Can a home country benefit from FDI?
A theoretical analysis

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Can a Home Country Benefit from FDI? A Theoretical Analysis.

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The effects of outward FDI on home country’s growth remain an open question. The growth of outward FDI has renewed this attention. By allowing for endogenous decisions of firms on both whether to conduct FDI and whether to flow capital returns back to the home country, we have found several interesting results. First, as long as the probability of conducting FDI is positive, a higher proportion of entrepreneurs may harm economic growth of the home country in short-run and long-run. The ambiguous effects of transaction costs and MRS between domestic and foreign consumption on the home country’s economic growth result from the role of financial intermediaries. If the effect via inflow probability dominates, conducting FDI in a host country with a more liberalized capital account, or with a higher capital return rate may promote the home country’s economic growth rate. This is consistent with the findings in the outward FDI in European Union since 1970s.

JEL: E20, F32, G11, O16.

Keywords: outward FDI, economic growth, capital returns, financial intermediaries

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1. INTRODUCTION

It has been well-documented that the more developed the country is, the more liberalized capital accounts the country has. A more liberalized capital account would definitely attract more capital flows, both inflow and outflow. Among different resources of capital flows, the actual flow, especially investment in production, tends to relatively clear to measure. The more advanced technology, which has lowered both communication costs and transportation costs, makes it easier for firms to conduct the internationalization of production, including contracts, outsourcing, offshoring, and foreign direct investment (FDI).

From 1980 to 2004, the FDI flows have counted for 2% to 10% of global fixed capital formation. The flows due to FDI have account for 14% and 20% of manufacturing jobs in US and Europe, respectively, which have been the main sources of world outward FDI since World War II in 1960s and remain significant after developed market economies joining the source of outward FDI in 1980s. Although it has been well documented that inwards FDI and economic growth are positive correlated in hosting countries, the relationship between outward FDI and growth in home countries remains unclear [Eichengreen (2004)].

One view is that outward FDI have negative effects on home country’s growth by applying the same logic of positive effects of inward FDI on growth of hosting countries. For hosting countries, the inward FDI stimulate employment, and investment, and hence, economic growth. Applying the same logic to home countries implies that the outward FDI would damage growth of home countries by taking away investment, employment and output production. This is the general concern of the public in home countries. A study by Marin (2004) has supported this concern. There are several opposite views stating positive effects of outward FDI on economic growth of home countries. One is that the growth of outward FDI is mainly due to a more liberalized capital accounts, which attracts capital inflows and promotes economic growth, especially in high income countries. However, this view does not hold when capital outflows, such as outward FDI, outweights capital inflows. This is one of the reasons why outward FDI have renewed attention worldwide [Blomstrom and Kokko (1994)].

3 By investigating 2,200 investment projects of 660 firms in 1990-2001 in Germany and Austria to East Europe, Marin (2004) has shown that these outward FDIs have caused 90,000 direct job loss in Germany and 24,000 job loss in Austria, which count for 0.25% and 0.75% of total employment in Germany and Austria, respectively.
Another is that by categorizing FDI into two types, horizontal and vertical, one can show that both types of outward FDI could promote economic growth of home countries. While horizontal FDI is market seeking and substitutes host countries' employment and output for home country, vertical FDI is cost minimizing and complement host country’s employment and output for home country. However, it could be difficult to clearly categorize each foreign direct investment project. When an investment project is a mix of both horizontal and vertical FDI, it is unclear whether the conclusion remains. There has been several major outward FDI in 2005 to 2007\textsuperscript{4}. Take Burberry’s relocation as an example. It is unclear whether FDI of Burberry is horizontal or vertical\textsuperscript{5}. Similar logic applies to other cases, in which it is hard to define the type of FDI of the project. However, the effects of FDI on economic growth are related to employment, output and wage rates of the country. Thus, if we could find the effect of FDI on employment, output and wage rates, the effect of FDI on economic growth becomes clear.

To address this issue, it is important to clarify the benefits and costs involved for an outward FDI. Costs include the lost of employment and investments, and benefits include the possible future capital returns to flow back to home countries. To have a simple model to begin with, inelastic labour supply is assumed, so it is the effect via output that we focus in this exercise. By extending an overlapping generation model of Bencivenga and Smith (1991), Schreft and Smith (1998), and Chang (2005), and allow financial intermediaries to play a role, we are able to examine the effect of FDI on the output of home country. Moreover, it is assumed costly for the capital flowing back to home country, and that the employment in host country is perfect substitutes for home country while the output produced in both countries is imperfect substitutes. This is to address individual's preference of "home-made" product. Note that it is the output effect of FDI the focus of this work, rather than the employment effect.

Consequently, we find that a higher capital share and a higher proportion of entrepreneurs

\textsuperscript{4}The well known one is the relocation of Burberry from South Wales (United Kingdom) to China in 2006, and other cases includes Blundstone Boots from Tasmania (Australia) to Thailand and India in 2007, Autolive and Kraft from Melbourne (Australia) to South Korea and China in 2005 and 2006, respectively, Heinz from Britain to Holland in 2006, and Legos from United States to Mexico and from Denmark to East Europe.

\textsuperscript{5}In terms of production, Burberry has its headquarter and design departments remained in London while having the manufacturing firms relocated in China. This implies that unskilled labours in host country are substitutes for home country while unskilled labours are complements for home country. So it could be defined as either vertical or horizontal FDI in terms of employment and production. However, in terms of output, it is clear that the Burberry bags whether made in China or made in UK are sold at the same price in the market, so the output of Burberry in host country is substitute for home country; therefore, it is horizontal FDI in terms of output. Overall, it is unclear whether Burberry’s FDI horizontal or vertical.
in the host country would generate higher capital returns and stimulate outward FDI in home country and damage economic growth of home country. Interestingly, the more control of capital accounts in the host country, although reducing the probability of future capital flows and damage economic growth, would increase the investment in illiquid investment and improve economic growth in home country. A smaller marginal rate of substitution between domestic and foreign consumption decreases relocated entrepreneurs’ incentive to flow capital returns back, but also increases domestic illiquid investment, so its overall effect in economic growth remains unclear. Overall, it is clear that the conditions of host country is crucial for home country’s economic growth. When the host country has higher capital share, and higher capital returns, it is more likely for FDI to promote economic growth of home country. This is consistent with the findings of Konings (2004) where he finds positive correlation in growth between host and home country when the host country is a high-income country.

The structure of this paper is as follows. Section II describes the general model as well as the specifications of the model. Section III shows the equilibrium and conditions, followed by conclusion and extension in Section IV.

2. THE MODEL

This two-country model extends both Bencivenga and Smith (1991) and Chang (2009) to address how FDI (foreign direct investment) affects capital outflows and inflows, employment, and the economic growth. These two countries are a home country, the source of FDI, and a host country, the destination of FDI. These two countries are symmetric, except for the populations, and the host population \((N^*)\) is assumed larger than that of the home country \((N)\): 
\[N^* = (1 + n)N, \text{ where } n > 0.\] It is assumed that both countries are under full employment before FDI takes place. Bencivenga and Smith (1991) have shown that an economy could achieve higher growth with financial intermediaries than without. The existence of the financial intermediaries in to serve as middlemen and as portfolio managers. To simplify the model, the access to investment in each country is restricted to financial intermediaries only.
2.1. The General Environment

The economy consists of an infinite sequence of three-period-lived overlapping generations, young (age 1), middle-aged (age 2), and old (age 3), plus an initial old generation and an initial middle-aged generation. Let $t = 0, 1, 2 \ldots$ etc index time. In each period, the economy contains a continuum of agents, and the population growth is assumed to be constant. There are two types of goods in the economy: consumption goods and capital goods. Each of the initial old generation is endowed with $k_0$ units of capital goods at period 0 while each of the initial middle-aged generation is endowed with $k_1$ units of capital goods at period 1. The capital goods are not consumable and depreciate within a period if not used. Both capital goods and labor are the inputs required to produce the consumption goods.

2.2. Agents

All individuals of the same generation are born and remain identical till the realizations and activities at age 2 that divide them into three states: non-entrepreneurs, non-relocating entrepreneurs and relocating entrepreneurs. Let $c_{1,t}$ denote consumption at age 1 at period $t$ for generation $t$, and let the variables $(c_{2,(t+1)})$ and $(c_{3,(t+2)})$ denote consumption at age 2 and age 3 of generation $t$, respectively. The difference between non-entrepreneurs and entrepreneurs is that the former values $(c_{2,(t+1)})$ only while the latter, whether relocating or not, values both $(c_{2,(t+1)})$ and $(c_{3,(t+2)})$ equally. The difference between non-relocating and relocating entrepreneurs is that the latter conducts FDI while the former does not. The details of timing are described as follows [also depicted in Figure 1].

- At age 1: each young agent is endowed with one unit of labor, which is supplied inelastically. Each young agent, however, does not value $c_{1,t}$, so it saves all income for future consumption. This saving decision will not be influenced by the financial structures.

- At age 2: without labor endowment, there is no extra labour income for any middle-aged agent. Two realizations arrive for every middle-aged agent: (i) whether he is an entrepreneur; (ii) if he is, whether he will conduct FDI at age 3. The exogenous probability for an agent to be an entrepreneur at age 3 is $\pi$, and the probability of an entrepreneur to conduct FDI is $\beta$, which could be determined endogenously. Both realizations are private information to the agent.
only. Once the realizations arrive, a non-entrepreneur withdraws all his deposits at age 2 while an entrepreneur will not do so until age 3.

- At age 3: similarly, without labour endowment, there is no extra labour income for any old-aged agent. At this age, an entrepreneur withdraws all its deposits and starts production. While a non-relocating home entrepreneur produces in the home country, a relocating entrepreneurs conducts FDI and produces in the host country. All capital goods owned by an entrepreneur will be invested in production. The amount of capital goods an old-aged agent might receive will be described in the next section. Note that no debt is not allowed. After completing the production in the host country, the relocating home entrepreneurs will decide whether to transfer their host capital returns back to the home country and consume in the home country.

The utility function of a young generation is in the form of:

\[
\begin{align*}
\phi(c_{2,t+1}, c_{3,t+2}; \phi, \sigma) &= - \frac{[c_{2,t+1} + \phi(c_{3,t+2} + \sigma c_{3,t+2}^*)]^{-\gamma}}{\gamma},
\end{align*}
\]

where \( \gamma > -1 \) represents the elasticity of the substitution of intertemporal consumption, \( 0 < \sigma < 1 \), and \( (1/\sigma) \) shows the marginal rate of substitution (MRS) between domestic consumption \( (c) \) and foreign consumption \( (c^*) \). A smaller \( \sigma \) implies that more unites of \( c^* \) are required to replace a unit of \( c \) in order to achieve the same utility level. The value of \( \phi \) is an individual-specific variable, which reflects an individual’s weight at \( c_{3,t+2} \) relative to \( c_{2,t+1} \), and has the probability distribution as follows:

\[
\phi = \begin{cases} 
0, & \text{a non-entrepreneur, with probability } (1 - \pi) \\
1, & \text{an entrepreneur, with probability } \pi 
\end{cases}
\] (2)

2.3. Case One: Without FDI

2.3.1. Production Function and Labor Market

The labor input for production, comes from the young generation at that period. Except for the initial old and initial middle-age generations, the only source of extra capital goods is the returns of the matured illiquid investment. Without direct access to investments, an agent
can only obtain capital goods via financial intermediaries by not withdrawing until age 3 since it takes the illiquid assets 2 periods to mature. Therefore, only a subset of age-3 agents owns capital goods and will use all in production. It is assumed that the capital goods obtained by each old agent are sufficient for production. Thus, no finance is needed, either internal or external finance, and there is no rental market for capital goods.

In this section, we will demonstrate the cases with and without FDI. The analysis will emphasize on the changes in employment, capital returns, and the home growth. The population difference may cause gaps on wage rates and capital returns across countries, which may motivate FDI to take place. In turn, FDI may change capital returns across countries and affect the home growth in the short run and the long run.

The Home Country Let \( k_t \) denote the capital goods owned by a home entrepreneur at date \( t \), and let \( \bar{k}_t \) denote the average capital goods per entrepreneur at date \( t \). In order to produce consumption goods, an entrepreneur would have to hire labours as inputs as well. Let \( L_t \) denote the amount of labour demand by a home entrepreneur. The production function can be written as:

\[
Y_t = \bar{k}_t^\delta k_t^\theta L_t^{1-\theta},
\]

where \( \theta \in [0,1] \), and \( \delta = 1 - \theta \). The value of \( \delta \) represents the scale effect, which implies that the social capital level could have positive externality on individual’s production. Each home entrepreneur will choose \( L_t \) to maximize the profit:

\[
\max_{L_t} \{ \bar{k}_t^\delta k_t^\theta L_t^{1-\theta} - w_t L_t \}.
\]

The first order condition gives labor demand of each entrepreneur:

\[
L_t = k_t \left( \frac{(1 - \theta)\bar{k}_t}{w_t} \right)^{1/\theta}.
\]

By normalizing the population of a home country \( (N = 1) \), the full employment assumption determines the labor supply to each entrepreneur: \( L_t = \frac{1}{\bar{k}_t} \). The labor market clearing condition then determines the home market wage rate:

\[
w_t = (1 - \theta) (\pi)^\theta \bar{k}_t.
\]
By substituting equations (4) and (5) into equation (3), the profit gained from production defines the capital returns of an entrepreneur. The home capital returns earned by a home entrepreneur by investing \( k_t \) units of capital goods is \( \theta (\pi)^{\theta - 1} k_t \), so the rate of the home capital returns is:

\[
ROR = \theta \left( \frac{1}{\pi} \right)^{1-\theta}.
\]

(6)

The Host Country  The host country is symmetric to the home country, except the population is larger than the home country. Since \( N = 1 \), \( N^* = 1 + n \). Let asterisk represent the variables of the host country. The host market wage rate is:

\[
w^*_t = (1 - \theta^*)(\frac{\pi^*}{1 + n})^{\theta^*} (k^*_t),
\]

(7)

and the host capital returns of \( k^*_t \) units of capital goods is: \( \theta^* \left( \frac{1 + n}{\pi^*} \right)^{1-\theta^*} k^*_t \). So the rate of host capital returns is:

\[
ROR^* = \theta^* \left( \frac{1 + n}{\pi^*} \right)^{1-\theta^*}.
\]

(8)

Since the derivations are similar to the home country, the details are described in the appendix.

Proposition 1. Compare equations (5)-(6) to (7)-(8), if \( \theta = \theta^* \), \( k_t = k^*_t \), \( \bar{k}_t = \bar{k}^*_t \), and \( \delta = \delta^* \), the host wage rate is lower than the home wage rate \( w_t < w^*_t \), and the host rate of capital return is higher than that of home capital returns, \( ROR^* > ROR \). Moreover, the larger population gap across countries, the larger the gap of rate of capital returns across countries.

Proof. see the appendix.

Proposition 1 shows that when all conditions between two countries are the same, the population gap alone could create the wage gap and the gap of capital returns across countries. To be more specific, the larger the population of a country, the lower the wage rate and the higher rate of capital returns will be. These gaps may motivate the home country’s entrepreneurs to conduct FDI in the host country, but not the other way around. The next case will be about when FDI takes place.
2.3.2. Investment

There are two types of assets available for investment, liquid and illiquid assets. The returns of the liquid assets are $v > 0$ units of consumption goods at $t + 1$ or $t + 2$ per unit of consumption goods invested at $t$. It takes one period for liquid assets to mature and two periods for illiquid assets to mature. The returns of the illiquid assets are $R > 0$ units of capital goods at $t + 2$ per unit of consumption goods invested at $t$. An early liquidation of illiquid investment is costly. The returns of an early liquidation of the illiquid assets are $\chi$ units of consumption goods at $t + 1$ per unit of consumption goods invested at $t$, and $0 \leq \chi < v$. The access to these two assets is only available to financial intermediaries. Although the type of an individual is not yet known at the of age 1, one can deposit his income into financial intermediaries, which can allocate all deposit between two types of assets based on the the public information $\pi$. An individual’s expected utility by depositing in the financial intermediaries can be shown higher than the expected utility by making investment portfolio himself before knowing his own type[Bencivenga and Smith (1991)]6.

2.3.3. The Home Financial Intermediaries

The banking system is assumed to be a competitive market and the financial intermediaries have zero profit and no reserve requirement. As soon as getting deposits from the young, the financial intermediaries allocate all deposits between liquid or illiquid assets and set the promised repayment to depositors at the time of deposit. Let $z_t \in [0, 1]$ denote the fraction of deposits invested in the liquid assets; and let $q_t \in [0, 1]$ denote the fraction of deposits invested in the illiquid assets. Without the reserve requirement, all deposits will be invested in assets, $z_t + q_t = 1$.

The amount and the type of goods received for repayment depend on the time of withdrawal. Only the late withdrawals (two periods after deposit) could obtain capital goods from the returns of illiquid assets. Let $r_{1t}$ denote the amount of consumption goods repaid to the depositor for one unit of good deposit for the early withdrawal (one period after deposit). Let $r_{2t}$ and $\tilde{r}_{2t}$ denote

---

6 This can be proved easily by thinking the case when an individual have the access to both investment, and there is no financial intermediaries. In order to count for the uncertainly of an individual’s type, one has to invest in both liquid and illiquid assets. After realizing the type, a non-entrepreneur has to liquidate the illiquid assets, and an entrepreneur has to re-invest the liquid assets. Given the pubic information, the probability of one to be an entrepreneur, an individual can receive higher return by investing all its income into financial intermediaries.
the amount of capital goods and consumption goods, respectively, repaid to the depositor for one unit of good deposit for the late withdrawals. In order to meet demand deposit requirement, the financial intermediaries must allocate the portfolio properly, and liquidate sufficient amount of assets to meet the required repayments. Let $\alpha_{1t}$ and $\alpha_{2t}$ denote the fractions of the liquid assets and illiquid assets that are liquidated one period after deposit, respectively. The budget constraints of the financial intermediaries can be written as:

$$ (1 - \pi) r_{1t} = \alpha_{1t} z_t v + \alpha_{2t} q_t \chi, $$

$$ \pi r_{2t} = (1 - \alpha_{2t}) R q_t, $$

$$ \pi \tilde{r}_{2t} = (1 - \alpha_{1t}) z_t v. $$

Equation (9) shows the asset liquidation in response to the early withdrawals by the non-entrepreneurs. Both equations (10) and (11) show the asset liquidation in response to the late withdrawals by the entrepreneurs. While equation (10) is in the form of capital goods, equation (11) is in the form of consumption goods.

The financial intermediaries is assumed to serve as a central planner. They choose the variable set $\{z_t, q_t, \alpha_{1t}, \alpha_{2t}, r_{1t}, r_{2t}, \tilde{r}_{2t}\}$ to maximize the expected utility of a representative agent. Without discrimination in repayment to individuals, the problem faced by financial intermediaries is:

$$ \max_{\{z_t, q_t, \alpha_{1t}, \alpha_{2t}, r_{1t}, r_{2t}, \tilde{r}_{2t}\}} \left\{ - \frac{1 - \pi}{\gamma} (r_{1t} w_t)^{-\gamma} - \frac{\theta}{\gamma} [\theta (\pi) \beta^{\theta-1} r_{2t} w_t + \tilde{r}_{2t} w_t]^{-\gamma} \right\}. $$

Equation (12) shows the expected utility of a representative young agent, who could become either a non-entrepreneur or an entrepreneur at age 2. Caring only about the age-2 consumption, a non-entrepreneur would withdraw and consume all his deposit at age 2, which is $r_{1t} w_t$. An entrepreneur, however, cares both consumption at age 2 and age 3. Since the returns of the matured illiquid assets could provide the capital returns, an entrepreneur would prefer to withdraw at age 3 in order to receive capital goods to produce. The repayment received by an entrepreneur at age 3 is $(r_{2t} + \tilde{r}_{2t}) w_t$, where $r_{2t} w_t$ is paid in capital goods, and will pro-
vide capital returns \( \theta \left( \frac{1}{\pi} \right)^{1-\theta} k_{t+2} \) units of consumption goods for an entrepreneur. The task of the financial intermediaries is to choose the variable set to maximize equation (12), subject to equations (9)-(11).

2.3.4. Equilibrium

**Home Portfolio Decision without FDI** By taking into account both the returns of assets and the repayments to depositors, one equilibrium is as shown in Proposition 2.

**Proposition 2.** (Home Portfolio Decision without FDI) If \( \theta (\pi)^{\theta-1} R \geq \nu > 0 \), then \( \alpha_{1t} = 1, \alpha_{2t} = 0, \tilde{r}_{2t} = 0 \). This gives equilibrium \( q_t = \hat{q}_t = \frac{\Phi}{1+\Phi}, z_t = 1 - \hat{q}_t = \frac{1}{1+\Phi}, \hat{r}_{1t} = \left( \frac{\nu}{1-\pi} \right) (1 - \hat{q}_t), \) and \( \hat{r}_{2t} = \left( \frac{R}{\pi} \right) \hat{q}_t \), where \( \Phi = \left( \frac{\nu}{1-\pi} \right) \left( \theta (\pi)^{\theta-1} \right) \).  

**Proof.** see the appendix.

Proposition 2 shows that if the rate of capital returns are larger than the rate of returns of the liquid assets, the financial intermediaries would choose to liquidate all liquid assets, \( \alpha_{1t} = 1 \), and not to liquidate any illiquid assets prematurely, \( \alpha_{2t} = 0 \), in response to the early withdrawals. So for the late withdrawals, the repayment will be paid in capital goods only, \( \tilde{r}_{2t} = 0 \). This helps pin down the equilibrium investment allocation \( q_t \) and \( z_t \), and determines the repayments to depositors at different periods \( r_{1t} \) and \( r_{2t} \).

Due to the equal weights on both \( c_2 \) and \( c_3 \), if the capital returns gained by the late withdrawals are higher than the repayment received at the early withdrawals, \( \left( \frac{\theta (\pi)^{\theta-1} R}{\pi} \right) \hat{q}_t > \left( \frac{\nu}{1-\pi} \right) (1 - \hat{q}_t) \), the entrepreneurs would prefer not to withdraw until age 3.

**Home Growth Rate** There are two ways to calculate the home economic growth rate. One is to calculate the consumption goods growth rate, and the other is to calculate the capital goods accumulation rate. In this paper, we adopt the capital goods accumulation rate since one source of extra consumption goods to the economy is through production, which requires capital goods obtained from the matured illiquid assets investment. So the growth rate of the capital goods in two-period time is \( k_{t+2} = \tilde{r}_{2t} w_t = \left( \frac{R \hat{q}_t}{\pi} \right) w_t \). By plugging equation (5) into and \( k_{t+2} \) and
dividing by \( k_t \), the home economic growth rate can be defined as:

\[
\mu \equiv R(1 - \theta) \left( \frac{1}{\pi} \right)^{1-\theta} \hat{q}_t = R(1 - \theta) \left( \frac{1}{\pi} \right)^{1-\theta} \left( \frac{\Phi}{1 + \Phi} \right), \tag{13}
\]

which has the properties: \( (\partial \mu / \partial \hat{q}_t) > 0 \).

**Proposition 3. (Effects on Economic Growth \( \mu \))** If direct effects dominate, \( \pi \) affects \( \mu \) negatively, and \( (R, \upsilon) \) affect \( \mu \) positively.

**Proof.** see the appendix.

Both \( R \) and \( \pi \) have direct effects on \( \mu \) and indirect effects on \( \mu \) through \( \hat{q}_t \) while \( \upsilon \) has positive indirect effect on \( \mu \) only. The direct and indirect effects of \( R \) on \( \mu \) are opposite forces, and so do the direct and indirect effects of \( \pi \) on \( \mu \). An increase in the value of \( R \) helps accumulate the capital goods, helps the home entrepreneurs to produce more consumption goods, and promotes home growth \( \mu \). An increase on \( \pi \), however, would decrease the home capital returns and damage \( \mu \). Indirectly, an increase in \( R \) or a decrease in \( \pi \) would lower \( \hat{q}_t \) and damage \( \mu \). As indicated in Proposition 3, when direct effects dominate, \( R \) affects \( \mu \) positively while \( \pi \) affects \( \mu \) negatively. The return of liquid assets \( \upsilon \) will increase investment on illiquid assets \( \hat{q}_t \), and promote \( \mu \).

2.4. Case Two: With FDI in the Short-Run (SR)

2.4.1. Production Function and Labor Market

*The Home Country*  The gap on the capital returns across countries would attract FDI from the home country to the host country. In fact, despite the difference in the rate of capital returns, there are home entrepreneurs remaining in the home country without conducting FDI. Let \( \beta \ (0 < \beta < 1) \) denote the proportion of the home entrepreneurs conducting FDI. In this model, the host country’s entrepreneurs have no incentive to conduct FDI, so \( \beta^* = 0 \).

Because the relocating home entrepreneurs have carried capital goods with them to conduct FDI in the host country, the average capital stock per home entrepreneur after FDI becomes \((1 - \pi \beta)\bar{k}_t\). This changes the production function of a home entrepreneur to: \( Y_t' = (1 - \pi \beta)\bar{k}_t^\delta L_t^\theta L_t'^{1-\theta} \), where \( L_t' \) represents the new labour demand of a home entrepreneur. Then the
profit maximization problem faced by a non-relocating home entrepreneur becomes:

$$\text{Max}_{L_t} \left\{ (1 - \pi \beta) k_t^e k_t^Q L_t^{1-\theta} - w_t L_t \right\}. \quad (14)$$

The first order condition gives:

$$L_t' = k_t \left[ (1 - \theta)(1 - \pi \beta) k_t^e \right]^{1/\theta}. \quad (15)$$

In the short run, the wage rate, however, may not adjust instantaneously in response to the change of labour demand. Assume that the home wage rate remains the same in the SR after FDI takes place. This assumption implies that the labour demand per home entrepreneur decreases after FDI takes place, $L'_t < L_t$, due to the loss of the capital goods to the host country. The decrease in labour demand would cause extra home unemployment in addition to the unemployment resulted from home relocating entrepreneurs. These newly unemployed labours may not find jobs for a short period of time.

**Proposition 4.** (Home Unemployment and the change on home ROR) In the SR, when the home wage rate cannot respond instantaneously to the outward FDI, the decrease on the labour demand per home entrepreneur and the relocating home entrepreneurs will both contribute to an increase on the home unemployment. This gives the home unemployment $UE = 1 - (1 - \beta)(1 - \pi \beta)^{1/\theta}$. The home capital returns become $\left[ \theta \left( \frac{1}{\pi} \right)^{1-\theta} (1 - \pi \beta)^{1/\theta} k_t \right]$, and the rate of home capital returns is:

$$\text{ROR}' = \theta \left( \frac{1}{\pi} \right)^{1-\theta} (1 - \pi \beta)^{1/\theta}, \quad (16)$$

which is lower than the rate of home capital returns without FDI, $\text{ROR}' < \text{ROR}$ [equation (6)].

**Proof.** see the appendix.

The proposition shows that FDI has caused the rate of home capital returns to decrease. The higher $\pi$ and $\beta$, the lower rate of home capital returns after FDI, and the larger the gap of the rate of home capital returns before and after FDI, $\theta \left( \frac{1}{\pi} \right)^{1-\theta} [1 - (1 - \pi \beta)^{1/\theta}]$. Therefore, to reduce the gap of the home capital returns before and after FDI, it is important to decrease the values of $\pi$ and $\beta$. 

13
The Host Country After FDI takes place, the \( k_t \) units of capital goods carried by each relocating home entrepreneur to the host country would contribute to the host capital stock. The average capital stock per entrepreneur located in host country after FDI becomes 

\[
\bar{k}_t^* = \bar{k}_t + \pi \beta \bar{k}_t.
\]

By assuming \( \bar{k}_t^* = \bar{k}_t \), the profit maximizing problem faced by an entrepreneur locating in the host country is:

\[
\max_{L_t^*} \left\{(1 + \pi \beta)(\bar{k}_t^*)^{\delta^*} (L_t^*)^{1-\theta^*} - w_t^* L_t^*\right\}.
\]

The labour demand per entrepreneur in the host country becomes:

\[
L_t^* = k_t^* \left( (1 - \theta^*)(1 + \pi \beta)(\bar{k}_t^*)^{\delta^*} \right)^{1/\theta^*}.
\]

With the inward FDI and an increase on labour demand, the host country remains full employment. However, the constant population implies that the labour supply to each entrepreneur in the host country will be reduced to: 

\[
L_t^* = \frac{1 + n}{\pi^* + \pi \beta},
\]

and that the host wage rate will increase to:

\[
w_t^* = (1 - \theta^*)(1 + \pi \beta) \left( \frac{\pi^* + \pi \beta}{\pi^* + \pi \beta} \right)^{\theta^*} (k_t^*) > w_t^*.
\]

The host capital returns by investing \( k_t^* \) units of capital goods are:

\[
\theta^* (1 + \pi \beta) \left( \frac{\pi^* + \pi \beta}{\pi^* + \pi \beta} \right)^{\theta^* - 1} k_t^*,
\]

so the rate of the host capital returns with inward FDI is:

\[
ROR_t^* = \theta^* (1 + \pi \beta) \left( \frac{1 + n}{\pi^* + \pi \beta} \right)^{1-\theta^*}.
\]

**Proposition 5.** If the condition, 

\[
\frac{1 + \pi \beta}{(\pi^* + \pi \beta)^{1-\theta^*}} < \frac{1}{(\pi^*)^{1-\theta^*}},
\]

holds, the rate of the host capital returns with inward FDI is lower than that without FDI, \( ROR_t^* < ROR^* \). This is the case that the unidirectional FDI from the home to the host country may stop when the gap of the capital returns across countries is eliminated.

**Proof.** See the appendix.

If the above condition does not hold, the unidirectional FDI could enlarge the gap of the capital returns across countries and stimulate even more outward FDI from the home country till all home entrepreneurs to relocate in the host country, \( \beta = 1 \).

If the condition in the above proposition holds, then FDI may not increase the rate of the host capital return \( ROR_t^* \). The main reason is the inward FDI shares the already full employed host labour supply and causes the labour supply to each entrepreneur to decrease.

When the
effect of the labour supply on \( ROR^* \) is sufficiently strong, \( ROR^* \) may be lower than \( ROR^* \) and the gap of the capital returns across countries may be narrower. This means that the continuing inward FDI may cause the unidirectional FDI from the home to the host country to discontinue at the time when the gap of the capital returns across countries is eliminated. This is the case we are interested.

2.4.2. Capital Inflow

This section is to examine the possibility that the relocating home entrepreneurs flow their host capital returns back to the home country and consume. It is assumed that a relocating home entrepreneur has choices on what to do with his host capital returns. One is to flow the host capital returns back to the home country by paying fixed transaction sunk costs \((\tau)\) per transfer, and to consume in the home country. Another is to remain in the host country and to consume in the host country. Given the imperfect substitute of the home and the host consumption, a relocating home entrepreneur will transfer its host capital returns back if the following condition holds.

**Condition 1:**

\[
\left\{ \theta^* (1 + \pi \beta) \left( \frac{\pi^* + \pi \beta}{1 + \pi^*} \right)^{\theta^* - 1} k^* - \tau \right\} < \sigma \left\{ \theta^* (1 + \pi \beta) \left( \frac{\pi^* + \pi \beta}{1 + \pi^*} \right)^{\theta^* - 1} k^* \right\}
\]

The relocating home entrepreneur will not flow his host capital returns back to the home country and will stay in the host country and consume.

By rearranging the above condition, one can obtain: 

\[
k^* \leq \left( \frac{\tau}{1 - \sigma} \right) \left( \frac{1}{ROR^*} \right) \equiv x,
\]

where \( x \) is defined as the probability that a relocating home entrepreneur would keep his host capital returns in the host country and consume there. One implicit assumption is that an entrepreneur will consume in the country to where his capital returns flow. If the entrepreneur flows capital returns back to the home country, then he will have the home consumption only. If he does not, then he will have the host consumption only.

The value of \( x \) is an increasing function of \( \tau \) or on \( \sigma \), but a decreasing function of \( ROR^* \). That means that a decrease on \( \tau \) or on \( \sigma \) (a increase on MRS between \( c \) and \( c^* \)), and an increase on \( ROR^* \) would decrease \( x \), and help the relocating home entrepreneurs flow the host capital returns back to the home country. The effects of \( \tau \) and \( \sigma \) are obvious. An increase on \( ROR^* \) would help the host capital returns earned by the relocating home entrepreneurs pass the threshold to flow.
the host capitals returns back to the home country and help the relocating home entrepreneurs to earn more utility. Therefore, the incentive for the relocating home entrepreneurs to flow their host capital returns back would increase. In order to encourage the relocating home entrepreneurs to flow their host capital returns back to the home country, it is important that the value of $\tau$ is low, and the value of $ROR^*$ in the host country, and the value of MRS is high (a low $\sigma$) for the home entrepreneur.

2.4.3. The Home Financial Intermediaries

The outward FDI from the home country would two things on the problem faced by the home financial intermediaries. First, only employed workers receive wage income and can deposit. The amount of employed workers is $(1 - \beta) (1 - \pi \beta)^{1/\theta}$, which includes entrepreneurs. Second, the home financial intermediaries will need to count for the capital returns of the relocated home entrepreneurs [the third term in equation (20)]. So the objective function faced by the financial intermediaries becomes:

$$\max\left\{\left[-\frac{(1-\beta)(1-\pi\beta)^{1/\theta}-\pi}{\gamma} r_{1t} w_t\right]^{-\gamma} \right\},$$

and the first budget constraint [equation (9)] is revised to:

$$\left(1 - \beta\right) (1 - \pi \beta)^{1/\theta} - \pi r_{1t} = \alpha_{1t} z_t v + \alpha_{2t} q_t \chi$$

In this case, the outward FDI reduces the capital returns earned by a non-relocating home entrepreneur to $\theta \pi^{\theta - 1} (1 - \pi \beta)^{1/\theta} k_{t+2}$ units of consumption goods, and increases the host capital returns earned by a relocating home entrepreneur up to $\{\theta^* (1 + \pi \beta) \left(\frac{\pi + \pi \beta}{1 + \pi \beta}\right)^{\theta^* - 1} k^*_{t+2}\}$ units of consumption goods. With probability $(1 - x)$, a relocating home entrepreneur would transfer his host capital returns back to the home country to consume; otherwise, he will stay and consume.
his host capital returns in the host country.

2.4.4. Equilibrium

The gap of the rates of capital returns across countries would motivate the home entrepreneurs to conduct FDI in the host country. The larger the gap of the capital returns across countries, the larger the amount of FDI and the higher the relocation rate will be from the home to the host country. If FDI shrinks the gap of the capital returns across countries [as shown in Proposition 3], the equilibrium relocation rate \(0 < \beta < 1\) of the home entrepreneurs can be determined when this gap is eliminated, \(ROR'[equation (16)]= ROR''[equation (19)]\):

\[
\left[\frac{\pi^* + \pi \hat{\beta}}{1 + n}\right]^{1-\theta'} \frac{1}{\pi^{1-\theta}} = \frac{\theta^* (1 + \pi \hat{\beta})}{\theta (1 - \pi \hat{\beta})^{1/\theta}},
\]

where \(\hat{\beta}\) represents the equilibrium relocation rate of the home entrepreneurs to the host country, and is an increasing function of \(\pi\) and \(n\), and a decreasing function of \(\pi^*\) if \((1 - \theta) \frac{(1 - \pi \hat{\beta})^{1/\theta}}{(\pi^* + \pi \hat{\beta})} > [(1 + n)\pi]^{1-\theta} \left[1 + \frac{1}{2} \left(\frac{1 + \pi \hat{\beta}}{1 - \pi \hat{\beta}}\right)\right].\) While a decrease on \(\pi\) would increase \(ROR'\), a decrease on \(n\) or an increase on \(\pi^*\) would keep \(ROR'\) unchanged, but all three (a decrease in \(\pi\) or \(n\), and an increase on \(\pi^*\)) would reduce \(ROR''\) and shrink the gap of the capital returns across countries. As a consequence, less FDI will be conducted and the equilibrium \(\hat{\beta}\) can be achieved when the gap is eliminated.

**Portfolio Decisions of the Home Financial Intermediaries** By taking the first order conditions of equation (20), subject to equations (10), (11), and (21), the following proposition can be obtained.

**Proposition 6. (Short-Run Portfolio Decision with FDI)** If \(\theta \pi^{d-1}(1 - \pi \beta)R \geq v > 0\) and

\[
\{\theta^* \left(\frac{\pi^* + \pi \hat{\beta}}{1 + n}\right)^{\theta-1} (1 + \pi \beta)R \geq v > 0, \ then \alpha'_{t1} = 1, \alpha'_{t2} = 0, r'_{t2} = 0. \ This \ gives \ one \ equilibrium:
\]

\[
q_t = q'_t = \frac{\Phi'}{1+\Phi'}, \ z'_t = 1 - q'_t = \frac{1}{1+\Phi'}, \ r'_{t1} = \left(\frac{v}{1-\pi}\right) (1 - q'_t), \ and \ r'_{t2} = \left(\frac{R}{\pi}\right) q'_t, \ where
\]

\[
\Phi' \equiv \left(\frac{\pi}{(1 - \beta) (1 - \pi \beta)^{\gamma} - \pi}\right)^{1/(1+\gamma)} \left(\frac{v}{R}\right)^{\gamma/(1+\gamma)} \left\{\frac{(1 - \beta)(ROR')^{-\gamma}}{(EUR) (ROR'')} + \beta \left((EUR) (ROR'') - \frac{(1-\beta)\tau}{q_{t2} R'} \right)^{\gamma-1}(EUR) (ROR'') \right\}^{1/(1+\gamma)}
\]
and $EUR \equiv (1 - x + x\sigma)$. The properties of $\Phi'$ are shown in the appendix.

Proof. see the appendix.

It can be shown that the effect of $x$ on $\Phi'$ via $EUR$ is smaller than the effect associated with $\tau$, so $\Phi'$ is a decreasing function in $x$. That implies that if the host capital returns earned by the relocating home entrepreneurs are less likely to flow back to the home country, the home financial intermediaries will reduce their investment in illiquid assets $\Phi'$ in response. The factors that might result in an increase in $x$, and hence, a reduction in $\Phi'$, were discussed above under Condition 1. Both $ROR$ and $ROR^*$ affect $\Phi'$ negatively. Because an increase in $ROR$ or $ROR^*$ indicates that the home financial intermediaries don’t have to invest as much on illiquid assets as before and can still help the home entrepreneurs, both relocating and non-relocating, obtain the same rate of the capital returns. An increase on outward FDI ($\beta$) from the home country would stimulate $\Phi'$ to increase in order to raise the home capital returns as well as the host capital returns earned by the relocating home entrepreneurs to pass the threshold, so that they have higher incentive to flow their host capital returns back to the home country.

Similar to the previous case, the conditions for the home entrepreneurs not to withdraw until age 3 are: 

$$
\left( \frac{\theta \pi^{-1} (1 - \pi \beta) R}{\pi} \right) \tilde{q}_t^{\pi} > \left( \frac{\nu}{1 - \pi} \right) (1 - \tilde{q}_t) $$

for the non-relocating home entrepreneurs, and

$$
\left( \frac{\theta^* (1 + \nu) 1 - \nu (1 + \pi \beta) R}{\pi (\pi^* + \pi \beta)} \right) \tilde{q}_t^{\pi^*} > \left( \frac{\nu}{1 - \pi} \right) (1 - \tilde{q}_t) $$

for the relocating home entrepreneurs.

Home Growth Rate Since the asset portfolios held by the home financial intermediaries in the case of FDI may differ from the cases without FDI, despite the fixed home wage rate in the short run, the home economic growth rate is revised according:

$$
\mu' \equiv R(1 - \theta) \left( \frac{1}{\pi} \right)^{1 - \theta} \left( \frac{\Phi'}{1 + \Phi'} \right). 
$$

Proposition 7. (Effects on Economic Growth $\mu'$) Because of the inflexible wage rate in the SR, the direct effects of the variable ($\pi, R, \nu$) on $\mu'$ remain the same as the case without FDI in terms of directions. However, since $\Phi'$ is decreasing in $x$, and so is $\mu'$, any policies that could promote the host capital returns earned by the relocating home entrepreneurs to flow back to the
home country could promote the home growth rate. As mentioned above, through $x$, the economic conditions of the FDI’s host country ($\tau$, $ROR^*$) could matter for the home growth rate.

Proof. see the appendix.

In addition to the existing direct effects of $(\pi, R, \nu)$, Proposition 7 adds the effects of the variables of the FDI’s host countries ($\tau, ROR^*$) on $\mu$ via $x$, and then via $\Phi'$. Recall that $x$ is increasing in $\tau$ and $\sigma$, but decreasing in $ROR^*$. The lower $\tau$ or a higher $ROR^*$ in the host country, the lower $x$ will be (more likely for the relocating home entrepreneurs to flow their host capital returns back to the home country). As a result, the home financial intermediaries would invest more on illiquid assets and promote the home growth rate. Therefore, the economic conditions of the FDI’s host countries may matter for the home growth. An increase in MRS (a decrease in $\sigma$) implies that it takes more units of host consumption to replace one unit of home consumption, so the relocating home entrepreneurs are more likely to flow back their host capital returns to the home country and have the home consumption. This will increase $\Phi'$, and hence, increases $\mu'$.

2.5. Case 3: With FDI in the Long-Run (LR)

In the LR, the home wage rate is fully flexible. If we consider the case that the home country returns to full employment in the presence of outwards FDI in the LR, the labour supply to each non-relocating home entrepreneur changes to $(L_i)^\ast = \frac{1}{\pi(1-\beta)}$. That means that the LR home wage rate changes to $(w_i)^\ast = (1-\theta)(1-\pi\beta^\ast)(\pi(1-\beta^\ast))^\theta k_i$; the LR rate of home capital returns changes to $ROR^\ast = \theta\pi^\ast(1-\beta^\ast)^{\theta-1}(1-\pi\beta^\ast)$; and the LR rate of host capital returns changes to $ROR^\ast = \theta\pi^\ast(1+\pi\beta^\ast)^{\theta-1}$, where $\beta^\ast$ is re-determined when $ROR^\ast = ROR^\ast$, and $x^\ast$ is redefined accordingly $x^\ast = \left(\frac{\tau}{1-\sigma}\right) \left[\frac{1}{\pi(1-\beta^\ast)}\right]$.

Note that $\beta^\ast$ has an extra positive effect on $ROR^\ast$ in the LR, in addition to its negative effect due to the loss of capital goods from outward FDI. That is because the flexible home wage rate allows the non-relocating home entrepreneurs to pay less labour cost and share more labour supply. This would increase $ROR^\ast$. If $(1-\theta)(1-\pi\beta^\ast) > \theta\pi(1-\beta^\ast)$, then the LR rate of home capital returns is greater than the SR rate, $ROR^\ast > ROR^\ast$. This implies that in the LR, the gap of the capital returns across countries is narrower, and that the LR relocation rate $\beta^\ast$ could
be smaller than the SR rate $\hat{\beta}$.

This LR home capital returns and relocation rate will then affect the problem faced by the home financial intermediaries and the home growth rate. Similar to previous cases, one equilibrium we could obtain is as follows. All details of derivation are described in the appendix.

**Proposition 8.** (LR Home Portfolio Decision and Growth Rate) If $\theta \left[ \frac{1}{\pi(1-\beta)} \right]^{1-\theta} (1 - \pi \beta) R \geq v > 0$ and $\{\theta^* (\pi^{\alpha} + \pi^{\beta})^{\theta^{-1}} (1 + \pi \beta) R \} \geq v > 0$, then $(\alpha_1)^n = 1$, $(\alpha_2)^n = 0$, $(\tilde{\tau}_{2t})^n = 0$. This gives equilibrium $q_t = (\tilde{q}_t)^n = \frac{\Phi^n}{1 + \Phi^n}$, and $(\tilde{z}_t)^n = 1 - (\tilde{q}_t)^n = \frac{1}{1 + \Phi^n}$ as well as $(r_{1t})^n = \left( \frac{v}{\pi} \right) \left( \frac{1}{1 + \Phi^n} \right)$ and $(r_{2t})^n = \left( \frac{v}{\pi} \right) \left( \frac{1}{1 + \Phi^n} \right)$, where $\Phi^n$ is determined by the following equilibrium condition.

$$\Phi^n = \left( \frac{\pi}{1 - \pi} \right) \left( \frac{v}{R} \right)^{\gamma/(1+\gamma)} \left\{ (1 - \beta^n) (ROR^n)^{-\gamma} + \beta^n \left\{ (EUR^n) (ROR^n*) - \frac{(1 - x^n) \pi \gamma}{(v/P) \gamma \pi R} \right\}^{-\gamma -1} (EUR^n) (ROR^n*) \right\}^{1/(1+\gamma)},$$

where $EUR^n \equiv 1 - x^n + x^n \sigma$. The LR home economic growth rate becomes:

$$\mu^n \equiv R (1 - \theta) \pi^{\theta - 1} [1 - \pi (\beta)^n] [1 - (\beta)^n] \left( \frac{\Phi^n}{1 + \Phi^n} \right).$$

**Proof.** See the appendix. ■

3. CASES COMPARISON AND POLICY IMPLICATIONS

In the short run with fixed home wage rate, by comparing Proposition 2 and Proposition 6, FDI may promote the home growth rate ($\mu'$) if the investment on the illiquid assets in the presence of FDI (\Phi') is higher than that in the absence of FDI (\Phi). Note that the home unemployment caused by the outward FDI may stimulate the home financial intermediaries to increase their investment on illiquid assets. This increase in \Phi' would promote the home growth rate (\mu').

According to Proposition 7, policies that could increase the probability of flowing back the capital returns by relocated entrepreneurs (1 - x) would increase (\Phi') and promote the home country's economic growth (\mu'). To increase (1 - x), it is important that the host country has lower transaction cost of capital flows ($\tau$) and that the domestic consumption and foreign
consumption are more imperfect substitute (a higher MRS, $1/\sigma$). Additionally, the fraction of entrepreneurs who conduct FDI ($\beta$) may matter for $\Phi'$. A higher $\beta$, one one hand, reduces $RORt$ and stimulate financial intermediaries to increase $\Phi'$, and promote $\mu'$. On the other hand, a higher $\beta$ increases $RORt^*$, discourages $\Phi'$ and harms $\mu'$. The ambiguous effect of $\beta$ on $\mu'$ implies that it is important for the policy maker to evaluate whether the effect of $\beta$ via $ROR'$ or $ROR^*$ is stronger before conducting policies to affect $\beta$.

The difference between long run and short run when FDI take place is wage rate adjustment. In the short run, wage rate does not respond instantaneously to the decrease in labour demand due to relocation of entrepreneurs. In the long run, wage rate is reset in the new equilibrium, where labour demand equals labour supply. Comparing the economic growth rate of the home country in the short run [equation (23)] and long run [equation (37)], one may find that if the fraction of relocated entrepreneurs ($\beta$) and the fraction of illiquid investment ($\Phi'$) remain the same in the short run and in the long run, the economic growth rate of the home country is smaller in the long run ($\mu''$) than in the short run ($\mu'$). One of the reasons is the lower wage rate level caused by lower labour demand. Compare equations (22) and (38), one may find that the effect of $\pi$ is smaller on $\beta''$ (long run) than on $\beta$ (short run). In the long run, while the home country adjust back to full employment level, $\Phi''$ decreases as a consequence, and harms the home country’s growth ($\mu''$). To restore the home country’s growth rate ($\mu''$) in the long run, the policies mentioned above, encouraging entrepreneurs to conduct FDI in the host country with a lower $\tau$ or a higher MRS, would work here. Additionally, if the direct effects dominate, another possible policy to promote $\mu''$ could be to discourage people to become entrepreneurs (a lower $\pi$).

4. CONCLUSION & EXTENSIONS

The purpose of this exercise is to illustrate how outward FDI affect home country’s growth when counting for the possible future capital inflow from relocated capital returns. By allowing for endogenous decisions of entrepreneurs, whether to conduct FDI and whether to flow capital returns to the home country, the results are quite interesting. First, as long as the probability of conducting FDI is positive, a higher proportion of entrepreneurs in the economy indicates
more capital taken away when FDI take place. This may harm economic growth of the home country. If FDI are conducted in the host countries where the transactions costs for capital flows are lower or where the MRS between two countries are higher, the probability for the capital returns to flow back to the home country will increase, and promote economic growth of the home country. The policies to encourage or to discourage FDI in the home countries requires evaluations on whether the effects via the capital returns in the home or the host country is stronger in order to determine whether it might promote the home country’s economic growth.

The unemployment for the home country caused by FDI may not harm home country’s growth in the short run, but the wage rate adjustment in response to the unemployment may harm the home country’s growth in the long run. This is consistent with the findings of Konings (2004), in which he finds the unemployment in the home country caused by FDI and a positive relationship of FDI in growth when the host country is a high-income country with more liberal capital account and lower transaction costs.

This framework allows us to have a simplified model to begin with in analyzing the effects of FDI on economic growth. It will be very interesting to analyze the effects of FDI on growth in different aspects and angles by extending current framework. For example, it could be interesting to allow entrepreneurs to borrow from the financial intermediaries or to allow individuals to have the access to investment. This may affect the capital returns across countries and individuals’ incentive to conduct FDI, and hence economic growth rates.

APPENDIX A

PROOF OF PROPOSITIONS AND THE DETAILS OF DERIVATIONS

The problem of the host country’s entrepreneurs and the derivation of wage rates in the case without FDI:

The host country is symmetric to the home country, except a larger population than the home country. The normalization of the home country’s population gives $N^* = 1 + n$. Each host entrepreneur employs $L^*_t$ units of labor to produce. Given the production function, $Y^*_t = (\bar{k}^*_t)^{\theta^*} (k^*_t)^{\theta^*} (L^*_t)^{1-\theta^*}$, where $\theta^* \in [0, 1]$, and $\delta^* = 1 - \theta^*$, each host entrepreneur chooses the
amount of labor units to maximize profit:

$$\max_{L_t^*} \left\{ (k_t^*)^{\delta^*} (k_t^*)^{\theta^*} (L_t^*)^{1-\theta^*} - w_t^* L_t^* \right\}. \tag{A1}$$

Similar to the derivation in the home country above, when the labor demand of each entrepreneur $L_t^* = k_t^* \left[ \frac{(1-\theta^*)(k_t^*)^{\theta^*}}{w_t^*} \right]^{1/\theta^*} \left( \frac{\pi}{1+\pi} \right)^{\theta^*} \left( k_t^* \right)$ equals to the labor supply to each entrepreneur under full employment, $L_t^* = \frac{1+n}{\pi}$, the labor market clearing condition determines the market wage rate of the host country:

$$w_t^* = (1-\theta^*) \left( \frac{\pi}{1+n} \right)^{\theta^*} \left( k_t^* \right). \tag{A2}$$

**Proof of Proposition 1:**

Since $n > 0$, when $\pi = \pi^*$, $ROR^* > ROR$. ■

**Proof of Proposition 2:**

By taking the first order condition of equation (12) with respect to , subject to equations (9)-(11), one can obtain $\Phi = \left( \frac{\pi}{\pi^*} \right) \left( \frac{n}{\pi} \right) \left[ \theta (\pi)^{\theta-1} \right]^{1+\gamma^*}$.

**Proof of Proposition 3:**

By taking partial differentiation of equation (13) with respect to variables and allowing the direct effect to dominate, one can find $(\partial \mu / \partial \pi) < 0$, $(\partial \mu / \partial R) > 0$, $(\partial \mu / \partial \nu) > 0$.

**Proof of Proposition 4:**

After FDI, equation (15) shows that total labour demand in the home country becomes

$$\pi (1-\beta) L_t' = (1-\beta) \left( (1-\pi) \left( 1-\pi^* \right) \right)^{1/\theta^*}. \tag{10}$$

So the home unemployment is $UE = 1-(1-\beta) \left( (1-\pi) \left( 1-\pi^* \right) \right)^{1/\theta^*}$.

Since both $0 < \pi, \beta < 1$, $ROR' < ROR$. ■

Moreover, $ROR'$ has the properties: $(\partial ROR' / \partial \pi) < 0$, $(\partial ROR' / \partial \beta) < 0$

**Proof of Proposition 5:**

By comparing equations (19) to equation (8), if $\frac{1+\pi \beta}{(\pi^* + \pi \beta)^{1/\gamma}} < \frac{1}{(\pi^*)^{1/\gamma}}$ holds, $ROR'^* < ROR^*$. ■

Moreover, $ROR'^*$ has properties: $(\partial ROR'^* / \partial n) > 0$, $(\partial ROR'^* / \partial \pi) < 0$, $(\partial ROR'^* / \partial \pi^*) < 0$, and $(\partial ROR'^* / \partial \beta) < 0$, if $(1-\theta^*) \left( \frac{1+\pi \beta}{(\pi^* + \pi \beta)} \right) > 1$.

**Condition 1:**

Followed by Condition 1, where the properties of $x$ are: $(\partial x / \partial \tau) > 0$, $(\partial x / \partial \sigma) > 0$, and
\( \frac{\partial x}{\partial ROR^*} < 0. \)

**Proof of Proposition 6:**

By taking the first order condition of equation (20) with respect to \( \dot{q}_t \), subject to equations (10), (11), and (21), we can get the following properties for \( \Phi' : (\partial \Phi' / \partial ROR^*) < 0, (\partial \Phi' / \partial EUR) < 0, (\partial \Phi' / \partial x) < 0. \) The direct effects of variables on \( \Phi' \) are \( (\partial \Phi' / \partial \pi) > 0, (\partial \Phi' / \partial \beta) > 0, (\partial \Phi' / \partial v) > 0. \)

\( EUR \equiv 1 - x(1 - \sigma) \) has the following properties: \( (\partial EUR / \partial x) < 0, (\partial EUR / \partial \sigma) > 0 \)

**Proof of Proposition 7:**

**Derivations for Case 3: LR effects of FDI on the home country**

In the long run, the labour supply to each non-relocated home entrepreneur becomes \( (L_t)^n = \frac{1}{\pi(1 - \beta^n)} \), which equals to labour demand of each entrepreneur: \( L_t^\prime = k_t \left[ \frac{(1 - \theta)(1 - \pi_\beta^n)k_t^2}{w_t} \right]^{1/\theta}. \) The labour market clearing condition gives the long-run home wage rate: \( w_t^\prime = (1 - \theta)(1 - \pi_\beta^n)[\pi(1 - \beta^n)]^\theta k_t, \) and the long-run rate of home capital returns \( ROR^* = \theta[\pi(1 - \beta^n)]^\theta - 1(1 - \pi_\beta^n). \) This equilibrium equation for \( \beta^* \) is determined when \( ROR^* = ROR^* : \)

\[
\left( \frac{1 + \pi_\beta^n}{1 - \pi_\beta^n} \right) \theta = \left( \frac{\pi^n + \pi_\beta^n}{(1 - \beta^n)(1 + n)} \right)^{1 - \theta},
\]

where the properties of \( \beta^* \) are: \( \left( \frac{\partial \beta^*}{\partial \pi} \right) > 0, \left( \frac{\partial \beta^*}{\partial \pi^*} \right) < 0, \) if \( \left| \pi(1 - \beta)[1 - \theta \pi(1 - \beta) - (1 + \pi_\beta)(1 - \theta)] \right| \geq \left| (\pi^n + \pi_\beta)^{-\theta} \pi^n + \pi_\beta - (1 - \pi_\beta)(1 - \theta) \right|. \)

By revising condition 1 based on the LR home capital returns, the probability of the host capital returned earned by the relocated home entrepreneurs to stay in the host country becomes:

\[
k_t^* \leq \left( \frac{\pi}{1 - \sigma} \right) \left[ \frac{(\pi^n + \pi_\beta^n)^{1 - \theta}}{(\theta(1 + n) + \pi_\beta^n)} \right] \equiv x^* .
\]

The financial intermediaries choose the variable set to maximize the expected utility of a representative agent. The problem faced by financial intermediaries becomes:

\[
\begin{align*}
& \text{Max}_{\{z_t, q_t, \alpha_t, \alpha_t, (r_{1t})^n, (r_{2t})^n, (\bar{r}_{2t})^n\}} \left\{ \begin{array}{c}
- \left( \frac{1 - \pi}{\gamma} \right) \left( (r_{1t})^n (w_t)^n \right)^{-\gamma} \\
- \pi^{(1 - \beta^n)} \left[ \theta[\pi(1 - \beta^n)]^{\theta - 1}(1 - \pi_\beta^n)(r_{2t})^n (w_t)^n + (\bar{r}_{2t})^n (w_t)^n \right]^{-\gamma} \\
\frac{\pi^{\beta^n}}{\gamma} \left( (\bar{r}_{2t})^n (w_t)^n + (1 - x)[\theta^*(\pi^{\beta^n})^{\theta - 1}(1 + \pi_\beta^n)(r_{2t})^n (w_t)^n - \tau] \\
+ x^n \sigma [\theta^*(\pi^{\beta^n})^{\theta - 1}(1 + \pi_\beta^n)(r_{2t})^n (w_t)^n]^{-\gamma} \right) \end{array} \right. \\
\end{align*}
\]

(A4)

24
**Proof of Proposition 7**

By taking the first order condition of equation (37) with respect to \((\hat{q}_t)\), subject to the budget constraints [equations (9)-(11)], the equilibrium \((\hat{q}_t^{''})\) can be solved as shown in Proposition 7.

That means that the following condition will hold for the home entrepreneurs not to withdraw until age 3.

The non-relocated home entrepreneurs will always withdraw at age 3, if

\[
\left(\frac{\nu}{1-\pi}\right) \left(1 - (\hat{q}_t)''\right) > \left(\frac{\nu}{1-\pi}\right) \left(1 - (\hat{q}_t)''\right).
\]

The relocated home entrepreneurs will always withdraw at age 3, if

\[
\left(\frac{\nu}{1-\pi}\right) \left(1 - (\hat{q}_t)''\right) > \left(\frac{\nu}{1-\pi}\right) \left(1 - (\hat{q}_t)''\right).
\]
REFERENCES


Figure ONE: Timing of events.