\) will be the most used set. The coordinates may involve each of the parts, but that is not inconsistent with regarding the errors as being associated with an orthogonal set of directions in the space.

Table 1 gives details of the dates of the cycle periods in the data used for distinguishing behavioural patterns in the graphs. These follow NBER trough dates except the final date of the first period, shifted twelve months before the trough to match a change in behaviour in the data. The final date is not a trough date, but the end of the data in an increasing phase of the cycle.

<table>
<thead>
<tr>
<th>Cycle</th>
<th>Starts</th>
<th>Ends</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1975 Jan</td>
<td>1979 July</td>
<td>A</td>
</tr>
<tr>
<td>2</td>
<td>1979 Aug</td>
<td>1982 Nov</td>
<td>B</td>
</tr>
<tr>
<td>3</td>
<td>1982 Dec</td>
<td>1991 Mar</td>
<td>C</td>
</tr>
<tr>
<td>4</td>
<td>1991 Apr</td>
<td>2001 Nov</td>
<td>D</td>
</tr>
<tr>
<td>5</td>
<td>2001 Dec</td>
<td>2009 Jun</td>
<td>E</td>
</tr>
<tr>
<td>6</td>
<td>2009 Jul</td>
<td>2017 Aug</td>
<td>F</td>
</tr>
</tbody>
</table>

Note: See text for period 1 date.

3.1 Partitioning variance

A first step in analysis is to get an overview of the main sources of variation in the ratio structure of this data. Much of the literature has been concerned with apportioning the observed variation in shares of the parts between sources. The coordinate systems we have used describe changes in the ratios of the parts. Table 2 below shows just how dependent the partitioning of the variance is on the conceptual framework being used and the value of having orthogonal data models.

Using the ratios between the parts gives a different perspective to the absolute values. The same absolute variation will give a small relative change for large shares, and large relative change for small shares. For a set of equal absolute changes, the relative changes will depend on the magnitude of the parts affected. However using the log ratios gives a set of values where equal changes for any coordinate represent equal percentage changes.

Each of the alternative models in Section 2.3 gives a complete partition of the variance in log ratios of the parts in the data and a different way of examining it. The \(clr\) transform of the data is in a space of six dimensions, and has a total Aitchison
variance of 0.0993. Each set of ilr coordinates has the same total Aitchison variance, but with components in orthogonal directions. Table 2 gives the percentage of the variance for each of nine alternative ways of representing the data using matrices $H_1$ to $H_9$.

In this table, the $z_i$ variables refer to the balance associated with row $i$ of the listed contrast matrix so can represent different concepts in each row.

Table 2: Percentage of Variance for Contrasts of Labour Force Status

<table>
<thead>
<tr>
<th>Focus</th>
<th>Matrix</th>
<th>$z_1$</th>
<th>$z_2$</th>
<th>$z_3$</th>
<th>$z_4$</th>
<th>$z_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emp</td>
<td>$H_1$</td>
<td>30.3</td>
<td>49.6</td>
<td>3.6</td>
<td>3.7</td>
<td>13.7</td>
</tr>
<tr>
<td>LF</td>
<td>$H_2$</td>
<td>14.9</td>
<td>65.0</td>
<td>3.6</td>
<td>3.7</td>
<td>13.7</td>
</tr>
<tr>
<td>Unemp</td>
<td>$H_3$</td>
<td>74.6</td>
<td>5.3</td>
<td>3.6</td>
<td>3.7</td>
<td>13.7</td>
</tr>
<tr>
<td>Emp</td>
<td>$H_4$</td>
<td>30.3</td>
<td>49.6</td>
<td>2.8</td>
<td>9.4</td>
<td>7.9</td>
</tr>
<tr>
<td>LF</td>
<td>$H_5$</td>
<td>14.9</td>
<td>65.0</td>
<td>2.8</td>
<td>13.4</td>
<td>3.8</td>
</tr>
<tr>
<td>Unemp</td>
<td>$H_6$</td>
<td>74.6</td>
<td>5.3</td>
<td>2.8</td>
<td>3.1</td>
<td>14.1</td>
</tr>
<tr>
<td>Emp</td>
<td>$H_7$</td>
<td>2.8</td>
<td>18.1</td>
<td>21.6</td>
<td>41.2</td>
<td>16.3</td>
</tr>
<tr>
<td>LF</td>
<td>$H_8$</td>
<td>2.8</td>
<td>20.9</td>
<td>7.4</td>
<td>38.4</td>
<td>30.4</td>
</tr>
<tr>
<td>Unemp</td>
<td>$H_9$</td>
<td>2.8</td>
<td>50.0</td>
<td>27.8</td>
<td>9.4</td>
<td>10.0</td>
</tr>
</tbody>
</table>

This table is very informative. The contrasts for the population represent combined effects over both genders. For all models $H_1$ to $H_6$ the sum of $\text{var}(z_1) + \text{var}(z_2)$ gives 79.9 percent of the variance. The remaining 20.1 percent of the variance is associated with gender balances and different ways of describing them. Having a single overarching framework for examining the variance is very important. Measures such as those based on variance in a single part of the source data such as unemployment fail to recognise the importance of the multivariate character of the shares within the simplex constraint.

$H_1$ splits the variance based on those in $(E, U + N)$ with 30.3 percent associated with change in employment participation, and 49.6 percent with the changed search behaviour within the subpopulation $(U, N)$. The fact that there are larger relative changes in job search is not surprising since the sum of the parts without a job is smaller than the total with a job. The numbers suggest large changes on both sides of this market. Changes which modify the number employers choose to employ, generate a large response in search behaviour of those without jobs. $H_4$ shows that change in both emp and jos contribute almost equally to the gender variance in $z_4$ and $z_5$. $H_7$ splits the $(30.3 + 9.4)$ for emp components in $H_4$ between males 18.1 and females 21.6 and the $(49.6 + 7.9)$ for jos components between males 41.2 and females 16.3. This large difference is likely to be based on a higher proportion of male employees being the primary earner in a household.

The model $H_2$ has 14.9 percent associated with changes in the level of the labour force, and 65.0 percent associated with changes in the proportion of the labour force in work. While the labour force shows significant variance the large fluctuations are taking place in the balance between employment and search within the labour force. Using the labour force to provide a summary clearly includes a part of the variance but detracts from the employers hire decisions which play a big role in variance within the labour force. This model is useful for comparison with the measures in
Elsby et al (2015) since the labour force log ratio is the inverse of the log ratio of being in $N$. Model $H_5$ shows that variance of participation in the labour force had a population component of 14.9% and a gender component of 13.4% and a total of 28.3%. These are clearly quite large shares of the change and consistent with the conclusions of Elsby et al. Model $H_8$ splits the variance in $H_5$ into components within the gender groups and shows there was more change in male labour force participation than female, and within the labour force both genders experienced large changes in participation in employment with rather more variance for males.

The model $H_3$ provides another approach. With 74.6 percent of the variance in the total population constrasts associated with the log ratio of unemployment to the other categories ($E,N$) and only 5.3 percent left for variance in the log ratio between those two categories. Whatever is driving change makes large changes in the balance between unemployment and other categories, but little change in the balance between market and non market work. Among those not searching for a job, there are relatively slow trends in the balance between $E$ and $N$. Unemployment is the major aspect of change in this data, but that provides little by way of explanation. Changes in the log ratio of $U$ with $E$ and with $N$ both usually move in the same direction. The unemployment perspective combines them in a single coordinate. This framework leads us to seek explanations which generate change in unemployment under labour supply conditions which change slowly over time, and an institutional framework which leads to rationing of work among those whose reservation wage means they would accept work, or seek it, if the demand is high enough.

3.2 The employment focus

Names for the coordinates help explanation and understanding. They are a brief way of providing a reference to the data concept underlying each coordinate, and are used in both text and the diagrams. For the employment focus we use the names $(emp, jos, eg, ug, ng)$ for model $H_1$ $(emp, jos, gen, empg, josg)$ for model $H_4$ $(gen, empm, empf, josm, josf)$ for model $H_7$.

Using the data model $H_1$ from Section 2.3 all five coordinates of $z$ are portrayed as a time series plot in Figure 1. The balances of employment $emp$ and of job search, $jos$ are at the top and the bottom. The three lines associated with gender in the centre of the plot show the decline in the balance for males among those employed ($eg$), a similar pattern of decline in the balance of males among unemployment ($ug$), and a strong trend increase in the balance for males among those in non market work ($ng$). The first and third of these gender variables show a pattern without a large business cycle component.

Figure 2 shows the similarity of the inverse movements in employment and search. The sign of coordinate $z_2(jos)$ is reversed so the movements in Figure 2 are parallel rather than in the inverse directions in Figure 1. The pro cyclical employment variable $emp$, and the log ratio of not seeking work are almost parallel and procyclical.

Inverting the ratio back to $jos$ we have the lowest log ratio of being unemployed at the business cycle peak and strong counter cyclical behaviour. A focus on the
Figure 1: Employment and Search Balances with Gender Groups, Monthly Business Cycle Data, 1975(1)-2017(8), Model $H_1$. For date details see Table 3. Cycle troughs are marked by vertical lines.

Figure 2: Coordinates of employment ($z_1$) and inverse of job search (N/U )($-z_2$) for Monthly Business Cycle Data 1975(1)-2017(8), Model $H_1$.

A feature of the diagram is the occurrence of long sequences with a nearly linear relationship between $emp$ and $jos$ gives Figure 3 where the horizontal axis is the log ratio of having a job and the vertical axis is the log ratio of seeking work.
pattern associated with each of the business cycle periods. Within this diagram each business cycle segment is identified to illustrate the near linear relationship. For more clarity the diagram can be enlarged in a pdf viewer. Figure 3 shows the pattern for the whole period. An alternative comparison is shown in Figure 6 with a separate panel for each cycle and a fitted line for that cycle. It illustrates the differences in gradient and location.

Trends in the gender balance are clearly shown in Figure 1. The factors determining these trends appear to have been almost independent of the business cycle fluctuation. Over the last forty years gender balance in unemployment has shown a broad trend similar to male participation in employment, but with some business cycle dependence and the share of males higher in the trough.

The model $H_4$ includes two terms which look at the gender balance for emp and jos, the fourth and fifth variables, $empg$ and $josg$. Figure 4 shows the rapid change in the second and third cycle periods, and the slower continuing reduction in the odds for males in total employment. There may be a procyclical element in the decline, but it is obviously modest. Figure 5 which looks at the search behaviour shows that as the level of search jos declines the log ratio that the person searching is male josm also declines. The gender balance in employment appears to be modestly affected over the business cycle, but there are larger changes in the gender balance of those seeking work. The balance of males among those unemployed tends to follow the balance for employment and is counter cyclical.

### 3.2.1 Relations in Levels

One approach to analysis of this data uses a simple cointegration model. Four of the series in the matrix $ilr1$ have augmented Dickey Fuller statistics within the 5
per cent point, and the remaining one is within the 1 percent point so we need to consider most as having unit roots. If we fit a simple linear relationship between the levels of emp and jos with shifts in both level and gradient between the business cycles, we obtain a relationship which has stationary residuals though with some autocorrelation structure. It is therefore a cointegrating relationship in the Engel-Granger sense, and represents behaviour maintaining a relationship between the parts even though both series are I(1) processes. In the following subsection we develop a detailed model allowing for the residual autocorrelation.
Table 3: Coefficients by Gender for Business Cycle Time Segments: Data for males \((josm, emp_m)\), for females \((josf, emp_f)\) for total \((jos, emp)\).

<table>
<thead>
<tr>
<th>Cycle</th>
<th>Males</th>
<th>Females</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(b_0)</td>
<td>(b_1)</td>
<td>(b_0)</td>
</tr>
<tr>
<td>A</td>
<td>1.059</td>
<td>-1.348</td>
<td>-1.325</td>
</tr>
<tr>
<td>(se)</td>
<td>0.055</td>
<td>0.034</td>
<td>0.036</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.972</td>
<td>0.849</td>
<td>0.974</td>
</tr>
<tr>
<td>B</td>
<td>0.863</td>
<td>-1.233</td>
<td>0.231</td>
</tr>
<tr>
<td></td>
<td>0.041</td>
<td>0.028</td>
<td>0.052</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.981</td>
<td>0.974</td>
<td>0.991</td>
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<tr>
<td>C</td>
<td>1.187</td>
<td>-1.492</td>
<td>-0.902</td>
</tr>
<tr>
<td></td>
<td>0.021</td>
<td>0.014</td>
<td>0.022</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.992</td>
<td>0.940</td>
<td>0.983</td>
</tr>
<tr>
<td>D</td>
<td>1.542</td>
<td>-1.772</td>
<td>-0.423</td>
</tr>
<tr>
<td></td>
<td>0.029</td>
<td>0.018</td>
<td>0.022</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.987</td>
<td>0.969</td>
<td>0.994</td>
</tr>
<tr>
<td>E</td>
<td>0.759</td>
<td>-1.349</td>
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</tr>
<tr>
<td></td>
<td>0.049</td>
<td>0.032</td>
<td>0.027</td>
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<tr>
<td>(R^2)</td>
<td>0.952</td>
<td>0.986</td>
<td>0.974</td>
</tr>
<tr>
<td>F</td>
<td>1.393</td>
<td>-1.887</td>
<td>0.459</td>
</tr>
<tr>
<td></td>
<td>0.024</td>
<td>0.018</td>
<td>0.027</td>
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<tr>
<td>(R^2)</td>
<td>0.992</td>
<td>0.986</td>
<td>0.990</td>
</tr>
</tbody>
</table>

Table 3 gives the coefficients of the model

\[ jos_t = b_{0c} + b_{1c}emp_t + \epsilon_t \]  

with coefficients varying by cycle \(c\) with \(c = (1, ..., 6)\). The levels data are from \(ilr7\) for males and females, and \(ilr1\) for the total population. The results show that within each group, there are significant differences for many comparisons of the periods and for the gender groups within the cycles, though there is more gender similarity over the most recent cycles.

These are extremely simple relationships involving no other variables. The values of \(R^2\) higher than 0.95 are evidence that, over periods of several years associated broadly with business cycle periods, changes in the level of employment are reflected within the same month in the level of search behaviour by both males and females. Individuals are very sensitive to changes in the number of jobs, and there is a rapid behavioural response within the population. Figure 6 shows the observed and fitted values by cycles for the total data of Table 3.

Figure 3 at the total population level shows the linearity in the log ratio patterns over extended periods, and the variation associated with a nearly linear change both
in the odds of having a job, and the odds of searching for a job if you do not have one. Some may claim that this is solely due to having U in the denominator of one ratio and the numerator of the other, but the ratios used are chosen because they represent meaningful orthogonal directions in the log ratio space. Movement along the curve is associated with a change in the participation in employment, and a corresponding change in search behaviour by those without a job.

3.2.2 Relations in first differences

It is common to examine data in levels and in first differences in time series analysis. Two simple descriptive variables, the first differences of emp and jos characterise some features of the monthly changes over more than forty years of data as shown in Figure 7.

Allowing for cointegration, a model of the form

$$\Delta jos_t = a + (b_j jos_{t-1} + b_e emp_{t-1}) + \sum_{i=0}^{p} \beta_i \Delta emp_{t-i} + \sum_{i=1}^{q} \gamma_i \Delta jos_{t-i} + \epsilon \quad (6)$$

examines the short term response of job search to changes in employment. This formulation of cointegration and short run adjustment offered by Pesaran, Shin & Smith (2001) is known as auto-regressive distributed lag (PSS-ARDL) cointegration where the series in levels can be any combination of I(1) or I(0), and the lag lengths for the differenced series are chosen to render the error non-auto-correlated.
The PSS method first tests the joint hypothesis $H_{je} : b_j = 0$ and $b_e = 0$ using the F statistic in conjunction with the critical bounds applicable for this non-standard test. Non-rejection supports the absence of a levels relationship. If the hypothesis is rejected, proceed to testing $H_j : b_j = 0$ using the t-statistic and the associated critical values that PSS offer. Rejection of this hypothesis supports the presence of a levels relationship (which is non-degenerate if $b_e \neq 0$).

The long run equilibrium (cointegrating error = 0) gives $0 = a + (b_j jos_{t-1} + b_e emp_{t-1})$ so

$$jos_{t-1} = \frac{\hat{a}}{b_j} - \hat{b}_e emp_{t-1}$$

where the long run coefficients appear. The short run coefficients are those applying to the differenced covariates. Since there is cointegration, and regression (6) addresses both serial correlation and endogeneity of the covariates, standard tests apply to both short and long run coefficients (Pesaran & Shin(1999) Theorems 2.2 & 3.2).

Using cycle dummies to allow cycle-specific response to $jos$ and $emp$ and cycle specific constants in the differenced equation, and sufficient dynamics to model autocorrelation, the equation estimated is,

$$\Delta jos_t = a_1 + \sum_{i=2}^{6} a_{i1}d_i + (b_j jos_{t-1} + b_e emp_{t-1} + \sum_{i=2}^{6} b_{ei}emp_{t-1}d_i)$$

$$+ \beta_1 \Delta emp_t + \sum_{i=2}^{6} \beta_{i1} \Delta emp_t d_i + \theta \Delta emp_{t-1} + \gamma jos_{t-1} + \epsilon$$  (7)
With an $R^2 = 0.83$ and an adjusted $R^2 = 0.82$ for the change in job search $\Delta jos$, this model of the data gives a remarkable description of the behaviour over more than 40 years. The residuals are not serially correlated. The hypothesis of no levels relationship is rejected at the 1% level of significance by the PSS bounds test.

Table 4: Response coefficients for levels Equation 7

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Estimate</th>
<th>Std-error</th>
<th>$t$ statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_1$</td>
<td>-0.9233</td>
<td>0.0923</td>
<td>-10.01</td>
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<tr>
<td>$b_{12}$</td>
<td>-0.5194</td>
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<td>$b_{13}$</td>
<td>-0.1738</td>
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<td>-1.65</td>
</tr>
<tr>
<td>$b_{14}$</td>
<td>-0.5154</td>
<td>0.1039</td>
<td>-4.96</td>
</tr>
<tr>
<td>$b_{15}$</td>
<td>-0.4861</td>
<td>0.1212</td>
<td>-4.01</td>
</tr>
<tr>
<td>$b_{16}$</td>
<td>-0.8804</td>
<td>0.1031</td>
<td>-8.54</td>
</tr>
</tbody>
</table>

Table 5: Response coefficients for first differences Equation 7

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Estimate</th>
<th>Std-error</th>
<th>$t$ statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>-0.0607</td>
<td>0.0271</td>
<td>-2.24</td>
</tr>
<tr>
<td>$a_{12}$</td>
<td>0.1589</td>
<td>0.0468</td>
<td>3.39</td>
</tr>
<tr>
<td>$a_{13}$</td>
<td>-0.0703</td>
<td>0.0317</td>
<td>2.22</td>
</tr>
<tr>
<td>$a_{14}$</td>
<td>0.1803</td>
<td>0.0392</td>
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<tr>
<td>$a_{15}$</td>
<td>0.1590</td>
<td>0.0441</td>
<td>3.61</td>
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<td>$a_{16}$</td>
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<tr>
<td>$\beta_1$</td>
<td>-1.4600</td>
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<td>$\beta_{12}$</td>
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<td>-2.99</td>
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<td>$\beta_{13}$</td>
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<td>0.0205</td>
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<td>$\beta_{14}$</td>
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<td>-2.83</td>
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<td>$\beta_{15}$</td>
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<td>-2.19</td>
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<td>$\beta_{16}$</td>
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</tr>
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<td>$\theta$</td>
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<td>-3.66</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>-0.2014</td>
<td>0.0424</td>
<td>-4.75</td>
</tr>
</tbody>
</table>

The estimated responses in the level of $jos$ given by the error correction term appearing in parentheses in equation 7 are listed in Table 4. Expressed with $jos$ as the dependent variable, the response to $emp$ in cycle 1 is

$$b_1 = -\left(\frac{\hat{b}_{1\times}}{b_j}\right) = -0.9233$$

In the situation where the odds of ending the period with a job are the lowest, because there are few jobs, the proportion whose aspirations for work and the social and economic status that it brings is clearly greatest. As the balance of having a job increases, and the odds of ending the period with a job rise, the relative demand for additional market income falls, and the level of search is reduced.
The other estimates in Table 4 give the difference made to this response by different cycles. In the last cycle

\[ b_{16} = -\left( \frac{b_e}{b_j} \right) = -0.8804 \]

Thus the last cycle shows the strongest response of jos to emp of \( b_1 + b_{16} = -1.8037 \). These responses indicate a highly significant variation over the cycles with \( F_{5,489} = 27.04 \).

The response coefficients for the first differences and deterministic components are shown in Table 5. The estimated constants (the \( a \) coefficients) indicate change in the mean short term response of search to changes in employment. The cycle specific variations in these shifts are significant at the 1% level \( (F_{5,489} = 5.80) \). The estimated shifts are positive in all cycles except the first.

The inverse relationship between job search and employment is again reflected in the response of \( \Delta \text{jos} \) to \( \Delta \text{emp} \) (the \( \beta \) coefficients). These responses also vary significantly over the cycles \( (F_{5,489} = 4.19) \). The coefficients \( \theta \) and \( \gamma \) modelling the dynamics in \( \Delta \text{jos} \) are also highly significant.

### 3.3 The labour force focus

With the data model \( H_5 \) we use the coordinate names \( lfb, lfe, gen, lfbg, lfeq \). The balance between the two components defining the labour force is \( lfb, lfe \) is the balance for being in employment if in the labour force, \( gen \) is a population gender difference, and \( lfbg \) and \( lfeq \) give the gender difference in the first two contrasts.

This focus uses the matrices \( H_2, H_5 \) and \( H_8 \) with \((E,U)\) as the numerator group and \( N \) as the denominator group. Figure 8 shows the time series using \( H_5 \). With those in \( E \) and \( U \) in the labour force, the product of the ratio of these groups to \( N \) behaves counter cyclically. \( (E/N) \) changes little over the cycle and \( (U/N) \) is counter cyclical and the combined effect is a decrease in the ratio of the parts in the labour force to those in non market work. The balance for search which was central in the employment focus, plays an important role in the labour force focus, generating a decrease in \( lfb(z_1) \), the balance of being in the labour force as employment participation rises. This gives the result that the balance of being in non market work (the negation of \( lfb \)) is procyclical. \( lfe(z_2) \) gives the balance for being in work if you are in the labour force and moves counter to \( lfb \).

The direction of change of the labour force with the cycle has been a subject of debate in the literature. Using ratios of all three parts gives a very different perspective. Figure 9 shows how the balance of being in the labour force \( (z_1) \) moves counter cyclically and its inverse the balance of being in \( N \) moves procyclically. This is a different result to the conventional analysis with the aggregation of proportions in the parts. It arises within the range of proportions of the parts in this data.

What is happening is that as the proportionate change in employment occurs, changes in all three parts occur. On this measure, after allowing for gender, the log ratio for the labour force contributes 15 per cent of the variance, with 85 per cent associated with the changes of odds of being in work if you are in the labour force. This is illustrated from another perspective in the next section.
Figure 8: Labour Force Balances, Model $H_5$

Figure 9: Labour Force $lfb(z_1)$ and Employment Balance in the Labour Force $lfe(z_2)$ for Monthly Business Cycle Data, Model $H_5$.

Equation(5) but with $emp$ and $jos$ replaced by variables $lfb(z_1)$ and $lfe(z_2)$ of the $ilr5$ coordinates has a very strong observed pattern, with $R^2$ of 0.983 and a standard error of the residual of 0.033. Again there is a consistent pattern, with the balance for the labour force ($lfb$) highest when within the cycle the balance of employment within the labour force ($lfe$) is at its lowest values. There are big
changes between cycles in the level and magnitude of the response.

3.4 The unemployment focus

This focus is used in models $H_3, H_6$ and $H_9$. The balances for the model $H_6$ are labelled $un, enu, gen, ung, enug$. The term $enu$ is used to indicate the log ratio of $E/N$ of the parts other then $U$.

![Balances for Unemployment Focus](image)

Figure 10: Unemployment Focus Balances, Model $H_6$

Figure 10 shows the main cyclical fluctuations are associated with the unemployment balance, $un$ ($z_1$). However the business cycle has some impact with the other variables. $un$ reflects the ratio of those seeking to change their position, the unemployed, to those in each of the other two categories. From this perspective $un$ summarises the effects of changes in U. $enu$ ($z_2$) is the balance of currently having a job if you are among those employed or not searching for one. While $un$ changes with the business cycle, we observe $enu$, the balance between E and N, increased until the end of the century, but has trended down since and shown more rapid decline after the GFC.

$gen$ ($z_3$) gives the increasing role of females in the parts of the survey population. $ung$ ($z_4$) is the gender balance of $un$. The ratio with males in the numerator peaks during the trough, and generally turns up again as the next trough is approached. It has some similarity with the pattern in $un$. $enug$ ($z_5$) slowly trends down over the whole observation period, reflecting the decreasing balance of males in employment and their increase in $N$. Figure 11 shows the covariation of $un$ and $enu$ and another perspective on the strong pattern within each cycle.

Employment is a social and economic imperative, but unemployment as search behaviour is also influenced by decisions of those doing non market work. The unemployment or search focus leads to considering both $E$ and $N$ as alternatives. If you
are unemployed and do not find a job, you may continue to search or pause, moving to $N$. Options within the household include choosing $E$, exchanging participation in $E$ with another member, continuing search or stopping search and settling for $N$ until a new period. A successful search leads to $E$ or some change within the household in the $E,N$ balance.

The levels equation 5 can be adapted to the unemployment contrast using $un(z_1)$ and $enu(z_2)$ from $ilr6$. The results are in Table 6. Note the positive sign of $b_{16}$ and very different behaviour over the most recent period. Figure 11 shows $enu$ has the smallest variance all of the aggregate components for the three focus groups. The unemployment balance captures nearly all of the variance of the largest principal component of the $clr$ data. The unemployment focus shows very clearly the change in the gender structure as the unemployment varies in Figure 12. As the level of unemployment decreases the balance that those searching for work are male also falls. However Figure 13 shows that the trend of decreasing male balance for those

![Trends for un vs enu](image)

Figure 11: Unemployment Balance $un(z_1)$ and Employment vs Non Market Balance $enu(z_2)$ for Monthly Business Cycle Data, Model $H_6$.

Table 6: Coefficients for Model $H_6$ and form Equation 5

<table>
<thead>
<tr>
<th>Term Cycle</th>
<th>Coeff $b_{16}$</th>
<th>Std Err</th>
<th>Coeff $b_{1c}$</th>
<th>Std Err</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.271</td>
<td>0.042</td>
<td>-0.296</td>
<td>0.015</td>
<td>0.913</td>
</tr>
<tr>
<td>2</td>
<td>0.376</td>
<td>0.018</td>
<td>-0.075</td>
<td>0.007</td>
<td>0.745</td>
</tr>
<tr>
<td>3</td>
<td>0.060</td>
<td>0.019</td>
<td>-0.210</td>
<td>0.007</td>
<td>0.903</td>
</tr>
<tr>
<td>4</td>
<td>0.427</td>
<td>0.009</td>
<td>-0.086</td>
<td>0.003</td>
<td>0.852</td>
</tr>
<tr>
<td>5</td>
<td>0.434</td>
<td>0.023</td>
<td>-0.070</td>
<td>0.008</td>
<td>0.463</td>
</tr>
<tr>
<td>6</td>
<td>0.598</td>
<td>0.012</td>
<td>0.036</td>
<td>0.004</td>
<td>0.438</td>
</tr>
</tbody>
</table>
in $E$ plus $U$ has moved with a different pattern to their combined share.

![Figure 12: Unemployment Balance $un(z_1)$ and Gender difference $ung(z_4)$, Model $H_6$](image)

![Figure 13: Employment Balance E/N $enu(z_2)$ and Gender difference $enug(z_5)$, Model $H_6$](image)

### 3.5 Issues in Modelling Composition Data

So far we have attempted to model the behaviour within the composition without reference to the external variables influencing it. We have shown that there are very systematic relationships between the variables at the same time using both levels and differences. An examination of some lags showed no significant improvement for lags
longer than two. If we regard the level of employment as exogenously determined then the remaining patterns in the data make sense as resulting from rapid reactions to a changing employment situation. In all the subgroups observed, increases in the proportions searching only occur as the proportion employed falls. Arguing that unemployment is a result of inadequate search activity is totally contrary to the pattern in the data. The number searching increases when there are not enough jobs. Reducing search does not increase the number of jobs.

In the search paradigm, the level of vacancies, or measures such as the help wanted index play an important role. If it is regarded as an indicator of changes in the demand side of the market, then it tells us about the desired direction of change in $E$. We explore that in Section 4.

To model composition data $X$ the objective is to estimate a point $x$ as a function of a set of independent variables. If $Z = \text{irl}(X, H)$ we can estimate a model for each column of $Z = (z_1, z_2, ..., z_{D-1})$. A linear model would be

$$Z = V \cdot B + \epsilon$$

where $V$ is a matrix of covariates, $B$ is a matrix of coefficients and $\epsilon$ is a matrix of random errors. To model the composition, given $V$, we could then make the inverse transform from $Z$ to $X$. In the economic literature the problem of modelling the level of employment has proved extremely intractable, and while the major transformation in the gender balance plays a role in much commentary, it has proved very difficult to construct adequate models of both the magnitude and speed of the change in the number of jobs and the gender balance of those holding them. The balance statistics provide a new way of exploring those relationships.

Any comprehensive model of this market must include factors which determine the changes in the number of jobs. In the absence of such a model we regard the employment contrast as a estimate of the reduced form value of that coordinate determined in an environment with sufficient competition for the available jobs to enable employers to largely match the desired number of jobs. This is consistent with both the comment of Shimer that the observations are close to what he calls short term equilibrium, and with the large monthly changes in the statistics. In the presence of a set of social, cultural and legal factors, changes in the number of jobs will generate changes in the log ratios between employment and the other categories. The patterns in those ratios are consistent with changes in employment being the main driver of change within each business cycle and the observed levels of search a consistent pattern of response to the employment changes.

This leads to a strategy of modelling the compositions, using a covariate for each busines cycle and treating the employment component of the composition as determined by other exogenous factors. The analysis in the preceding subsections can be viewed from that framework, as changes in the composition in response to changes in the number of jobs and supply side effects changing with some delay when the number of jobs falls rapidly from the peak, which is the common cycle pattern. That means we treat coordinate $z_1$ in model $H_1$ as the effect of external variables on this market.

When we come to the more detailed data on the gross flows we can consider an extension of the model. We would expect that the supply side of this market is influenced by factors in addition to the log odds of employment which are ob-
viously important in influencing the search behaviour. If we assume a recursive framework, and regard $jos$ as a function of $emp$ and other exogenous variables then the remaining variables can be considered as determined by an equation of the form

$$z = \alpha_0 + \alpha_1 emp + \alpha_2 jos + \sum \beta_r v_r + \epsilon$$

with $v_r$ exogenous variables. Section 4 uses that model to study the gross flows data.

4 Gross Flows Data

4.1 Models for the gross flows data

The gross flows tables give a rich picture of change. The analysis below is restricted to the monthly de-seasonalised data for 1990-2016. The data categorize all individuals by their initial and final status categories:

$$(EE, EU, EN, UE, UU, UN, NE, NU, NN)$$

The initial status provides three groups. The two genders, three groups and three end of month status alternatives give an 18 category partition of the population. Within month transitions may occur, but the focus is on month to month outcomes. Persons who are not in the sample population for the whole month are omitted. The factors generating transitions in or out of the survey population are often demographically determined and likely to be modified by quite different variables from those determining behaviour within the sample population.

The usual approach to the gross flows data considers the parts as a framework of continuing states. We interpret the data consistently with the emphasis in Section 1 as a set of current responses to a changing situation. Persons in each of the parts may make changes in any direction, purely on the basis of what they think is the best response to a current situation. Their situation may change and their response may change without any long term change in their attitude towards interaction with the market economy.

For the SBP process construct an employment contrast (Emp) and job search (Jos) contrast for the whole population by expanding the framework used in Section 2. The Appendix has the matrix $B_{18}$ listing the components in each of 17 contrasts to construct an ilr transform of the data. The first component is a contrast between the genders for all cells. The next two contrasts are Emp an initial employment based partition for all cells, and Jos, a search based contrast for all cells of those not initially in jobs. These are followed by an Emp and a Jos partition, for the total population in each initial status subgroup. The final eight contrasts are gender differences for each of the eight contrasts already defined. This gives a partition of the variance into seventeen components, one for the balance in the population between the genders, and the remainder for differences in employment and job search position. This structure is more complex than the SBP process, and to convert each row of $B_{18}$ to the required $H$ matrix use coefficients $(2,-1,-1)$ for the parts in the employment contrasts and a divisor of the square root of the sum of squared items. Label the variables generated by these contrasts

$$G, Emp, Jos, EEmp, EJos, UEmp, UJos, NEmp, NJos,$$

This matrix $H_{18}$ and the source data in clr form give a data matrix ilr18 consisting of 17 variables.

Table 7: Variance of Contrasts

<table>
<thead>
<tr>
<th>Contrast</th>
<th>Variance Total</th>
<th>Variance Gender</th>
<th>Percent Total</th>
<th>Percent Gender</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>n.a. 0.01050</td>
<td>n.a. 2.90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emp</td>
<td>0.05973 0.00480</td>
<td>16.49 1.32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jos</td>
<td>0.07239 0.00734</td>
<td>19.98 2.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EEmp</td>
<td>0.00438 0.00224</td>
<td>1.21 0.62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EJos</td>
<td>0.02537 0.00657</td>
<td>7.00 1.81</td>
<td></td>
<td></td>
</tr>
<tr>
<td>UEmp</td>
<td>0.06648 0.00531</td>
<td>18.35 1.47</td>
<td></td>
<td></td>
</tr>
<tr>
<td>UJos</td>
<td>0.03683 0.00690</td>
<td>10.17 1.90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NEmp</td>
<td>0.02810 0.00215</td>
<td>7.78 0.59</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NJos</td>
<td>0.02089 0.00224</td>
<td>5.77 0.62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subtotal</td>
<td>0.3142 0.0480</td>
<td>86.74 13.26</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>0.3622</td>
<td>100.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The ilr variables give a partition of the variance associated with the gross flows data. Table 7 gives the components and total variance. The first feature of the table is that 87 percent of the variance is associated with the gross flow pattern ignoring gender, and only 13 percent with the differences between male and female over this 25 year period. It is not surprising that there is a smaller share for gender differences than in Table 2 as we noted earlier that some of the largest changes were in the period up to 1990. Section 3 showed evidence of a convergence of male and female behaviour. This data shows that when there are changes in employment, they affect both genders and both respond depending on their current status. The population pattern for Emp and Jos accounts for 36.5 percent of the total variance, split almost equally between changes in employment and search. The subgroup U contributes a further 28.5 percent with two thirds associated with odds of employment. The changing market situation has bigger effects on their odds of employment than on their search. Changes in the experience of those in N account for a further 13.5 percent, and most of the variance of those in E is associated with their search behaviour at 7.0 percent.

If instead we aggregate over the Emp and Search contrasts we find that 43.8 and 42.9 percent of the variance is associated with the population totals, 4.0 and 6.4 percent from gender differences and the remaining 2.9 percent with changes in the gender balance. This is evidence of the way in which these two contrasts affect the population both at a total level, and also differentially for all subgroups.

Similar tables can be constructed for the labour force and unemployment focus models. They show that the gross flows model of subgroup data has large changes in variance like those in Table 2 for the gender data on the levels.
4.2 Job finding and rationing process for subgroups

In the job rationing process, those in different initial positions have different outcome patterns. For those with a job, the odds of retaining or being able to move directly to another job are high. All looking for a job or doing non market work have outcomes dependent on the cycle. The literature has an emphasis on the job finding rates. For those without a job, the Emp balance is a scalar times the log ratio of the geometric means for parts with a job at the end of the period and for parts without a job so reflects changes in the job finding rate. The Jos balance for the unemployed is a mean of the log ratios that they will continue searching if they did not get a job, and for those in $N$ the mean of the log ratios they have started searching if they did not get a job. Both have a strong cyclical pattern.

The data provides a link between the outcomes for different groups and their search behaviour in response to two actions over which they have limited control, the action to terminate a job and the hire which terminates looking for a job. We can plot the data for the four $Emp$ versus $Jos$ pairs on a single diagram. Figure 14 shows those in E have the highest balances of having a job at the end of the period by a very large margin, those unemployed have modest balances for changing to employment, and those in Non market work have an even lower likelihood of ending the period employed. The balance for searching is highest for those initially
unemployed, considerably lower for those who lose a job or quit, and lowest for those currently in non market work. The figure illustrates that within each subgroup there is an association between the balance for being employed at the end of the period, and the odds of search. But it is a striking pattern. The amount of search is highest when the probability of success is lowest. The pattern observed at the total level, that the balance for search declines as the balance for a person having a job at the end of the period increases also applies within each subgroup.

The source data on numbers provides a table which is important in interpreting the log ratios in this diagram. If we close the data on gross flow numbers for each year, and find the mean pattern we get Table 8. Of the 2.57 percent moving to new jobs, 63 percent come from $N$, and the very much smaller odds of getting a job from $N$ are offset by the much larger pool from which they are drawn.

Table 8: Gross flow shares

<table>
<thead>
<tr>
<th>Status(t)</th>
<th>Status</th>
<th>$E_{t+1}$</th>
<th>$U_{t+1}$</th>
<th>$N_{t+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_t$</td>
<td>59.21</td>
<td>0.87</td>
<td>1.69</td>
<td></td>
</tr>
<tr>
<td>$U_t$</td>
<td>0.96</td>
<td>2.16</td>
<td>0.86</td>
<td></td>
</tr>
<tr>
<td>$N_t$</td>
<td>1.61</td>
<td>0.94</td>
<td>31.72</td>
<td></td>
</tr>
</tbody>
</table>

To provide further analysis we focus on the eight coordinates of gross flows for the total population, which are associated with 87 per cent of the variance. The correlation matrix of these coordinates is given in Table 9 and shows there is a great deal of structure. All cells $(i, j)$ where one index is odd and the other even have a negative sign, reflecting the inverse relationship between employment and job search examined in Section 3. All others are positive. Within the total population, and the three subgroups, the $Emp$ variables are positively correlated. Among those with a job, Unemployed or doing Non-market work, the odds of having a job at the end of the period on average move in the same direction as for the whole population. Similarly the odds of search move up or down across all subgroups.

The striking similarities in the behaviour of the contrasts for each initial status group are illustrated in Figures 15 and 16. These show clearly that the behaviour in

Table 9: Correlation Coefficients between Gross Flow Coordinates

<table>
<thead>
<tr>
<th></th>
<th>Emp</th>
<th>Jos</th>
<th>EEmp</th>
<th>EJos</th>
<th>UEmp</th>
<th>UJos</th>
<th>NEmp</th>
<th>NJos</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emp</td>
<td>1.00</td>
<td>-0.777</td>
<td>0.280</td>
<td>-0.455</td>
<td>0.901</td>
<td>-0.707</td>
<td>0.863</td>
<td>-0.765</td>
</tr>
<tr>
<td>Jos</td>
<td>-0.777</td>
<td>1.00</td>
<td>-0.658</td>
<td>0.815</td>
<td>-0.697</td>
<td>0.839</td>
<td>-0.854</td>
<td>0.839</td>
</tr>
<tr>
<td>EEmp</td>
<td>0.280</td>
<td>-0.658</td>
<td>1.00</td>
<td>-0.688</td>
<td>0.384</td>
<td>-0.609</td>
<td>0.486</td>
<td>-0.552</td>
</tr>
<tr>
<td>EJos</td>
<td>-0.455</td>
<td>0.815</td>
<td>-0.688</td>
<td>1.00</td>
<td>-0.370</td>
<td>0.727</td>
<td>-0.642</td>
<td>0.767</td>
</tr>
<tr>
<td>UEmp</td>
<td>0.901</td>
<td>-0.697</td>
<td>0.384</td>
<td>-0.370</td>
<td>1.00</td>
<td>-0.683</td>
<td>0.823</td>
<td>-0.648</td>
</tr>
<tr>
<td>UJos</td>
<td>-0.707</td>
<td>0.839</td>
<td>-0.609</td>
<td>0.727</td>
<td>-0.683</td>
<td>1.00</td>
<td>-0.742</td>
<td>0.705</td>
</tr>
<tr>
<td>NEmp</td>
<td>0.863</td>
<td>-0.854</td>
<td>0.486</td>
<td>-0.642</td>
<td>0.823</td>
<td>-0.742</td>
<td>1.00</td>
<td>-0.855</td>
</tr>
<tr>
<td>NJos</td>
<td>-0.765</td>
<td>0.839</td>
<td>-0.552</td>
<td>0.767</td>
<td>-0.648</td>
<td>0.705</td>
<td>-0.855</td>
<td>1.00</td>
</tr>
</tbody>
</table>
all of these groups is closely linked to the behaviour at an aggregate level. However there are differences in scale for each of these coordinates. The similarity of shape suggests very close linkages between them.

In view of the analysis in Sections 2 and 3 it is natural to consider a model with changes in Emp as central to the behaviour in each of the subgroups. We will regard it as a variable determined by exogenous factors. JOS is strongly associated with Emp, but the data shows it contains some additional information relevant to behaviour in the subgroups. We treat it as a part of a recursive system, possibly also affected by other variables exogenous to the labour market.

Search behaviour is not solely dependent on the participation in jobs as measured by Emp, but may be influenced by a large range of other social factors, income, prices, expectations and life cycle aspects of demographics. There may also be level changes between business cycle periods. Together these elements give equation 9 as a model for the data. The literature has given a great emphasis to vacancies, which are a demand side analogue of unemployment. We have included the log of the Help Wanted Index LHwi constructed and maintained by Barnichon(2010) and expressed as a log of the variable to maintain consistency with using logarithmic comparisons. The numbers used are from his revision and concept changes to 2016(12). They have not been modified to a population rather than labour force basis. To assess if
Search contrasts for Gross Flows Data

![Trace of Search Contrasts for Initial Status Groups](image)

Figure 16: Trace of Search Contrasts for Initial Status Groups

vacancies are associated with any of the gross flow categories we initially consider models with $b_{3r}$ set to zero, where

$$z_r = a_0 + \sum_{j=5}^{6} a_{jr}d_j + b_{1r}Emp + b_{2r}Jos + b_{3r}lHwi + \epsilon$$  \hspace{1cm} (9)

$z_r$ for ($r = 4, \ldots, 9$) refers to the six initial status based $emp$ and $jos$ contrasts in the data. The coefficients from fitting the model and imposing $b_3 = 0$ are given in Table 10.

Given the high degree of correlation between the variables in Table 9 we need to remember that the coefficients in Table 10 are the partial effect having allowed for all interactions with the remaining included variables. For those in E the balance $EEmp$ showed the least variance of all the coordinates in Table 7. Table 8 shows it varies little with changes in $Emp$ though there is a small positive association, with a slow rise. The two slope coefficients generate offsetting change because of the different signs of the associated variables. Among those in E who no longer have a job, the log ratio for the search contrast is not as high as among the unemployed, but behaves in the usual inverse way relative to the log odds of employment. $EJos$, $UJos$ and $NJos$ are strongly associated with the aggregate measure $Jos$, consistent with the view that the macro features have a major effect for all groups in the market.
Table 10: Regression results Model 9 excluding Help Wanted Index

<table>
<thead>
<tr>
<th>Equation</th>
<th>(a_0)</th>
<th>(a_5)</th>
<th>(a_6)</th>
<th>(b_1)</th>
<th>(b_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EEmp</td>
<td>4.365</td>
<td>-0.030</td>
<td>-0.099</td>
<td>-0.381</td>
<td>-0.396</td>
</tr>
<tr>
<td></td>
<td>0.022</td>
<td>0.006</td>
<td>0.010</td>
<td>0.027</td>
<td>0.018</td>
</tr>
<tr>
<td></td>
<td>0.666</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EJos</td>
<td>0.408</td>
<td>-0.052</td>
<td>-0.148</td>
<td>-0.037</td>
<td>0.506</td>
</tr>
<tr>
<td></td>
<td>0.042</td>
<td>0.011</td>
<td>0.020</td>
<td>0.053</td>
<td>0.035</td>
</tr>
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<td></td>
<td>0.782</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>9.7e8</td>
<td>-4.775</td>
<td>-7.380</td>
<td>-0.700</td>
<td>14.551</td>
</tr>
<tr>
<td>UEmp</td>
<td>-1.348</td>
<td>-0.178</td>
<td>-0.355</td>
<td>0.179</td>
<td>-0.438</td>
</tr>
<tr>
<td></td>
<td>0.043</td>
<td>0.011</td>
<td>0.020</td>
<td>0.054</td>
<td>0.036</td>
</tr>
<tr>
<td></td>
<td>0.913</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-31.537</td>
<td>-15.976</td>
<td>-17.360</td>
<td>3.324</td>
<td>-12.312</td>
</tr>
<tr>
<td>UJos</td>
<td>2.016</td>
<td>0.052</td>
<td>0.033</td>
<td>-0.047</td>
<td>0.567</td>
</tr>
<tr>
<td></td>
<td>0.057</td>
<td>0.015</td>
<td>0.027</td>
<td>0.072</td>
<td>0.047</td>
</tr>
<tr>
<td></td>
<td>0.723</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>35.451</td>
<td>3.514</td>
<td>1.198</td>
<td>-0.658</td>
<td>12.000</td>
</tr>
<tr>
<td>NEmp</td>
<td>-2.372</td>
<td>0.018</td>
<td>-0.063</td>
<td>0.195</td>
<td>-0.360</td>
</tr>
<tr>
<td></td>
<td>0.037</td>
<td>0.010</td>
<td>0.018</td>
<td>0.047</td>
<td>0.031</td>
</tr>
<tr>
<td></td>
<td>0.844</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NJos</td>
<td>-2.352</td>
<td>-0.045</td>
<td>-0.175</td>
<td>-0.566</td>
<td>0.116</td>
</tr>
<tr>
<td></td>
<td>0.036</td>
<td>0.009</td>
<td>0.017</td>
<td>0.046</td>
<td>0.030</td>
</tr>
<tr>
<td></td>
<td>0.800</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-64.654</td>
<td>-4.763</td>
<td>-10.081</td>
<td>-12.346</td>
<td>3.842</td>
</tr>
</tbody>
</table>
Table 11: Regression Coefficients Model 9 including Help Wanted Index

<table>
<thead>
<tr>
<th>Equation</th>
<th>Coef</th>
<th>SE</th>
<th>t</th>
<th>R²</th>
<th>t</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>EEmp</td>
<td></td>
<td>4.404</td>
<td>0.003</td>
<td>-0.087</td>
<td>-0.381</td>
<td>-0.304</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.022</td>
<td>0.008</td>
<td>0.010</td>
<td>0.026</td>
<td>0.024</td>
</tr>
<tr>
<td>EJos</td>
<td></td>
<td>0.320</td>
<td>-0.128</td>
<td>-0.176</td>
<td>-0.035</td>
<td>0.296</td>
</tr>
<tr>
<td>UEmp</td>
<td></td>
<td>-1.268</td>
<td>-0.110</td>
<td>-0.329</td>
<td>0.178</td>
<td>-0.247</td>
</tr>
<tr>
<td>UJos</td>
<td></td>
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<td>-0.011</td>
<td>-0.044</td>
<td>0.243</td>
</tr>
<tr>
<td>NEmp</td>
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<td>-2.349</td>
<td>0.038</td>
<td>-0.055</td>
<td>0.194</td>
<td>-0.304</td>
</tr>
<tr>
<td>NJos</td>
<td></td>
<td>-2.427</td>
<td>-0.109</td>
<td>-0.199</td>
<td>-0.565</td>
<td>-0.064</td>
</tr>
</tbody>
</table>

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The balance that a person in U will get employment in the period moves in the same direction as Emp and shows the same negative effect associated with Jos. Again there is a much stronger association with Jos than with Emp. The external effects of job search are large. As would be expected, the odds of moving to a job are lower if more are searching, even given the current level of the proportion who have jobs. Persons in U have the highest levels of job search at the end of the period, clearly indicating their commitment to finding a job.

For persons in the N group, the balance of finishing the period with a job is strongly associated with the level of Emp as in the other groups. There is a similar pattern in the partial coefficients, to that for those in U. The level of job search shows a large negative coefficient on Emp showing reduced search as higher employment levels are attained.

It is a feature of Table 10 that the data shows a strong behavioural pattern based on the level of Emp and Jos and any association with vacancies ignored. The balance for getting a job in each group depends on Emp but the negative dependence in the case of EEmp is unexpected. Search behaviour in each group appears to be largely determined by the aggregate search behaviour, except for those in N where it moves strongly inversely with Emp.

When the number of jobs changes, every aspect of behaviour in this market shows some response. The gradient differences demonstrated in Figure 6 show that there are also significant changes in search behaviour which is a supply response. The Jos contrast provides the best single predictor for several variables, but the equations in Table 10 show that the Emp contrast is the primary factor in each of the other Emp contrasts and that the search behaviour contrasts are likewise dominated by Jos.

4.3 What is the role of Vacancies?

Vacancies have a central role in models in the Mortenson-Pissarides tradition as a signal about the demand side of the market. They can arise from changes in the number of jobs available, the result of quits or fires or changes in the rate at which new hires are achieved. They can change independently of the current stock of jobs but empirically are highly correlated with it. Emp is based on the current stock of jobs, relative to the level of search behaviour and the share in non market work. HHwi relates to the remaining variables in the same general way as Emp and is positively correlated with all Emp variables and negatively correlated with all Jos variables. Table 11 gives the coefficients for equation 9 and shows HHwi does improve the fit in EEmp, UEmp and in the three search equations for continuing in search at the end of the period.

The variable HHwi is strongly procyclical. When it is higher, it is associated with an increase in EEmp which is exactly what we would expect. Employers want additional staff and in many cases are able to add them, making the balance that a person employed at time $t$ will be employed at $t+1$ higher relative to the expectation with only Emp and Jos information. However there is also evidence it makes a difference to the odds of moving to a job for those in U and perhaps also in N. This contrasts with the result that in each group, higher HHwi leads to lower levels of search. The balances for having a job appear to be determined by the
combination of Emp and Jos. Increased help wanted advertising reduces the level of search possibly through reducing search costs or through reduced time searching before obtaining a hire but the overall structure of dependence on the level of Emp and Jos as central variables appears to be little affected by including \( lHwi \).

5 Conclusions and Contrasts

The recent literature on this market has focused on two approaches. One has explored the gross flows data and generated hazard rates and job finding rates using macro data. The other has used detailed micro data from the CPS to explore heterogeneity and behaviour within subgroups. By using a model giving each labour force status a parallel role, and macro data we have shown that a simple labour demand based model can explain a very large proportion of the variance in all the observed categories. Studies by Shimer (2012, 2014), Barnichon and Figura (2015, 2015a), Hall and Schulhofer-Wohl (2018) and Elsby, Hobijn and Sahin (2015) have illustrated the need to consider all three categories and to recognise the heterogeneity in the population. However they have not adopted the consistent multivariate log ratio framework used here. The results provide a new perspective for viewing their work, and a framework for a wide range of further studies.

By using a model integrating all of the parts, it is possible to construct a better overview of what is happening on the supply and demand side of the labour market. The responses observed are consistent with a model of a changing number of jobs, rationing achieved by employers filling available jobs, and persons making a decision about further job search on the basis of their rationed situation. Since search is costly, the current employment situation provides a guide to the expected return, and changing decisions.

Elsby et al refer to “the often neglected empirical regularity that worker flows between unemployment and non participation display prominent cyclical fluctuations”. Sections 3 and 4 showed that the decisions in all current status groups are strongly affected by the current level of employment participation and this generates cyclicality. Elsby et al emphasize the impact of these changes on the labour force participation margin, but because of the simplex properties, any driving variable will impact on the whole vector of proportions of the parts. Our analysis using a set of orthogonal coordinates, has shown that changes in labour force participation are directly associated with the variance of unemployment but place them in the context of a model including all three parts. Elsby et al go further to suggest “that future research should focus less on cyclical variation in unemployment and instead direct attention towards fluctuations in employment”. This paper has shown that a model in which fluctuations in employment are regarded as exogenously driven is consistent with the level of unemployment being strongly associated with behavioural responses to varying job availability. The pattern of search behaviour rising as employment participation falls is found for all groups distinguished in this study. It provides compelling evidence of the the social impact of changes in the total number of jobs.

In a series of papers Barnichon and Figura have argued that the decomposition of changes in unemployment has multiple significant components. They argue there are both long term and business cycle frequency components with a different primary mechanism. This paper has used that distinction, and observed shifts be-
between cycles as periods with stable patterns of search in response to changes in job numbers between cycle troughs. The level of recruitment from those in $N$ is strong evidence that at all times, the conventional labour force is only a lower bound on the available supply at current employment conditions. The changes in the level of search observed between cycles may or may not be associated with changes in the supply curve of labour. They may be associated in a complex way with many social and behavioural variables. Factors influencing the level of search are an important area for further research.

Many including Shimer, Krusell et al, Elsby et al and Barnichon and Figura make extensive use of the notion of an equilibrium process to determine parameters of their models. It is clearly true that the changes are a dynamic process, and may for short periods move in a consistent direction, but the framework of our model is of continuing shocks and rapid response to them with the process essentially stochastically driven. The stability of the pattern of immediate response in all of the simple models we have explored emphasizes the behavioural impact of changes in job numbers. Using an equilibrium process to justify patterns of transition may be imposing unjustified restrictions given the rapid response we have observed.

Hall and Schullhofer-Wohl study recent experience and heterogeneity of job length, and effects on the job-finding rate. Their study provides strong evidence of the way in which detailed history influences individual decisions, with heterogeneity going down to an almost personal level. Barnichon and Figura(2015) also explore time varying heterogeneity. It would be very surprising if there were not significant changes over time for all age groups given the pace of change in the economic environment, in technology, and major social and psychological drivers. The motivation for entering or leaving the labour force at the individual level can change both rapidly and substantially over time. Elsby, Hobijn and Sahin(2015) make many interesting adjustments to the data to give a better short term picture of participation in the labour market from a traditional perspective. We use the framework of composition models to explore the behavioural response to recent change and experience studied by these authors from a different perspective.

Diamond and Sahin(2014) have shown that there have been substantial historical shifts in the Beveridge curve. This highlights a major difficulty of the Beveridge curve. What we know from $(E, U, N)$ is that the proportion willing to work $P_s$ satisfies $P_s > E + U$. If there is a change in the behaviour which leads to a change in the level of search at any given Emp there will be a change in the Beveridge curve. It can be entirely independent of other institutional or technological change in job finding. The facts that higher levels of vacancies are strongly associated with the level of employment, and that higher levels of search result from lower levels of job availability do not provide a sufficient case for an observed pattern of association playing a production function role in modelling the labour market.

Shimer’s work(2005, 2012) also looks at shares of the variation in unemployment. Unemployment is a very significant feature of behaviour but is not the only important component in the three parts. While he emphasizes job finding, you can only find a job if it exists in the sense that an employer makes an offer. Variation in the number of jobs available determines the number of jobs to be found. That number will vary with processes of termination, resignation and job creation all of which are part of the flux in the market. Using ‘job finding’ as the main descriptor...
suggests a search based process as playing an important part in determining the number of jobs. However our data consistently shows that the level of search is greatest when there is a smaller stock of jobs to be shared, and declines as the stock of jobs increases and a greater proportion of the population have their desire for a job satisfied. Differences in ‘job finding’ arise in all groups including those currently with a job. For those in U and N the balances for finding a job are pro cyclical, and strongly associated with the total population balances of having a job, so are consistent with a rationing model.

Faced with the levels of unemployment in the Great Depression, Keynes (1936) argued that the demand for labour was not determined in the labour market. His analysis was based on personal observation and experience. We now have rich bodies of empirical data and new tools of analysis which demonstrate that a model which is based on the level of jobs being determined by other markets, provides a foundation for a large proportion of the observed variation in the shares of population in each of the labour force status categories.

Much recent work has tended to focus on new longitudinal data sets of individual data, and the interaction of personal characteristics and labour supply. The results in this study show significant changes in average patterns with changing macro conditions and evidence of search behaviour reflecting unsatisfied supply objectives and rationing of available jobs. In that situation the assumption that the current observed employment experience identifies long run supply behaviour is clearly invalid.

This paper has only explored these approaches in the context of a single country and at a general level. There are many ways of applying these models to other individual characteristics with data sets like the CPS. The tools also provide a framework for a wide range of new research using these labour market measures to examine the impact or association with other financial and technical variables, and studying issues which arise in applying them across countries with different social, legal and institutional environments.

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References


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Pearson, E.S (1897)Mathematical contributions to the theory of evolution. On the form of spurious correlation which may arise when indices are used in the measurement of organs. Proceedings of the Royal Society of London. LX, 489-502.


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Appendix

The array $B_{18}$ gives a set of coefficients for log linear contrasts of the logarithms of 18 variables in the gross flow data by gender. Converting each row to unit length gives a transform matrix $H_{18}$ to construct 17 orthogonal components describing the composition.

$$B_{18} = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
2 & 2 & -1 & -1 & -1 & -1 & -1 & -1 & 2 & 2 & 2 & -1 & -1 & -1 & -1 & -1 & -1 \\
0 & 0 & 0 & 1 & 1 & 1 & -1 & -1 & -1 & 0 & 0 & 1 & 1 & 1 & -1 & -1 & -1 \\
2 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 2 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 2 & -1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\
2 & 2 & -1 & -1 & -1 & -1 & -1 & -1 & 2 & -2 & -2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 & -1 & -1 & -1 & 0 & 0 & 0 & -1 & -1 & -1 & 1 & 1 & 1 \\
2 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
\end{pmatrix}$$