Capital controls, capital flows, and banking crises

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Capital Controls, Capital Flows, and Banking Crises

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Abstract: Capital controls have been adopted by emerging economies to change the volume and the composition of capital flows and to protect the economy from sudden stops. The effectiveness measured by empirical studies has remained inconclusive, due to the limitation of the available data. This paper adopts a theoretical model to examine whether capital controls could achieve these goals effectively. Consequently, this paper finds that capital controls on outflows and inflows may not achieve the goals on changing the volume and the composition of capital flows and on protecting the economy from banking crises and sudden stops. To be more specific, controls on capital outflows and inflows could change the volume of capital flows at the time when the controls are imposed. However, the ability of capital controls on changing composition of capital flows and to protect the country from banking crises and sudden stops is limited, regardless of symmetric or asymmetric controls across countries. It is concluded that capital controls may not be the way to protect the economy from sudden stops. It is overcoming the liquidity problems and offering affordable rates, rather than competitive rates, that are crucial to protect the economy from crises and sudden stops.

JEL Classification: F02, F32, F34.

Keywords: international capital flows, capital controls, bank runs, banking crises

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1 Introduction

Capital controls have been adopted widely before 1970s, and have been relaxed gradually to promote flows. Since 1990s and before the Asian Financial Crisis in 1997, quite a few countries adopted capital controls again [Edwards (2009), Johnson et al (2007)]. During this period, most controls were on inflows, such as Thailand, Malaysia, Philippines, Indonesia, Czech Republic, Colombia, and Brazil, and one country which started controlling outflows was Spain. After the crisis in 1997, more countries, such as Argentina, have joined the group to controls capital flows, or implement controls on both inflows and outflows, such as Thailand, Malaysia, and Brazil.

Capital controls is regarded as part of prudential regulations and is used to restrict capital flows from certain directions. The reason behind this restriction is from the the fear that one free capital flows may magnify the contagion effects at the time of international crises. Therefore, it is the hope that capital controls can restrict and/or restructure capital flows and prevent the economy from crises. This might explain why capital controls have become one popular policy that many countries have reconsidered or have implemented, especially the emerging economies.

While capital controls have become more popular, its effectiveness on affecting capital flows and on preventing crises is still under debates. The earlier studies on capital flows are limited to the data, which often has the net flows available, but not gross flows. The recent studies, although overcoming the limitation on net flows and able to develop methods to analyze gross capital flows, have its difficulties to conclude the effectiveness of capital controls. For example, Forbes and Warnock (2012) find that capital controls have little association to foreign-driven capital flows. El-Shagi (2011) discovers that capital controls can restructure capital flows without distortion.

One main reason why the effectiveness of capital controls in empirical studies remains inconclusive is because of the different aspects on which each individual research focuses. Some focus on certain periods to looks at waves of flows [Forbes and Warnock (2012)]; some focus on panel data of certain countries [Ding and Jinjarak(2012), El-Shagi (2011)], and others focus on the aggregate flows of one single country, such as Brazil, Malaysia...etc. Moreover, the definitions of capital accounts and flows in different countries could be different. The results of capital flows in empirical studies are sensitive to the measurement and econometric methods adopted in the analysis. That is,
depending on the measurement and econometric methods, the empirical studies on the same capital controls in the same country may have opposite conclusions on the effectiveness of the controls. These existing apple-to-orange problems related to capital controls in empirical studies might be resolved by a unified theoretical framework, as Magud and Reinhart (2007) suggest. It is the goal of this paper to construct a theoretical framework which incorporates the key factors of capital flows to provide a mechanism through which capital controls affect international capital flows and may (or may not) prevent/cause banking crises. The contribution of this paper provides the insights of the debates on the effectiveness of capital controls by analyzing macroeconomic consequences of controls and by offering explanations of why capital controls are effective in some research but not in others.

According to current studies, successful capital controls must achieve the following outcome: affecting the volume and the composition of capital flows, and preventing the economy from international financial crises. These outcome will be the criteria of this paper to determine the effectiveness of capital controls. Since most of sudden stops are associated with banking crises, rather than currency crises [references], it is banking crises the main focus of this paper. Regarding to the framework, this paper develops an open-economy overlapping generations model with both the debt and the equity markets. By incorporating the characteristics of both credit markets, this paper analyzes the macroeconomic consequences of the economy without controls and of the economies under different types of capital controls.

As a result, the challenges faced by the banks under different types of controls are different. Whether banks would run depends on the domestic conditions as well as capital flows. It is shown that symmetric controls are more effective in achieving all three outcomes: affecting both the volume and the composition and preventing the economy from crises, compared to the economy without controls. Symmetric controls means that both countries control on inflows, or that both countries control on outflows. Asymmetric controls, however, are not as effective to achieve the outcomes. Whether asymmetric controls could achieve the outcomes depends on the interest rate differentials, the controls of the other country, and the domestic conditions. If extending this framework to multi-countries to look at gross flows to a certain country, it is possible to find that
the flows from various countries may offset each other, and leave capital controls less relevant to capital flows. This might be what has been shown in Forbes and Warnock (2012). If focusing on the countries to which the domestic country has asymmetric controls in similar way, it is possible to find that interest rate differential and capital controls are linked to each other [El-Shagi (2011)].

The rest of the paper is organized as follows. Section 2 provides the environment of the model without capital controls, followed by the equilibrium in Section 3. Section 4 discusses the effects of capital controls on inflows and outflows and whether the goals have been achieved. Conclusions and possible conclusions are provided in Section 5.

2 The model without capital controls

This paper will start with an open economy with two countries and none has implemented any form of capital controls. Let us call these two countries the home country and the foreign country. The population growth rate \((n)\) of these two countries is assumed the same, \(N_t = nN_{t-1}\) and \(N^*_t = nN^*_{t-1}\), where \(N_t\) and \(N^*_t\) represent the populations of the home and the foreign country at period \(t\), respectively. The economy of each country is composed of households, firms, and financial intermediaries. The financial intermediaries serve as a middleman in both credit markets, the debt and the equity markets, and as a portfolio manager for the depositors. As a portfolio manager, the financial intermediaries could save duplication costs and share risks at some degree, so they could offer a higher return rate, which cannot achieved by each individual [Champ, Freeman and Haslag (2011), Bencivenga and Smith (1991)]. Therefore, individuals with rational expectation would deposit their income in the financial intermediaries rather than self-investing. Different from the traditional setup, the financial intermediaries would act as firms maximizing profits, and are subject to run, should they fail to meet demand deposit\(^1\).

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\(^1\)The conventional setup which often assume zero profit for the financial intermediaries has its difficulties in addressing the possibilities of bank runs. Therefore, this paper relaxes the zero-profit assumption for the financial intermediaries.
2.1 Households

Each individual is born identical and lives for three periods: young, middle-aged and old. The endowment of each individual is one unit of labor when young and nothing when middle-aged and when old. It is assumed that only the middle-aged \( (c_{2,t+1}) \) and old \( (c_{3,t+2}) \) consumption that will be valued by each individual. Thus, the entire labor income will be deposited into the financial intermediaries as soon as the income is earned when young. At every period, the financial intermediaries have two types of short-term accounts available, one is saving accounts and the other is investment accounts. Both types of accounts take one period to mature, and may offer different return rates. That is mainly because saving accounts are for the debt market while the investment accounts are for the equity market. More details will be provided in the later subsections.

At middle-aged, the individuals would grow to two different types: investors and entrepreneurs. The type will be learned by each individual as soon as s/he turns middle-aged. The probability of becoming to either type is exogenous. With a probability \( \lambda \), an individual would become an investor at middle-age, and with a probability \( (1 - \lambda) \), an individual would become an entrepreneur. The type of an individual remains unchanged through his lifetime and is private information to the individual only. The distribution of types, however, is public information.

The investors and entrepreneurs are different mainly in two aspects: the skills and how they spend the withdrawals of their short-term accounts. In terms of skills, the entrepreneurs own the skills of obtaining funds in either credit market to finance projects, and the skills of operating firms to produce output. Therefore, if operating firms is more profitable than doing other investment, the entrepreneurs would spend his withdrawals on operating firms. The investors who do not have these skills of entrepreneurs would spend the withdrawals on reinvestment. The utility function of a young individual is assumed in the form of:

\[
U(c_{2,t+1}, c_{3,t+2}) = -\left(\frac{c_{2,t+1}^i + \sigma^i c_{3,t+2}^i}{\phi}\right)^{-\phi},
\]

where \( i = I \) (investor), \( E \) (entrepreneur), and \( \sigma^i \) represents the degree of patience regarding to how \( c_{3,t+2}^i \) is valued, relative to \( c_{2,t+1}^i \). This degree of patience depends on the individual’s type. It is assumed that \( 0 < \sigma^I < \sigma^E < 1 \). This indicates that investors (I) are less patient than entrepreneurs.
(E). After learning his own type, the middle-age household would visit the financial intermediaries to withdraw his matured SR accounts and spend as his own type. When old, the entrepreneurs would receive the profits from firm operation to consume, and the investors would receive returns from their reinvestment for their consumption in old age.

The return rates of different types of accounts are assumed different. Both accounts take one period to mature. The return rate of the saving accounts is the deposit rate \((1 + i^D_t)\), and the return rate of the investment accounts is the equity rate \((1 + i^E_{t+1})\). The only demand deposit is saving accounts, to which the deposit rate, \((1 + i^D_t)\), is determined by the financial intermediaries and is offered to the depositors at the time of deposit. The equity rate, \((1 + i^E_{t+1})\), however, is determined by the equity market clearing condition and will be offered to the equity holders after production is completed at the following period \(t + 1\).

As shown in Figure 3, a young individual would allocate between the two types of accounts: a fraction \(\alpha^I_t\) in the investment account and the rest \((1 - \alpha^I_t)\) in the saving accounts. At date \(t + 1\), the young who deposited at period \(t\) and becomes middle-aged could withdraw their matured accounts, and receives \(W^M_{t+1}\):

\[
W^M_{t+1} = (1 + i^D_t) (1 - \alpha^I_t) w_t + (1 + i^E_{t+1}) \alpha^I_t w_t \tag{2}
\]

### 2.1.1 Investors

As shown in Figure 4, the middle-age individuals of generation \(t - 1\) who learn their types would behave as their own types at period \(t\). That is, an investor would maximize his utility \((c^I_{2,t} + \sigma^I c^I_{3,t+1})\) by choosing whether to re-invest (RI) and then which country to re-invest. Due to the transaction costs, the investors would focus on one country to re-invest\(^2\) if there were no government restrictions, such as capital controls. When the expected return rates are the same across countries, the investors are assumed to re-invest in his domestic country. Moreover, given the constant transaction cost, the return rate is higher by investing the entire amount in the same country.

\(^2\)Providing fixed transaction costs, the rate of return is increasing in the investment amount. Therefore, the investors are better off by investing in one country. This is especially true when the risks of investment across countries are considered similar.
Let $\alpha_{t,Dj}^{IM}$ denote the fraction of reinvestment placed in the investment accounts in country $j$ by a domestic investor, and let . A domestic investor, who decides to re-invest in the domestic (foreign) country, would place a fraction, $\alpha_{t,DD}^{IM} \left( \alpha_{t,DF}^{IM} \right)$, of his/her total re-investment in investment accounts and the rest, a fraction $\left( 1 - \alpha_{t,DD}^{IM} \right) \left( 1 - \alpha_{t,DF}^{IM} \right)$, in saving accounts. Let $R_{t,DD}^{IM}$ denote the expected rate of return of reinvestment by a domestic investor in the domestic country at period $t$, $R_{t,DD}^{IM} \equiv \left[ \left( 1 + i^D_t \right) \left( 1 - \alpha_{t,DD}^{IM} \right) \right] + \left( 1 + i^{E+1}_t \right) \alpha_{t,DD}^{IM}$.

Compared to the rate of storage, as long as $R_{t,DD}^{IM} > 1$, the investors would always reinvest. Then the old consumption of an investor would become

$$c_{t+1}^{D} = W_{t+1,DD}^{IM} = R_{t,D}^{IM} \left( W_{t}^{IM} - c_{t,2}^{D} \right),$$

depending on the amount of reinvestment (RI) and the expected return rate of reinvestment.

### 2.2 Firms

The entrepreneurs are the only agents who have the special skills in operating firms. At period $t$, a middle-aged entrepreneur of generation $t-1$, after withdrawing his SR accounts, must transform part of his $W_{t}^{IM}$ into capital goods ($K_t$) to start production. The transformation from output goods to capital goods ($K_t$) is assumed to be one-to-one. The output production requires both capital goods ($K_t$) and labour ($L_t$) as inputs, and takes one period to complete. The production is in the Cobb-Douglas form with constant return to scale: $Y_{t+1} = A_t K_t^{\theta} L_t^{1-\theta}$, $0 < \theta < 1$, where $A_t$ represents production technology, and $Y_{t+1}$ represents total output goods produced at period $t+1$.

Moreover, it takes exogenous sunk cost $q_t$ to operate a firm, and the wage income of all labours has to be paid by the end of period $t$, which is before the completion of the production at period $t+1$. Therefore, it requires the amount $(q_t + w_t L_t + K_t)$ to start the production at period $t$. The assumption $q_t + w_t L_t + K_t > W_{t}^{IM}$ implies that any entrepreneur who plans to operate a firm must borrow to start production.

There are two resources available to the entrepreneurs to obtain funds. One resource is to file applications to financial intermediaries to obtain loans (the debt market). The other is to issue equities in the equity market. To simplify the model, it is assumed that entrepreneurs could raise funds in their own domestic credit markets only. Since the production takes one period to

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*Because of the regulations, the entrepreneurs who are eligible to issue equity in their own domestic countries may*
complete, the entrepreneurs would acquire only short-term fund.

### 2.2.1 Debt finance

To acquire a loan from the financial intermediaries, an entrepreneur must provide collateral \((B_t)\), which will not be returned until the loan is repaid. The amount of loan demanded by an entrepreneur is:

\[
b_t^D = q_t + w_t L_t - \left(W_t^M - c_{2,t}^E - K_t - B_t\right),
\]

where \(c_{2,t}^E\) represents the entrepreneur's middle-aged consumption. In the debt market, the source of loanable fund is limited to the sum of depositors' saving accounts at that period. When loan demand exceeds loan supply, the type 2 credit rationing will be the result\(^4\). That means that only a fraction \((\beta^D)\) of entrepreneurs would obtain loans from the financial intermediaries. Let \(S_t\) denote loanable funds which is the sum of all saving accounts at period \(t\). Let \(RI_t^D\) denote the sum of total re-investment, both saving and investment accounts, by the domestic investors, \(RI_t^D \equiv \left[\lambda + (1 - \beta^D - \beta^E)\right] N_t \left(W_t^M - c_{2,t}\right)\), and let \(RI_t^F\) denote the sum of total re-investment by the foreign investors, \(RI_t^F \equiv (1/e_t) \left[\lambda^* + (1 - \beta^{D*} - \beta^{E*})\right] N^* \left(W_t^{M*} - c_{2,t}^*\right)\). Let \(\gamma_t(1 - \gamma_t^*)\) denote the fraction of domestic (foreign) investors who re-invest in the domestic country. The loanable fund can be written as

\[
S_t \equiv (1 - \alpha_t^D) w_t N + \gamma_t \left(1 - \alpha_{t,DD}^M\right) RI_t^D + (1 - \gamma_t^*) \left(1 - \alpha_{t,FD}^{M*}\right) RI_t^F,
\]

where the first term is the total saving of the young, the second and the third terms are the total savings of the domestic and the foreign investors, respectively. As shown in Figure 5, the resource constraint for the debt market is

\[
S_t \geq \beta_t^D \left(1 + \lambda\right) N_t b_t^D.
\]

\(^4\) According to Blanchard and Fischer (1989, page 479), the type 2 credit rationing defined as follows: given an interested rate, some borrowers get credit but others do not, and all borrowers are identical. In this model, it requires the entrepreneurs to obtain sufficient amount of fund to operate firms, so it will be type 2 credit rationing.
The entrepreneurs who obtain loans are called debt-finance entrepreneurs, and would definitely start the production. Similarly, the entrepreneurs who obtain funds in the equity market are called equity-finance entrepreneurs.

The loan rate \((1 + i_{t}^{\text{loan}})\) is determined by the financial intermediaries at the time when the debt contract is constructed. The debt contract is designated that the debt payment must be affordable, \((1 + i_{t}^{\text{loan}}) b_t^D < Y_{t+1}\) and incentive compatible for the entrepreneurs to be willing to borrow funds to operate firms. Moreover, the debt contract would count for the uncertainty and ensure that the entrepreneurs would always be truth-telling\(^5\). Let \(p\) denote the probability that the production succeeds, the loan is repaid, and the entrepreneur earns his collateral back. Otherwise, with probability \((1 - p)\), the production fails, the entrepreneur cannot repay the loan, and would lose his collateral to the financial intermediaries.

The expected payoff of a debt-finance entrepreneur is his expected capital gains:
\[
E_t \Pi_{t+1}^{DF} = p \left[ Y_{t+1} - (1 + i_{t}^{\text{loan}}) b_t^D + B_t \right].
\]

The incentive constraint for an entrepreneur to obtain a loan to operate his firm is:
\[
c_{2,t}^E + \sigma^E E_t \Pi_{t+1}^{DF} \geq c_{2,t}^J + \sigma^E \max \{ W_{t+1,DD}, W_{t+1,DF} \},
\]  
where \(W_{t+1,DD}\) and \(W_{t+1,DF}\) represent the expected returns to an investor who invest in the domestic and the foreign country, respectively. This indicates that the entrepreneurs would start production only if the expected capital gains are higher than the maximum expected returns of an investor. In other words, if the expected capital gains were less than the returns of an investor, the entrepreneurs would prefer to become an investor, rather than start production.

### 2.2.2 Equity finance

The entrepreneur who do not obtain loans could issue equities in the equity market. The funds available to purchase equities is restricted to the sum of all investment accounts \((I_t)\),
\[
I_t = \alpha_t w_t N + \gamma_t \alpha_t^{IM} R_{t}^D + (1 - \gamma_t^*) \alpha_t^{IM*} R_{t}^F,
\]
\(^5\)That is, verification would take place whenever the loan repayment is not made. The verification would take away all entrepreneurs’ profits.
where the first term is the sum of investment accounts of the young, and the second and the third terms are the sum of the investment accounts of the domestic and foreign investors, respectively. Different from the debt market, the equity market does not require collateral \((B_t^E = 0)\), so the amount to borrow becomes \(b_t^E = q_t + w_t L_t - (W_t^M - K_t) < b_t^D\). The resource constraint for the equity market is:

\[ I_t \geq b_t^E (1 - \lambda) \beta^E N_t. \] (6)

Equation (6) shows that the limited amount of fund to purchase equity indicates that only a fraction \(\beta^E\) of entrepreneurs who could obtain sufficient funds to operate firms, and \(\beta^D + \beta^E \leq 1\). The entrepreneurs, who do not obtain funds via debt or equity finance, would become investors.

Another difference between the debt and the equity market is that the equity rate \((1 + i_{t+1}^E)\) is determined by the equity market clearing condition at the period \(t+1\) when the production is completed:

\[ (1 + i_{t+1}^E) = \frac{p \left( 1 + E_t i_{t+1}^E \right) (1 - \lambda) \beta^E N b_t^E}{I_t} = \frac{(1 - \lambda) \beta^E N p \psi_t Y_{t+1}^{EF}}{I_t}, \] (7)

where \(\psi_t\) represents the expected fraction of output which is extracted to repay the equity holders by the equity-finance entrepreneurs who have successful production. The equity rate \((1 + i_{t+1}^E)\) may not be the same as the deposit rate \((1 + i_t^D)\).

Similarly, the expected capital gain of an equity-finance entrepreneur is \(E_t \Pi_{t+1}^{EF} = p \left[ Y_{t+1} - (1 + E_t i_{t+1}^E) b_t^E \right]\).

The incentive constraint for an equity-finance entrepreneur to borrow to operate his firm is:

\[ c_{2,t}^E + \sigma^E E_t \Pi_{t+1}^{EF} \geq c_{1,t}^I + \sigma^E \left\{ W_{t+1,DD}, W_{t+1,DF} \right\}. \] (8)

Equation (8) shows that before raising funds in the equity market to operate firms, the entrepreneurs would form an expectation for the equity rate. Based on the expectationary equity rate, the equity-finance entrepreneurs make the decisions on the amount of capital goods to invest, and the amount of labour to hire.

### 2.2.3 Equilibrium Capital Gains to the Entrepreneurs

When the expectationary equity rate equals the loan rate, \(E_t i_{t+1}^E = i_t^{loan}\), the entrepreneurs, whether debt- or equity-finance would invest the same amount of capital goods and hire the same amount of
labour $K_t^{DF} = K_t^{EF}$, and $L_t^{DF} = L_t^{EF}$. The full employment assumption gives $w_t^{DF} = w_t^{EF}$, which shows no wage discrimination and no labour mobility across firms. Thus, when $E_t t_{t+1} = i_t^{loan}$, the equilibrium capital input ($\bar{K}_t^{DF}, \bar{K}_t^{EF}$), labour demand ($\bar{L}_t^{DF}, \bar{L}_t^{EF}$), and wage rate ($\bar{w}_t^{DF}, \bar{w}_t^{EF}$) are:

$$
\bar{K}_t^{DF} = \bar{K}_t^{EF} = \frac{1}{(1-\lambda)(\beta^D + \beta^E)} \left( \frac{A_t \theta}{1 + i_t^{loan}} \right)^{1/(1-\theta)},
$$

$$
\bar{L}_t^{DF} = \bar{L}_t^{EF} = \frac{1}{(1-\lambda)(\beta^D + \beta^E)},
$$

$$
\bar{w}_t^{DF} = \bar{w}_t^{EF} = \theta^{(1-\theta)}(1-\theta) \left( \frac{A_t}{1 + i_t^{loan}} \right)^{1/(1-\theta)}.
$$

The equilibrium values of the variables in equation (9) can be used to determine the equilibrium capital gains to entrepreneurs:

$$
E_t \bar{\Pi}_{t+1}^{DF} = p \left[ \bar{Y}_{t+1} - (1 + i_t^{Loan}) \bar{b}_t^D + B_t \right],
$$

$$
E_t \bar{\Pi}_{t+1}^{EF} = p \left[ \bar{Y}_{t+1} - (1 + E_t i_{t+1}) \bar{b}_t^E \right].
$$

By plugged equation (10) into equations (5) and (8), one can derive both the maximum loan rate and the equity rate which the entrepreneurs can accept:

$$
(1 + i_t^{Loan}) \leq \frac{\bar{Y}_{t+1} - \frac{1}{p} \left\{ W_t^{IM}_{t+1,DD}, W_t^{IM}_{t+1,DF} \right\}}{\bar{b}_t^D} \equiv \max (1 + i_t^{Loan}),
$$

$$
(1 + E_t i_{t+1}) \leq \frac{\bar{Y}_{t+1} - \frac{1}{p} \left\{ W_t^{IM}_{t+1,DD}, W_t^{IM}_{t+1,DF} \right\}}{\bar{b}_t^E} \equiv \max (1 + E_t i_{t+1}).
$$

### 2.3 Financial Intermediaries

Different types of accounts have different purposes. The saving accounts are for the debt market while the investment accounts are for the equity market. It is assumed that the financial intermediaries do not mix or misuse the funds of each type of accounts. Recall that saving accounts are the only type of demand deposits. Failing to repay demand deposits could result in bank runs. Therefore, the financial intermediaries must choose carefully the deposit rate. Any deposit rate must be sufficiently high to attract depositors and must be affordable for the financial intermediaries to meet demand deposit. The problem faced by the domestic financial intermediaries at date $t$ is
to maximize the expected payoff $E_t\Pi^{B}_{t+1}$ by choosing $i_t^D$ and $i_t^{loan}$:

$$E_t\Pi^{B}_{t+1} = \left\{ \left[ S_{t+1} - \beta_t^D (1 - \lambda) N b_t^D \right] + \left[ (1 - \lambda) \beta_t^D N \left[ p (1 + i_t^{loan}) b_t^D + (1 - p) B_t \right] - (1 + i_t^D) S_t \right] \right\}, \quad (12)$$

where the first bracket is to supply the new loans to the debt-finance entrepreneurs, and the second bracket is to use the loan repayment to meet the matured demand deposits. The liquidity constraint of the financial intermediaries at period $t+1$ is:

$$\left[ p \left( 1 + i_t^{loan} \right) b_t^D + (1 - p) B_t \right] (1 - \lambda) \beta_t^D N \geq (1 + i_t^D) S_t, \quad (13)$$

which is for the matured demand deposits, the saving accounts\textsuperscript{6}. Note that the left hand side of equation (13) is the assets while the right hand side is the liability for the financial intermediaries. Therefore, when the right hand side is higher than the left hand side, the financial intermediaries would have liquidity shortfalls and experience insolvency.

To attract the deposit from the young, the deposit rate must exceed the rate of return of storage, $(1 + i_t^D) > \left( 1 + i_t^{storage} \right) = 1$. Meanwhile, to attract the deposits from the investors, both domestic and foreign, the deposit rate must be competitive, compared to the foreign deposit rate. Recall that the investors would focus on one country to invest, the incentive constraints for the investors to re-invest in the domestic country requires that the expected domestic return rate exceeds the expected foreign return rate. Moreover, since the focus of this paper is not on the exchange rate, it is assume that the exchange rate between the domestic and the foreign country is one in this real economy. Therefore, the incentive constraints for the domestic investors [equation (14a)] and for the foreign investors [equation (14b)] to re-invest in the domestic country can be written as follows:

$$R_{t,DD}^{IM} \geq R_{t,DF}^{IM*}, \quad (14a)$$
$$R_{t,FD}^{IM} > R_{t,FF}^{IM*}, \quad (14b)$$

\textsuperscript{6}Since the returns of debt and equity are from successful production of the entrepreneurs, and the probability of a successful production is identical for all entrepreneurs, the degree of risks for debt and equity the same. That means that debt or equity is not riskier than the other, and the concept of capital requirement does not apply in this model. This is mainly because capital requirement divides assets into different groups based on the degree of risk, and asks the financial intermediaries to invest in less riskier assets. In this model, when all assets, debt the equity share the same degree of risk, the capital requirement cannot be applied in this model.
where $R_{IM}^{IM} \left( R_{IM}^{IM} \right)$ represents the expected domestic (foreign) return rate to a country $i$’s investor:

$$R_{IM}^{IM} \equiv \left(1 + i^{D}_t \right) \left( 1 - \alpha_{IM}^{IM} \right) + \left( 1 + E_{t+1}^{E} \right) \alpha_{IM}^{IM},$$

and

$$R_{IM}^{IF} \equiv \left[(1 + i^{F}_t) \left( 1 - \alpha_{IM}^{IM} \right) + \left( 1 + E_{t+1}^{E} \right) \alpha_{IM}^{IM} \right],$$

and

$$R_{IM}^{IF} \equiv \left[(1 + i^{F}_t) \left( 1 - \alpha_{IM}^{IM} \right) + \left( 1 + E_{t+1}^{E} \right) \alpha_{IM}^{IM} \right],$$

Note that it is possible for $\alpha_{IM}^{IM} \neq \alpha_{IM}^{IM}$ and/or $\alpha_{IM}^{IM} \neq \alpha_{IM}^{IM}$, which means that the domestic and/or foreign investors have different investment portfolios when re-investing in the domestic and foreign countries.

Provided the deposit rate which is determined by the financial intermediaries before taking the deposits, and the loan rate which has its maximum, equation (13) indicates that where $\beta^{D}_t = 1$, there is a maximum amount of deposit which the financial intermediaries can take:

$$S_t \leq \frac{p \max \left(1 + i^{loan}_t \right) b^D_t + (1 - p) B_t \left(1 - \lambda \right) N}{1 + i^{D}_t} = \max S_t. \quad \text{(15)}$$

The amount of deposit, which exceeds the maximum deposit may result in liquidity shortfalls in the following period. Should the liquidity shortfalls not be resolved, bank runs will be the result.

3 Equilibrium

3.1 Closed economy (CL)

In a closed economy (CL), there is no capital in or outflow, $\gamma_t = 1$, and $\gamma^{*}_t = 1$. The only source of deposits is from domestic agents. The amount of aggregate domestic saving is $S_{CL}^{CL} = (1 - \alpha_{t}^D) w_t N + \left(1 - \alpha_{IM}^{IM} \right) R^{D}$, and the amount of aggregate equity investment is $I_{CL}^{CL} = \alpha_{t}^D w_t N + \alpha_{IM}^{IM} R^{D}$. Since the investors could invest in their own countries only, the incentive constraints to attract investors to invest in the domestic country do not apply. The equity rate offered to the equity holders is restricted to the incentive constraint of the equity-finance entrepreneurs [equation (11)]. The deposit rate offered to the depositors must be affordable. That is, the total repayment to the depositors must be no more than the total loan repayment by the entrepreneurs.

$$\left(1 + i^{D(k)}_t \right) \leq \frac{p \max \left(1 + i^{loan(k)}_t \right) b^D_t + (1 - p) B_t \left(1 - \lambda \right) \beta^{D}_t N}{S^k_t}, \quad \text{(16)}$$

where the superscript $k$ represents the case. In this closed economy, $k = CL$. In this closed economy, the amount of deposits to receive can be easily predicted: $S_t = S_{CL}^{CL}$ when $\left(1 + i^{D(CL)}_t \right) > 1$, and $S_t = 0$ when $\left(1 + i^{D(CL)}_t \right) \leq 1$. Therefore, When there is no adverse shock which affects the
liquidity constraint, for the deposit rate that satisfies equation (16), it is less likely to have bank runs.

3.1.1 Banking crises

Similar to Chang (2012), this paper introduces a large adverse shock \((\varepsilon_t)\) on the successful rate of projects \((p)\) at period \(t\) to examine whether banking crises can be prevented in the closed economy and in an open economy with and without capital controls. This adverse shock is assumed sufficiently large to cause liquidity shortfalls and may lead to immediate bank runs at period \(t\) if the shortfalls were not overcome. Based on equation (13), this liquidity shortfalls at period \(t\) caused by the adverse shock in this closed economy is:

\[
SF^k_t \equiv \left(1 + i_{l-1}^{D(k)}\right) S_{l-1}^k - \left[p - \varepsilon_t\right] \left[1 + i_{l-1}^{loan(k)}\right] b_{l-1}^D + (1 - p + \varepsilon_t) B_{l-1} \left(1 - \lambda\right) \beta_{l-1}^D N > 0, k = CL.
\]

(17)

One source which is often used by the financial intermediaries to finance the shortfalls is the new deposits, which are from the young and from the investors. While the young would deposit when the deposit rate is greater than one, the investors would not deposit until receiving the repayments of their matured accounts. This implies that the deposits from the young is the only recourse which the financial intermediaries could use to finance the liquidity shortfalls.

Let \(S_{t,y}^{CL}\) denote the deposit of the young in the saving accounts, \(S_{t,y}^{CL} = (1 - \alpha_t) w_t N\). If \(S_{t,y}^{CL} < SF_t^{CL}\), there are investors not receiving repayments of their mature accounts. Without meeting demand deposits, the financial intermediaries will run immediately. Therefore, it requires \(S_{t,y}^{CL} \geq SF_t^{CL}\) to meet demand deposits and to prevent the immediate bank runs at period \(t\).

After overcoming the liquidity shortfalls and repaying the matured accounts, the financial intermediaries could now attract the investors to deposit. In the closed economy, the investors would re-invest when the deposit rate is greater than one, which is the same condition to attract the deposits from the young. So the total deposits would be \(S_t^{CL}\), and the loanable funds to the entrepreneurs would reduced to \((S_t^{CL} - SF_t^{CL})\), which would lower the likelihood for the entrepreneurs to obtain the loan \(\beta_t^D (1 - \lambda) N_t = (S_t^{CL} - SF_t^{CL}) / b_t^D\) [equation(4)]. Consequently, even if the
adverse shock is temporary, the liquidity shortfalls may still exist at period \( t + 1 \):

\[
SF^{k}_{t+1} \equiv (1 + i^{D}_t) S^{k}_t - \frac{p (1 + i^{\text{loan}}(k)) b^{D}_t + (1 - p) B_t}{b^{D}_t} \left( S^{k}_t - SF^{k}_t \right), k = CL. \tag{18}
\]

If not able to overcome the shortfalls \( SF^{CL}_{t+1} \) indicated in equation (18), the financial intermediaries would experience bank runs at the following period \( t + 1 \). One example is when the deposits from the young equals the shortfalls at period \( t \), \( S^{CL}_{t,y} = SF^{CL}_t \), the loanable fund would be the deposits from the investors, \( (S^{CL}_t - SF^{CL}_t) = (1 - \alpha^{IM}_t) R^{D}_t \). The fact that \( (1 + i^{D}_t) > 1 \) implies that in order for to overcome the liquidity shortfalls, \( SF^{CL}_{t+1} = 0 \), the loan rate must be sufficiently high and satisfy the following condition:

\[
\left( 1 + i^{\text{loan}}_t \right) \geq \frac{1}{p} \left[ \left( \frac{S^{k}_{t,y}}{S^{k}_t - SF^{k}_t} + 1 \right) - \frac{(1 - p) B_t}{b^{D}_t} \right] = \min \left( 1 + i^{\text{loan}}_t \right), k = CL \tag{19}
\]

However, the loan rate has its maximum value \( \max (1 + i^{\text{loan}}_t) \) [equation (11)]. Any loan rate charged by the intermediaries exceeds the maximum value \( (1 + i^{\text{loan}}_{CL}) > \max (1 + i^{\text{loan}}_t) \) would result in no entrepreneur apply for loans. Thus, the liquidity shortfalls would become

\[
SF^{CL}_{t+1} \equiv (1 + i^{D}_t) S^{CL}_t - (S^{CL}_t - SF^{CL}_t) = i^{D}_t S^{CL}_t + SF^{CL}_t \tag{20}
\]

which is greater than \( SF^{CL}_t \), and more difficult for the financial intermediaries to overcome. Therefore, it is more likely to have bank runs at period \( t + 1 \).

### 3.2 An open economy without capital controls (NC, original framework)

In an open economy without controls, the country in which the investors would re-invest depends on the relative expected return rate. The mobility of investors’ re-investment is a challenge to the financial intermediaries in to decide the deposit rate to offer. On one hand, the deposit rate must be sufficiently high in order to attract the investors. However, a higher deposit rate at date \( t \) which attract more deposits means a higher debt (demand deposit) to be repaid at period \( t + 1 \). On the other hand, a low deposit rate which attracts less deposits means less loanable fund. The lower amount of loanable fund would drive up the loan rate in order to repay the demand deposits. However, the loan rate has its maximum value [equation (11)], and any loan rate
exceeding \(\max(1 + i_t^{loan})\) would result in no entrepreneurs borrowing. Without loan repayment at the following period \(t + 1\), the financial intermediaries cannot overcome the liquidity shortfalls and must run at period \(t + 1\).

Recall that the investors would re-invest in the country where they were born when the expected return rate is the same across countries. This implies that it requires the expected foreign return rate higher than the expected domestic return rate in order for the domestic investors to feel indifferent in re-investing in either country, \(R_{t,D}^{IM*} = R_{t,D}^{IM} + \xi^D_t\) where \(\xi^D_t\) represents the risk premium for the domestic investors to invest in the foreign country, and vice versa, \(R_{t,F}^{IM*} = R_{t,F}^{IM} + \xi^F_t\), where \(\xi^F_t\) is similar to the risk premium for the foreign investors to invest in the domestic country. For simplicity, it is assumed that when \(R_{t,D}^{IM*} = R_{t,D}^{IM} + \xi^D_t\) holds for all \(0 < \alpha_{t,Dj}^{IM} \leq 1\), \(j = D, F\), it is true that \(R_{t,D}^{IM*} > R_{t,D}^{IM}\). Meanwhile, when \(R_{t,F}^{IM*} = R_{t,F}^{IM} + \xi^F_t\) holds for all \(0 < \alpha_{t,Fj}^{IM} \leq 1\), \(j = D, F\), it is true that \(R_{t,F}^{IM*} > R_{t,F}^{IM}\).

Let \(\gamma^NC_t (\gamma^NC^*_t)\) where \(\gamma^NC_t, \gamma^NC^*_t \in (0, 1)\) denote the fraction of the domestic (foreign) investors who re-invest in the domestic (foreign) country when feeling indifferent in re-investing in either country. Under the circumstance where \(R_{t,D}^{IM*} = R_{t,D}^{IM} + \xi^D_t\), \(\forall \alpha_{t,Dj}^{IM} \in [0, 1]\), \(j = D, F\), all foreign investors and a fraction \((1 - \gamma^NC_t)\) of domestic investors would reinvest in the foreign country. When the expected foreign return rate is relatively attractive, the capital outflows \((CO_{t,F})\) from and inflows \((CI_{t,F})\) to the domestic country are:

\[
CO_{t,F}^{NC} = (1 - \gamma^NC_t) R_{t,D}^D, \quad CI_{t,F}^{NC} = 0,
\]

respectively. Accordingly, the sum of saving accounts is \(S_t^{NC} = (1 - \alpha^I_t) w_t N + \gamma^NC_t \left(1 - \alpha_{t,D}^{IM}\right) R_{t,D}^D\) and the amount of equity fund is \(I_t^{NC} = \alpha^I_t w_t N + \gamma^NC_t \alpha_{t,D}^{IM} R_{t,D}^D\). When \(R_{t,D}^{IM*} > R_{t,D}^{IM} + \xi^D_t\), both domestic and foreign investors would invest in the foreign country, \(\gamma^NC_t = 0\), and capital outflows would become \(CO_{t,F}^{NC} = R_{t,D}^D\) while inflows remains the same, \(CI_{t,F}^{NC} = 0\). Therefore, the loanable and equity funds would be purely from the young: \(S_t^{NC} = (1 - \alpha^I_t) w_t N = S_t^{NC, y}\), and \(I_t^{NC} = \alpha^I_t w_t N\).

Under the circumstance where \(R_{t,F}^{IM*} = R_{t,F}^{IM} + \xi^F_t\), all domestic investors and a fraction \((1 - \gamma^NC^*_t)\) of foreign investors would invest in the domestic country. The volumes of capital
flows of the domestic country are:

\[ CO_{t,D}^{NC} = 0, CI_{t,D}^{NC} = (1 - \gamma_{t}^{NC}) R_{t}^{F}. \]

Accordingly, the amounts of loanable and equity funds are: \( S_{t,D}^{NC} = (1 - \alpha_{t}^{I}) w_{t}N + (1 - \alpha_{t,DD}^{IM}) R_{t}^{P} + (1 - \alpha_{t,FD}^{IM} + \gamma_{t}^{NC}) R_{t}^{F} \), \( I_{t,D}^{NC} = \alpha_{t}^{I} w_{t}N + \alpha_{t,DD}^{IM} R_{t}^{P} + \alpha_{t,FD}^{IM} (1 - \gamma_{t}^{NC}) R_{t}^{F} \). As shown in equation (15), provided the deposit rate \((1 + i_{t}^{D(NC)})\), and \( \beta_{t}^{I} = 1 \), the amount of saving which is affordable by the financial intermediaries must satisfy the following condition:

\[
S_{t}^{k} \leq \frac{[p \max (1 + i_{t}^{loan}) b_{t}^{D} + (1 - p) B_{t}] (1 - \lambda) N}{(1 + i_{t}^{D(k)})} \equiv \max S_{t}^{k}, k = NC. \tag{21}
\]

In equation (21), \( \max S_{t}^{k} \) represents the maximum amount of the savings which is affordable by the financial intermediaries. The value of \( \max S_{t}^{k} \) is decreasing in the deposit rate, \((1 + i_{t}^{D(k)})\). Any amount of saving exceeds this threshold \( (S_{t,D}^{NC} > \max S_{t}^{NC}) \) would result in insolvency at the following period since the received loan repayment is insufficient to repay the demand deposits. If such insolvency cannot be overcome, bank runs will be the result at the following period \( t + 1 \). This means that even without an adverse shock, the insolvency is possible when the financial intermediaries take the amount of deposits which exceeds the threshold. Moreover, when \( R_{t,DD}^{IM} > R_{t,FF}^{I} + \xi_{t}^{F} \), all domestic and foreign investors would re-invest in the domestic country, \( \gamma_{t}^{NC} = 1 \), and capital inflows would increase to \( CI_{t,D}^{NC} = R_{t}^{F} \). Therefore, it is important for the financial intermediaries to accept the amount of savings \( S_{t,D}^{NC} \) to prevent insolvency.

If \( S_{t,D}^{NC} > \max S_{t}^{NC} \), and the financial intermediaries accept savings \( S_{t,D}^{NC} \leq \max S_{t}^{NC} \), the amount \((S_{t,D}^{NC} - S_{t,D}^{NC})\) would move to the equity market, and increase the amount of equity fund, \( I_{t,D}^{NC} = \alpha_{t}^{I} w_{t}N + \alpha_{t,DD}^{IM} R_{t}^{P} + \alpha_{t,FD}^{IM} (1 - \gamma_{t}^{NC}) R_{t}^{F} + \left(S_{t,D}^{NC} - S_{t}^{NC}\right) \). This increase in the equity fund would cause the equity rate at the following period \((1 + \gamma_{t+1}^{E(NC)})\) to decrease.

### 3.2.1 Banking crises and capital flows

At the time of adverse shock \((\varepsilon_{t})\), the financial intermediaries face the liquidity shortfalls as shown in equation (17) with \( k = NC \). To attract the deposits from the young \((S_{t,y}^{NC})\) to finance the shortfalls, the deposit rate must be greater than one, \((1 + i_{t}^{D(NC)}) > 1\). Then it requires \( S_{t,y} \geq SF_{t}^{NC} \) to prevent immediate bank runs at period \( t \). However, using the new deposit to finance \( SF_{t}^{NC} \) might
lead to the liquidity shortfalls at period $t + 1$, $SF_{t+1}^{NC}$ [equation (18) with $k = NC$]. If $SF_{t+1}^{NC}$ cannot be overcome, bank would run at period $t + 1$. Whether $SF_{t+1}^{NC}$ can be overcome depends on the relative values of expected return rates and the loan rate and deposit rate.

Under the circumstance where $R_{t,DF}^{IM} = R_{t,DD} + \xi_t^D$, and $S_{t, y}^{NC} = SF_{t}^{NC}$, the loanable fund after financing $SF_{t}^{NC}$ is

$$ (S_{t,F}^{NC} - SF_{t}) = \gamma_t^{NC} (1 - \alpha_{t,DD}^{IM}) R_{t}^{D}, $$

which is less than that in the closed economy, $(S_{t,F}^{NC} - SF_{t}) < (S_{t,F}^{CL} - SF_{t})$ because $\gamma_t^{NC} < 1$. Substituting the value of $(S_{t,F}^{NC} - SF_{t})$ into equation (19) gives the minimum loan rate, $\min \left(1 + i_{t,F}^{\text{loan}(NC)}\right)$, which financial intermediaries must charge in order to meet the demand deposit. Moreover, one can find that $\min \left(1 + i_{t,F}^{\text{loan}(NC)}\right) > \min \left(1 + i_{t,F}^{\text{loan}(CL)}\right)$. Compared to equation (11), when $\min \left(1 + i_{t,F}^{\text{loan}(NC)}\right) > \max \left(1 + i_{t,F}^{\text{loan}}\right) > \min \left(1 + i_{t,F}^{\text{loan}(CL)}\right)$, the open economy without capital controls (NC) is worse than a closed economy in overcoming the liquidity shortfalls $SF_{t+1}^{k}$ and in preventing bank runs at period $t + 1$.

Moreover, when $R_{t,DF}^{IM} > R_{t,DD} + \xi_t^D$, all domestic investors re-invest in the foreign country, $\gamma_t^{NC} = 0$, like the foreign investors. So when $S_{t, y}^{NC} = SF_{t}^{NC}$, the loanable fund becomes $(S_{t,F}^{NC} - SF_{t}^{NC}) = 0$ after financing the liquidity shortfalls, $SF_{t}^{NC}$. This would lead to $(1 + i_{t,F}^{\text{loan}(NC)}) \to \infty$ [equation (19) with $k = NC$], which means that the liquidity shortfalls can not be overcome, and that bank runs at period $t + 1$.

Under the circumstance where $R_{t,FD}^{IM} = R_{t,FF} + \xi_t^F$ and $S_{t, y}^{NC} = SF_{t}^{NC}$, the loanable fund becomes

$$ (S_{t,D}^{NC} - SF_{t}^{NC}) = (1 - \alpha_{t,DD}^{IM}) R_{t}^{D} + (1 - \gamma_t^{NC*}) (1 - \alpha_{t,FD}^{IM}) R_{t}^{F}, $$

which is more than that in the closed economy, $(S_{t,D}^{NC} - SF_{t}) > (S_{t,D}^{CL} - SF_{t})$. As the amount of deposits is high, and part of it is used to finance the liquidity shortfalls, it is important to ensure that equation (21) with $k = NC$ is satisfied. This is especially true when $R_{t,FD}^{IM} > R_{t,FF} + \xi_t^F$, and $\gamma_t^{NC*} = 0$. Compared to case CL, since $(1 + i_{t,D}^{\text{D}(NC)}) > (1 + i_{t,D}^{\text{D}(NC)}) > 1$, the domestic deposit rate could also be higher than that in case CL, $(1 + i_{t,D}^{\text{D}(NC)}) > (1 + i_{t,D}^{\text{D}(CL)})$. This would lead to $\max S_{t,D}^{NC} < \max S_{t,D}^{CL}$. Should the financial intermediaries accept the amount of savings which exceeds the threshold, $S_{t,D}^{NC} > \max S_{t,D}^{NC}$, bank runs at period $t + 1$. In this aspect, the open
economy without capital controls would do worse than a closed economy in overcoming $SF_{t+1}^{NC}$ and in preventing bank runs at period $t + 1$.

3.3 An open economy with capital controls

The controls on capital flows are assumed to set up a ceiling to which each investor could invest in a particular country. For example, the controls on capital inflows are to set up a ceiling to which a foreign investor could invest in the domestic country, and the controls on capital outflows are to set up a ceiling to which a domestic investor could invest in the foreign country. Therefore, capital flows may be affected by capital controls of both countries or either country.

Since symmetric and asymmetric controls have different impacts on the volumes and the compositions of flows as well as the ability in preventing banking crises. This section would discuss symmetric and asymmetric controls separately. To be more specific, the symmetric controls are the case where both countries have controls on the same type of flows, regardless of the size of controls. All other cases are asymmetric controls.

The assumption that both countries are symmetric allows this paper to focus on the impacts on the domestic country only. The analysis for the foreign country can be easily applied by adding asterisk to each variable. The cases of symmetric controls are symmetric controls on outflows (SCCO), and on inflows (SCCI). The cases of asymmetric controls are: domestic controls on outflows vs. no foreign controls (ACON), domestic controls on outflows vs. foreign controls on inflows (ACOI), and domestic controls on inflows vs. no foreign controls (ACIN).

It is assumed that the controls are imposed at period $t$. This means that the values of all variables determined prior to period $t$ would be the same in all cases. For example, $S_{t-1} = S_{t-1}^{NC} = S_{t-1}^{k}$, and $(1 + i_{t-1}^{D}) = (1 + i_{t-1}^{D(NC)}) = (1 + i_{t-1}^{D(k)})$, where $k$ represents any open economy case. According to equation (17), since all component of $SF_{t}$ are the variables determined prior to period $t$, the values of $SF_{t}$ is the same in all open economy cases, $SF_{t} = SF_{t}^{NC} = SF_{t}^{k}$, $k = SCCO, SCC1, ACON, ACOI, ACIN$. 
3.3.1 Symmetric capital controls

Both countries control capital outflows (SCCO) In this case, the domestic country sets up the ceiling, $\tilde{\gamma}_t^{SCCO} > \gamma_t^{NC}$, for each domestic investor while for foreign country sets up the ceiling, $\tilde{\gamma}_t^{SCCO*} > \gamma_t^{NC*}$ for each foreign investor. Under the circumstance where $R_{t,FD}^{IM} \geq R_{t,DD}^{IM} + \xi_t^D$, a fraction $(1 - \tilde{\gamma}_t^{SCCO})$ of domestic investors and all foreign investors would invest in the foreign country. The volumes of capital flows of the domestic country are:

$$CO_{t,F}^{SCCO} = (1 - \tilde{\gamma}_t^{SCCO}) R_{t,F}^D, CI_{t,F}^{SCCO} = 0,$$

which the volume of capital outflows is less than that in case NC, $CO_{t,F}^{SCCO} < CO_{t,F}^{NC}$. The amount of loanable fund (saving accounts) is $S_{t,F}^{SCCO} = (1 - \alpha_t^I) w_t N + \tilde{\gamma}_t^{SCCO} \left(1 - \alpha_{t,DD}^{IM}\right) R_{t,D}^P$, and the amount of equity fund (investment accounts) is $I_{t,F}^{SCCO} = \alpha_t^I w_t N + \tilde{\gamma}_t^{SCCO} \alpha_{t,DD}^{IM} R_{t,D}^F$. Both funds are more than the case NC, $S_{t,F}^{SCCO} > S_{t,F}^{NC}$, and $I_{t,F}^{SCCO} > I_{t,F}^{NC}$. Since the deposit rate is predetermined, the change on the funds would affect the equity rate only. According to equation (7), this increase in equity fund lowers the equity rate at period $t + 1$, $(1 + i_{t+1,F}^{E(SCCO)})$, which pays after production is completed. This decrease in equity rate $(1 + i_{t+1,F}^{E(SCCO)})$ may shift the composition of the deposits towards to the loanable fund by decreasing $\alpha_{t+1,F,D}$ and/or $\alpha_{t+1}^I$.

Under the circumstance where $R_{t,F}^{IM} \geq R_{t,F}^{IM*} + \xi_t^F$, all domestic investors and a fraction $(1 - \tilde{\gamma}_t^{SCCO*})$ of foreign investors would re-invest in the domestic country. So capital outflows and inflows would be:

$$CO_{t,D}^{SCCO} = 0, CI_{t,D}^{SCCO} = (1 - \tilde{\gamma}_t^{SCCO*}) R_{t,D}^F,$$

where the capital inflows is less than the case without capital controls (NC), $CI_{t,D}^{SCCO} < CI_{t,D}^{NC}$. Accordingly, the amount of loanable fund would become $S_{t,D}^{SCCO} = (1 - \alpha_t^I) w_t N + (1 - \alpha_{t,DD}^{IM}) R_{t,D}^P + (1 - \tilde{\gamma}_t^{SCCO*}) \left(1 - \alpha_{t,F,D}^{IM*}\right) R_{t,D}^F$, and the amount of equity fund would become $I_{t,D}^{SCCO} = \alpha_t^I w_t N + \alpha_{t,D,D}^{IM} R_{t,D}^P + (1 - \tilde{\gamma}_t^{SCCO*}) \alpha_{t,F,D}^{IM*} R_{t,D}^F$. Both funds are less than those in case NC, $S_{t,D}^{SCCO} < S_{t,D}^{NC}$, and $I_{t,D}^{SCCO} > I_{t,D}^{NC}$. The lower amount of equity fund would drive up the equity rate $(1 + i_{t+1,D}^{E(SCCO)})$, and may shift the composition of deposits towards the equity fund by increasing the values of $\alpha_{t,F,D}^{IM*}$, $\alpha_{t,D,D}^{IM}$ and $\alpha_t^I$ may increase. The size of the changes on the composition is increasing in the gap between $\tilde{\gamma}_t^{SCCO}$ and $\gamma_t^{NC}$. The further $\tilde{\gamma}_t^{SCCO*}$ deviates from $\gamma_t^{NC*}$, the larger sizes of the changes on the compositions.
Banking Crises  At the time of the adverse shock ($\varepsilon_t$), the financial intermediaries face the liquidity shortfalls $SF_t$. By offering $\left(1 + i_t^{D(SCCO)}\right) > 1$, the financial intermediaries could attract the deposit from the young, $S_{t,y}^{SCCO}$, which can be used to finance $SF_t$. When $S_{t,y}^{SCCO} \geq SF_t$, the immediate bank runs can be prevented. However, doing so might lead to liquidity shortfalls at following period $t + 1$, $SF_{t+1}^{SCCO}$ as shown in equation (18) with $k = SCCO$. Whether $SF_{t+1}^{SCCO}$ can be overcome would depend on the expected return rates, the loan rate, and the amount of deposits accepted by the financial intermediaries.

When $R_{t,D}^{IM*} \geq R_{t,D}^{IM} + \xi_t^D$ and $S_{t,y}^{SCCO} = SF_t$, the available loanable fund after paying $SF_t$ becomes

$$S_{t,F}^{SCCO} - SF_t = (1 - \alpha_t^{IM}) \hat{\gamma}_t^{SCCO} R_{t,F}^D,$$

which is more than the fund in case NC, $\left(S_{t,F}^{SCCO} - SF_t\right) > \left(S_{t,F}^{NC} - SF_t\right)$ because of $\hat{\gamma}_t^{SCCO} > \gamma_t^{NC}$. Substituting the values of $S_{t,F}^{SCCO}$ and $S_{t,F}^{NC}$ into equation (19) gives $\min \left(1 + i_{t,F}^{loan(SCCO)}\right) < \min \left(1 + i_{t,F}^{loan(NC)}\right)$, which can be compared to the value of $\max \left(1 + i_{t,F}^{loan}\right)$ in equation (11). When $\min \left(1 + i_{t,F}^{loan(SCCO)}\right) < \max \left(1 + i_{t,F}^{loan}\right) < \min \left(1 + i_{t,F}^{loan(NC)}\right)$, the symmetric capital controls on outflows could overcome $SF_{t+1}^{SCCO}$, which cannot be achieved in the case without capital controls. Moreover, the capital outflows are restricted by the domestic country which has a lower expected return rate.

When $R_{t,D}^{IM*} \geq R_{t,D}^{IM} + \xi_t^F$, $\forall \alpha_t^{IM*} \in [0,1]$, $j = D, F$, and $S_{t,y}^{SCCO} = SF_t$, the available loanable fund after paying $SF_t$ is

$$S_{t,D}^{SCCO} - SF_t = (1 - \alpha_t^{IM}) \hat{\gamma}_t^{SCCO} R_{t,D}^F + (1 - \hat{\gamma}_t^{SCCO*}) (1 - \alpha_t^{IM*}) R_{t,F}^F,$$

which is less than the fund without capital controls, $\left(S_{t,D}^{SCCO} - SF_t\right) < \left(S_{t,D}^{NC} - SF_t^{SCCO}\right)$ because of $\hat{\gamma}_t^{SCCO*} > \gamma_t^{NC*}$. Since the deposit rate is relatively attractive and the amount of demand deposit is relatively high, it is important that $S_{t,D}^{SCCO}$ must satisfy equation (21) with $k = SCCO$. In the case when $\left(1 + i_t^{D(NC)}\right) = \left(1 + i_t^{D(SCCO)}\right)$, and $S_{t,D}^{SCCO} < \max S_t^{SCCO} = \max S_t^{NC} < S_t^{NC}$, capital controls could overcome the liquidity shortfalls and prevent banking crises which cannot be achieved in the case without capital controls.
Both countries control capital inflows (SCCI) In this case, the domestic country sets up the ceiling \(1 - \tilde{\gamma}^{SCCI}_{t}\) < \(1 - \gamma^{NC}_{t}\) for each foreign investor while the foreign country sets up the ceiling \(1 - \tilde{\gamma}^{SCCI}_{t}\) < \(1 - \gamma^{NC}_{t}\) for each domestic investor. Under the circumstance where \(R_{t,DF}^{IM} \geq R_{t,DD}^{IM} + \xi^D_{t}\), all foreign investors would re-invest in the foreign country but only a fraction \((1 - \tilde{\gamma}^{SCCI}_{t})\) of domestic investors would re-invest in the foreign country. The volumes of capital flows of the domestic country would become:

\[
CO_{t,F}^{SCCI} = (1 - \tilde{\gamma}^{SCCI}_{t}) RI_{t}^{D}, CII_{t,F}^{CCI} = 0,
\]

where the volume of capital outflows is also less than the case NC, \(CO_{t,F}^{SCCI} < CO_{t,F}^{NC}\) because of \((1 - \tilde{\gamma}^{SCCI}_{t}) < (1 - \gamma^{NC}_{t})\). Note that the volume of capital outflows is restricted by the foreign controls. The available loanable fund and equity fund are \(S_{t,F}^{SCCI} = (1 - \alpha^I_t) w_tN + \tilde{\gamma}^{SCCI}_{t} (1 - \alpha^{IM}_{t,DD}) RI_{t}^{D}\) and \(I_{t,F}^{SCCI} = \alpha^I_t w_tN + \tilde{\gamma}^{SCCI}_{t} \alpha^{IM}_{t,DD} RI_{t}^{D}\). Both funds are more than the case NC, \(S_{t,F}^{SCCI} > S_{t,F}^{NC}\) and \(I_{t,F}^{SCCI} > I_{t,F}^{NC}\). The increase in equity fund would decrease the equity rate \(\left(1 + \tilde{\iota}_{t+1}^{F(SCCCI)}\right)\), and decrease the values of \(\alpha^{IM}_{t+1,DD}\) and \(\alpha^I_{t+1}\). This means that the composition of deposit would shift away from the equity market.

Under the circumstance where \(R_{t,F,D}^{IM} \geq R_{t,F,F}^{IM} + \xi^F_{t}\), all domestic investors and a fraction \((1 - \tilde{\gamma}^{SCCO}_{t})\) of foreign investors would re-invest in the foreign country. The volumes of capital inflows of the domestic country are:

\[
CO_{t,D}^{SCCI} = 0, CII_{t,D}^{SCCI} = (1 - \tilde{\gamma}^{SCCI}_{t}) RI_{t}^{F},
\]

where the volume of capital inflows is less than the case NC, \(CII_{t,D}^{SCCI} < CII_{t,D}^{NC}\) because of \((1 - \tilde{\gamma}^{SCCI}_{t}) < (1 - \gamma^{NC}_{t})\). The amounts of loanable fund and equity fund become \(S_{t,D}^{SCCI} = (1 - \alpha^I_t) w_tN + \left(1 - \alpha^{IM}_{t,DD}\right) RI_{t}^{D} + \left(1 - \tilde{\gamma}^{SCCI}_{t}\right) \left(1 - \alpha^{IM}_{t,F,D}\right) RI_{t}^{F}\), and \(I_{t,D}^{SCCI} = \alpha^I_t w_tN + \alpha^{IM}_{t,DD} RI_{t}^{D} + \left(1 - \tilde{\gamma}^{SCCI}_{t}\right) \alpha^{IM}_{t,F,D} RI_{t}^{F}\). Both funds are less than the funds in the case NC, \(S_{t,D}^{SCCI} < S_{t,D}^{NC}\) and \(I_{t,D}^{SCCI} < I_{t,D}^{NC}\). This decrease in the equity fund would drive up the equity rate \(\left(1 + \tilde{\iota}_{t+1,D}^{E(SCCCI)}\right)\), and increase the values of \(\alpha^{IM}_{t,F,D}\), \(\alpha^{IM}_{t,DD}\) and \(\alpha^I_t\). This means that the compositions of the deposits, including capital inflows, would shift towards the equity market. The sizes of the changes on the compositions are increasing in the gap between \(\tilde{\gamma}^{SCCO}_{t}\) and \(\gamma^{NC}_{t}\).
Banking Crises  At the time of the adverse shock \( (\varepsilon_t) \), the financial intermediaries face the liquidity shortfalls \( SF_t \). By offering the deposit rate, \((1 + i_t^{D(SCCI)}) > 1\), the financial intermediaries attract the deposits of the young, \( S_{t,y}^{SCCI} \), which can be used to finance \( SF_t \). When \( S_{t,y}^{SCCI} > SF_t \), the liquidity shortfalls \( SF_t \) are overcome and the immediate bank runs are prevented. However, doing so might lead to liquidity shortfalls at following period \( t + 1 \), \( SF_{t+1}^{SCCI} \), as shown in equation (18) with \( k = SCCI \). Whether the shortfalls can be overcome would depend on the the expected return rates, the amount of demand deposits, and the loan rate.

When \( R_{t,DF}^M \geq R_{t,DD}^M + \xi_t^D \) and \( S_{t,y}^{SCCI} = SF_t \), the available loanable fund after paying \( SF_t \) becomes

\[
S_{t,F}^{SCCI} - SF_t = (1 - \alpha_t^{IM}) \tilde{\gamma}_t^{SCCI} RI_D, 
\]

which is more than the fund in case NC, \( \left( S_{t,F}^{SCCI} - SF_t \right) > \left( S_{t,F}^{NC} - SF_t \right) \) because of \( \tilde{\gamma}_t^{SCCI} > \gamma_t^{NC} \).

Substituting the values of \( S_{t,F}^{SCCI}, k = SCCI, NC \) into equation (19), one can obtain

\[
\min \left( 1 + i_t^{loan(SCCI)} \right) < \max \left( 1 + i_t^{loan} \right). 
\]

Compared to the value of \( \max \left( 1 + i_t^{loan} \right) \) in equation (11), when \( \min \left( 1 + i_t^{loan(SCCI)} \right) < \max \left( 1 + i_t^{loan} \right) < \min \left( 1 + i_t^{loan(NC)} \right) \), the symmetric capital controls again could overcome the liquidity shortfalls, which cannot be achieved in case NC.

When \( R_{t,DF}^M \geq R_{t,DF}^M + \xi_t^F \) and \( S_{t,y}^{SCCI} = SF_t \), the available loanable fund after paying \( SF_t \) would change to

\[
S_{t,D}^{SCCI} - SF_t = (1 - \alpha_t^{IM}) RI_D^D + \left( 1 - \tilde{\gamma}_t^{SCCI*} \right) (1 - \alpha_t^{IM*}) RI_t^F, 
\]

which is less than the fund in case NC, \( \left( S_{t,D}^{SCCI} - SF_t \right) < \left( S_{t,D}^{NC} - SF_t^{SCCO} \right) \) because of \( (1 - \tilde{\gamma}_t^{SCCI*}) < (1 - \gamma_t^{NC*}) \). According to equation (21) with \( k = SCCI \), when \( \left( 1 + i_t^{D(NC)} \right) = \left( 1 + i_t^{D(SCCI)} \right) \), and \( S_{t,D}^{SCCI} < \max S_{t,F}^{SCCI} = \max S_{t,D}^{NC} < S_{t,D}^{NC} \), this symmetric capital controls could overcome the liquidity shortfalls and prevent banking crises which cannot be achieved in case NC.

3.3.2 Asymmetric capital controls

Domestic controls on outflows vs. no foreign controls (ACON)  In this case, the domestic country sets up the ceiling \( (1 - \gamma_t^{ACON}) < (1 - \gamma_t^{NC}) \) for each domestic investor while no foreign control. Under the circumstance where \( R_{t,DF}^M \geq R_{t,DD}^M + \xi_t^D \), all foreign investors would re-invest in the foreign country, but only a fraction \( (1 - \gamma_t^{ACON}) \) of domestic investors would re-invest in the
foreign country. The volumes of capital flows of the domestic country are:

\[ CO_{t,F}^{ACON} = (1 - \gamma_t^{ACON}) R^D_t, CI_{t,F}^{ACON} = 0, \]

where the volume of capital outflows is less than the case NC, \( CO_{t,F}^{ACON} < CO_{t,F}^{NC} \) since \( (1 - \gamma_t^{ACON}) < (1 - \gamma_t^{NC}) \). The restriction on the volume of capital outflows is from the domestic controls. The available loanable fund and equity fund are \( S_{t,F}^{ACON} = (1 - \alpha_t^I) w_t N + \gamma_t^{ACON} (1 - \alpha_t^{IM}) R^D_t \) and \( I_{t,F}^{SCCI} = \alpha_t^I w_t N + \gamma_t^{ACON} \alpha_t^{IM} R^D_t \). These funds are more than the case NC, \( S_{t,F}^{ACON} > S_{t,F}^{NC} \) and \( I_{t,F}^{ACON} > I_{t,F}^{NC} \). The increase in the amount of equity fund would decrease the equity rate \( \left(1 + i^{E(t,ACON)}_{t+1}\right) \), and decrease the values of \( \alpha_{t+1}^{IM} \) and/or \( \alpha_t^I \). This means that the composition of deposit would shift away from the equity fund.

Under the circumstance where \( R_{t,F}^{IM} = R_{t,F}^{IM} + \xi_t^{F} \), all domestic investors and a fraction \( (1 - \gamma_t^{ACON^*}) \) of foreign investors would re-invest in the domestic country. The volumes of capital flows of the domestic country are:

\[ CO_{t,D}^{ACON} = 0, CI_{t,D}^{ACON} = (1 - \gamma_t^{ACON^*}) R^F_t, \]

where the volume of capital inflows is the same as the case NC, \( CI_{t,D}^{ACON} = CI_{t,D}^{NC} \) since \( (1 - \gamma_t^{ACON^*}) = (1 - \gamma_t^{NC^*}) \). The amount of loanable fund and equity fund are the same as the case NC, \( S_{t,D}^{ACON} = S_{t,D}^{NC} = (1 - \alpha_t^I) w_t N + \left(1 - \alpha_t^{IM} \right) R^D_t + (1 - \gamma_t^{ACON^*}) \left(1 - \alpha_t^{IM} \right) R^F_t \), and \( I_{t,D}^{ACON} = I_{t,D}^{NC} = \alpha_t^I w_t N + \alpha_t^{IM} \alpha_t^{IM*} R^D_t + (1 - \gamma_t^{ACON^*}) \alpha_t^{IM*} R^F_t \). Therefore, the equity rate \( \left(1 + i^{E(t,SCCO)}_{t+1}\right) \) would remain the same as before imposing capital controls, and so do the values of \( \alpha_t^{IM} \), \( \alpha_t^{IM} \) and \( \alpha_t^I \). This means that there is no change on the composition of the deposits. Note that when \( R_{t,F}^{IM} > R_{t,F}^{IM} + \xi_t^{F} \), all foreign investors would re-invest in the domestic country, \( \gamma_t^{ACON^*} = 0 \), and the volume of capital inflows would change to \( CI_{t,D}^{ACON} = R^F_t \). This is also the same as the case without controls.

**Banking Crises** At the time of the adverse shock \( (\varepsilon_t) \), the financial intermediaries face the liquidity shortfalls \( SF_t \). By offering the deposit rate \( \left(1 + i^{D(t,SCCI)}_{t}\right) > 1 \), the financial intermediaries attract the deposits from the young \( S_{t,y}^{ACON} \). When \( S_{t,y}^{ACON} \geq SF_t \), the financial intermediaries overcome \( SF_t \) and prevent the immediate bank runs. However, whether the liquidity shortfalls at
the following period $t+1$ can be overcome would depend on the the expected return rates, the amount of demand deposits, and the loan rate.

When $R_{t,DF}^{IM} \geq R_{t,DD}^{IM} + \xi_D^D$, and $S_{t,y}^{ACON} = SF_t$, the available loanable fund after paying $SF_t$ becomes

$$S_{t,F}^{ACON} - SF_t = (1 - \alpha_t^{IM}) \hat{z}_{t}^{ACON} RI_t^D,$$

where $\left(S_{t,F}^{ACON} - SF_t\right) > \left(S_{t,F}^{NC} - SF_t\right)$. According to equation (19), $\min\left(1 + i_t^{loan(ACON)}\right) < \min\left(1 + i_t^{loan(NC)}\right)$. Compared to equation (11), when $\min\left(1 + i_t^{loan(ACON)}\right) < \min\left(1 + i_t^{loan(NC)}\right)$, this asymmetric capital controls, ACON, could overcome the liquidity shortfalls, $SF_{t+1}^{ACON}$, which cannot be achieved in NC.

When $R_{t,FD}^{IM} = R_{t,FF}^{IM} + \xi_F^F$ and $S_{t,y}^{ACON} = SF_t$, the available loanable fund after paying $SF_t$ would change to

$$S_{t,D}^{ACON} - SF_t = (1 - \alpha_{t,DD}^{IM}) RI_t^D + (1 - \gamma_{t}^{ACON*}) (1 - \alpha_{t,FD}^{IM}) RI_t^F,$$

where $\left(S_{t,D}^{ACON} - SF_t\right) = \left(S_{t,D}^{NC} - SF_t\right)$ since $\gamma_{t}^{ACON*} = \gamma_{t}^{NC*}$ and $S_{t,D}^{ACON} = S_{t,D}^{NC}$. Moreover, when $R_{t,FD}^{IM} > R_{t,FF}^{IM} + \xi_F^F$, $\gamma_{t}^{ACON*} = \gamma_{t}^{NC*} = 0$, and $S_{t,D}^{ACON} = S_{t,D}^{NC}$. According to equation (21) with $k = ACON$, when $\left(1 + i_t^{D(NC)}\right) = \left(1 + i_t^{D(ACON)}\right)$, $S_{t,D}^{ACON} = S_{t,D}^{NC} < \max S_{t,D}^{ACON}$, the ability of capital control in overcoming the liquidity shortfalls and in preventing banking crises is the same as the case NC.

**Domestic controls on outflows vs. foreign controls on inflows (ACOI)** In this case, the controls are all for the domestic investors. While the domestic country sets up the ceiling $(1 - \hat{\gamma}_{t}^{ACOI}) < (1 - \gamma_{t}^{NC})$, the foreign country sets up the ceiling $(1 - \hat{\gamma}_{t}^{ACOI}) < (1 - \gamma_{t}^{ACOI})$ for each domestic investor. When $(1 - \hat{\gamma}_{t}^{ACOI}) < (1 - \gamma_{t}^{ACOI})$, the domestic controls are more restrictive for the domestic investors to invest in the foreign country. Under the circumstance where $R_{t,DF}^{IM} \geq R_{t,DD}^{IM} + \xi_D^D$, all foreign investors would re-invest in the foreign country but only a fraction $(1 - \hat{\gamma}_{t}^{ACOI})$ of domestic investors would re-invest in the foreign country. The volumes of capital flows of the domestic country are:

$$CO_t^{ACOI} = (1 - \gamma_{t}^{ACOI}) RI_t^D, CI_t^{ACOI} = 0,$$
where the volume of capital outflows is less than the case NC, \( CO_{t,F}^{ACOI} < CO_{t,F}^{NC} \) since \((1 - \gamma_{t}^{ACOI}) < (1 - \gamma_{t}^{NC})\). The available loanable and equity funds are \( S_{t,F}^{ACOI} = (1 - \alpha_{t}^{l}) w_{1}N + \gamma_{t}^{ACOI} \left( 1 - \alpha_{t,DD}^{IM} \right) R_{t}^{D} \)
and \( I_{t,F}^{ACOI} = \alpha_{t}^{l} w_{1}N + \gamma_{t}^{ACOI} \alpha_{t,DD}^{IM} R_{t}^{D} \), which are more than the case NC, \( S_{t,F}^{ACOI} > S_{t,F}^{NC} \) and \( I_{t,F}^{ACOI} > I_{t,F}^{NC} \). Therefore, both the equity rate \( (1 + i_{t+1}^{(ACOI)}) \), and the values of \( \alpha_{t+1,DD}^{IM} \) and/or \( \alpha_{t+1}^{l} \) would decrease. This means that the composition of deposit would shift away from the equity fund.

Under the circumstance where \( R_{t,F}^{IM} = R_{t,F}^{IM} + \xi_{t}^{F} \), all domestic investors and a fraction \((1 - \gamma_{t}^{ACOI})\) of foreign investors would re-invest in the domestic country. So the volumes of the capital flows of the domestic country would be:

\[
CO_{t,D}^{ACOI} = 0, CI_{t,D}^{ACOI} = (1 - \gamma_{t}^{ACOI}) R_{t}^{F}.
\]

Since \((1 - \gamma_{t}^{ACOI}) = (1 - \gamma_{t}^{NC})\), the volumes of inflows would be the same as the case NC, \( CI_{t,D}^{ACOI} = CI_{t,D}^{NC} \), and so do the amounts of loanable fund and equity fund: \( S_{t,D}^{ACOI} = S_{t,D}^{NC} = (1 - \alpha_{t}^{l}) w_{1}N + (1 - \alpha_{t,DD}^{IM}) R_{t}^{D} \), \( I_{t,D}^{ACOI} = I_{t,D}^{NC} = \alpha_{t}^{l} w_{1}N + \alpha_{t,DD}^{IM} R_{t}^{D} \). Consequently, the equity rate would remain the same as the case before imposing controls, \( 1 + i_{t+1,D}^{E(ACOI)} = 1 + i_{t+1,D}^{E(NC)} \), and the the values of \( \alpha_{t,F,D}^{IM} \) and \( \alpha_{t,DD}^{IM} \) do not change. Similar to case NC, when \( R_{t,F}^{IM} > R_{t,F}^{IM} + \xi_{t}^{F} \), all foreign investors would re-invest in the domestic country, \( \gamma_{t}^{ACON} = 0 \), and capital inflows would change to \( CI_{t,D}^{ACON} = R_{t}^{F} \).

**Banking Crises** At the time of the adverse shock (\( \varepsilon_{t} \)), the financial intermediaries face the liquidity shortfalls \( SF_{t} \). By offering the deposit rate \( 1 + i_{t}^{D(ACOI)} > 1 \) to attract the deposit from the young, the financial intermediaries could overcome \( SF_{t} \) and prevent immediate bank runs when \( S_{t,y}^{ACON} \geq SF_{t} \). Then it depends on the expected return rate and the loan rate to overcome the liquidity shortfalls at the following period \( t+1 \), \( SF_{t+1}^{ACOI} \), and to prevent the banking crises at period \( t+1 \).

When \( R_{t,F}^{IM} \geq R_{t,F}^{IM} + \xi_{t}^{D} \), and \( S_{t,y}^{ACOI} = SF_{t} \), the available loanable fund after paying \( SF_{t} \) becomes

\[
S_{t,F}^{ACOI} - SF_{t} = (1 - \alpha_{t}^{IM}) \gamma_{t}^{ACOI} R_{t}^{D},
\]

\(^7\)The reason why volumes of the flows is restricted by the domestic controls is because \((1 - \gamma_{t}^{ACOI}) < (1 - \gamma_{t}^{ACOI})\). When \((1 - \gamma_{t}^{ACOI}) > (1 - \gamma_{t}^{ACOI})\), the volumes of flows would be restricted by the foreign controls.
where \( S_{t,F}^{ACOI} - SF_t > (S_{t,F}^{NC} - SF_t) \). According to equation (19), \( \min \left( 1 + i_t^{loan(ACOI)} \right) < \min \left( 1 + i_t^{loan(NC)} \right) \). Compared to equation (11), when \( \min \left( 1 + i_t^{loan(ACOI)} \right) < \max \left( 1 + i_t^{loan(NC)} \right) \), the asymmetric capital controls again could overcome the liquidity shortfalls at period \( t + 1 \), which may not be achieved in the case without capital controls.

When \( R_{t,F}^{IM} = R_{t,F}^{IM*} + \xi_t^F \), and \( S_{t,\gamma}^{ACOI} = SF_t \), the available loanable fund after paying \( SF_t \) would change to

\[
S_{t,D}^{ACOI} - SF_t = (1 - \alpha_{t,DD}^{IM}) R_{t}^{D} + \left( 1 - \gamma_t^{ACOI*} \right) (1 - \alpha_{t,F}^{IM*}) R_{t}^{F}.
\]

Since \( \gamma_t^{ACOI*} = \gamma_t^{NC*}, S_{t,D}^{ACOI} = S_{t,D}^{NC} \). Similarly, when \( \left( 1 + i_t^{D(NC)} \right) = \left( 1 + i_t^{D(ACOI)} \right) \) and \( S_{t,D}^{ACOI} = S_{t,D}^{NC} < \max S_{t,D}^{ACOI} = \max S_{t,D}^{NC} \) [equation (21) with \( k = ACOI \)], the ability of capital control in overcoming the liquidity shortfalls and in preventing banking crises is the same as the case without capital controls. Under the circumstance where \( R_{t,F}^{IM} > R_{t,F}^{IM*} + \xi_t^F \), \( \gamma_t^{ACOI} = \gamma_t^{NC} \) and the conclusion remains the same.

**Domestic controls on inflows vs. no foreign controls (ACIN)** In this case, the domestic country sets up the ceiling \( (1 - \gamma_t^{ACIN*}) < (1 - \gamma_t^{NC*}) \) for each foreign investor and there is no foreign controls. Under the circumstance where \( R_{t,F}^{IM*} = R_{t,F}^{IM*} + \xi_t^D \), all foreign investors would re-invest in the foreign country but only a fraction \( (1 - \gamma_t^{ACIN}) \) of domestic investors would re-invest in the domestic country. The volumes of capital flows of the domestic country are:

\[
CO_{t,F}^{ACIN} = (1 - \gamma_t^{ACIN}) R_{t}^{D} + CI_{t,F}^{ACOI} = 0,
\]

Since \( \gamma_t^{ACIN} = \gamma_t^{NC} \), the volume of capital outflows is the same as case NC, \( CO_{t,F}^{ACIN} = CO_{t,F}^{NC} \), and so do the loanable and equity funds, \( S_{t,F}^{ACIN} = S_{t,F}^{NC} = (1 - \alpha_t) w_t N + \gamma_t^{ACIN} (1 - \alpha_{t,DD}^{IM}) R_{t}^{D} \), and \( I_{t,F}^{ACIN} = I_{t,F}^{NC} = \alpha_t w_t N + \gamma_t^{ACIN} \alpha_{t,DD}^{IM} R_{t}^{D} \). As a result, the equity rate and the values of \( \alpha_{t,DD}^{IM} \) and/or \( \alpha_t \) would remain the same as the case NC. In other words, the composition of flows does not change after imposing controls on inflows in this case. Under the circumstance where \( R_{t,F}^{IM*} > R_{t,F}^{IM*} + \xi_t^D, \gamma_t^{ACIN} = 0 \), and the volume of capital outflow changes to \( CO_{t,F}^{ACIN} = R_{t}^{D} \).

Under the circumstance when \( R_{t,F}^{IM} > R_{t,F}^{IM*} + \xi_t^F, \forall \alpha_{t,j}^{IM*} \in [0, 1], j = D, F, \) all domestic investors and a fraction \( (1 - \gamma_t^{ACIN*}) \) of foreign investors would re-invest in the domestic country.
So capital flows of the domestic country would be:

\[ CO_{t,D}^{ACIN} = 0, CI_{t,D}^{ACIN} = (1 - \gamma_{t}^{ACIN*}) R_{t}^{F}, \]

where the volume of capital inflows is less than the case NC, \( CI_{t,D}^{ACIN} < CI_{t,D}^{NC} \) since \((1 - \gamma_{t}^{ACIN*}) < (1 - \gamma_{t}^{NC*})\). Accordingly, the loanable fund and the equity fund reduced to \( S_{t,D}^{ACIN} = (1 - \alpha_{t}^{lM}) w_{t} N + (1 - \alpha_{t,DD}^{lM}) R_{t}^{D} + (1 - \gamma_{t}^{ACIN*}) (1 - \alpha_{t,FD}^{lM}) R_{t}^{F} \), and \( I_{t,D}^{ACOI} = I_{t,D}^{NC} = \alpha_{t}^{l} w_{t} N + \alpha_{t,DD}^{lM} R_{t}^{D} + (1 - \gamma_{t}^{ACIN*}) \alpha_{t,FD}^{lM} R_{t}^{F} \), which are less than the case NC, \( S_{t,D}^{ACIN} < S_{t,D}^{NC} \) and \( I_{t,D}^{ACIN} < I_{t,D}^{NC} \). The decrease in the equity fund would drive up the equity rate and increase the values of \( \alpha_{t,FD}^{lM}, \alpha_{t,DD}^{lM} \) and \( \alpha_{t}^{l} \). That is, the composition of deposit would shift towards to the equity market.

**Banking Crises** At the time of the adverse shock \((\epsilon_{t})\), the financial intermediaries face the liquidity shortfalls \( SF_{t} \). By offering the deposit rate \((1 + i_{t}^{D(ACIN)}) > 1\) to attract the deposit from the young, the financial intermediaries could overcome \( SF_{t} \) and prevent immediate bank runs when \( S_{t,F}^{ACIN} \geq SF_{t} \). Then it depends on the expected return rate, the amount of demand deposits, and the loan rate to overcome the liquidity shortfalls at the following period \( t + 1 \) and to prevent the banking crises at period \( t + 1 \).

When \( R_{t,DF}^{lM} = R_{t,DD}^{lM} + \xi_{t}^{D} \), and \( S_{t,F}^{ACIN} = SF_{t} \), the available loanable fund after paying \( SF_{t} \) becomes

\[ S_{t,F}^{ACIN} - SF_{t} = (1 - \alpha_{t}^{lM}) \gamma_{t}^{ACIN} R_{t}^{D}, \]

where \( (S_{t,F}^{ACIN} - SF_{t}) = (S_{t,F}^{NC} - SF_{t}) \) since \( \gamma_{t}^{ACIN} = \gamma_{t}^{NC} \). According to equation (19), \( \min(1 + i_{t}^{loan(ACIN)}) \leq \left(1 + i_{t}^{loan(ACIN)}\right) \). Therefore, the ability of this asymmetric capital controls, ACIN to overcome the liquidity shortfalls, \( SF_{t+1}^{ACIN} \), and to prevent banking crises is the same as the case NC. Similar to case NC, when \( R_{t,DF}^{lM} > R_{t,DD}^{lM} + \xi_{t}^{D} \), \( \gamma_{t}^{ACOI} = \gamma_{t}^{NC} = 0 \) and \( S_{t,F}^{ACIN} - SF_{t} = 0 \) which drives up \( \min(1 + i_{t}^{loan(ACIN)}) \rightarrow \infty \). This means that the liquidity shortfalls at period \( t + 1 \) cannot be overcome and bank will runs at the period \( t + 1 \).

When \( R_{t,FD}^{lM} = R_{t,FF}^{lM} + \xi_{t}^{F} \) and \( S_{t,F}^{ACIN} = SF_{t} \), the available loanable fund after paying \( SF_{t} \) would change to

\[ S_{t,D}^{ACIN} - SF_{t} = (1 - \alpha_{t,DD}^{lM}) R_{t}^{D} + (1 - \gamma_{t}^{ACIN*}) (1 - \alpha_{t,FD}^{lM}) R_{t}^{F}, \]
where \( \left( S_{t,D}^{ACIN} - SF_t \right) < \left( S_{t,D}^{NC} - SF_t \right) \). Equation (21) with \( k = ACIN \) shows that when 
\[
\left( 1 + i_t^{D(\text{NC})} \right) = \left( 1 + i_t^{D(\text{ACIN})} \right) \text{ and } S_{t,D}^{ACIN} < \max S_{t,D}^{ACIN} = \max S_t^{NC} < S_t^{NC},
\]
this asymmetric capital controls could overcome the liquidity shortfalls and prevent banking crises, which cannot be achieved in the case NC.

4 Discussion

The results of all cases are summaried in Table 1 and Table 2. Table 1 shows the results of the circumstance where the expected foreign return rates are more attractive while Table 2 shows the results of the circumstance where the expected domestic return rates are more attractive. In different circumstances, the financial intermediaries face different challenges to overcome the liquidity shortfalls and to prevent banking crises. Since it is the demand deposits (saving accounts) that are crucial to bank runs, the discussion will focus on the saving accounts (loanable funds).

In Table 1, when the expected foreign return rate is more attractive, the domestic country lose the deposits to the foreign country. The lower amount of loanable fund would drive up the loan rate in order to repay the demand deposits. However, the loan rate has its maximum value which the entrepreneurs could accept. Therefore, any required loan rate which exceeds the maximum loan rate means that no entrepreneur would apply for the loan. Without loan repayment at the following period, the financial intermediaries cannot repay the demand deposits, and bank runs will be the result. So in Table 1, it is the required loan rate which is compared to the maximum loan rate and the loan rate in case NC to identify the ability to overcome the liquidity shortfalls.
\[ R^{IM*}_{t,DF} = R^{IM}_{t,DD} + \xi^D_t \]

Volumes

\begin{align*}
\text{Case} & \quad \text{Composition} & \text{loan rate} \\
\text{CL} & \quad (1 - \gamma^{NC}_t) / 1 & 0 & 0 & \alpha^{IM}_{t+1,DD} & \downarrow & \max (1 + i^{loan}_t) \\
\text{NC} & \quad 0 & 0 & 0 & \alpha^{IM}_{t+1,DD} & \downarrow & \left(1 + i^{loan}(NC)ight) < \left(1 + i^{loan}(CL)\right) \\
\text{SCCO} & \quad (1 - \gamma^{SCCO}_t) & 0 & 0 & \alpha^{IM}_{t+1,DD} & \downarrow & \left(1 + i^{loan(SCCO)} \right) < \left(1 + i^{loan}(NC)\right) \\
\text{SCCI} & \quad (1 - \gamma^{SCCI}_t) & 0 & 0 & \alpha^{IM}_{t+1,DD} & \downarrow & \left(1 + i^{loan(SCCI)} \right) < \left(1 + i^{loan}(NC)\right) \\
\text{ACON} & \quad (1 - \gamma^{ACON}_t) & 0 & 0 & \alpha^{IM}_{t+1,DD} & \downarrow & \left(1 + i^{loan(ACON)} \right) < \left(1 + i^{loan}(NC)\right) \\
\text{ACOI} & \quad (1 - \gamma^{ACOI}_t) & 0 & 0 & \alpha^{IM}_{t+1,DD} & \downarrow & \left(1 + i^{loan(ACOI)} \right) < \left(1 + i^{loan}(NC)\right) \\
\text{ACIN} & \quad (1 - \gamma^{ACIN}_t) / 1 & 0 & 0 & \alpha^{IM}_{t+1,DD} & \downarrow & -/ \left(1 + i^{loan(ACIN)} \right) \rightarrow \infty > \max (1 + i^{loan}_t) \\
\end{align*}

Table 1. When expected foreign rate is relatively attractive: \( R^{IM*}_{t,DF} \geq R^{IM}_{t,DD} + \xi^D_t \).

As shown in Table 1, when the expected foreign return rate is relatively attractive, except for case ACIN, both symmetric and asymmetric controls would affect the volume of capital outflows. It is important to note that the volume of capital outflows would be restricted by the domestic country when there is domestic capital controls on outflows. The volume of capital outflows, however, would be restricted by the foreign country when there is no domestic capital control or when domestic controls are on inflows. Regarding to the composition of the deposits, the decrease in capital outflows due to the controls would increase both the loanable and the equity funds. Since the deposit rate is pre-determined, the equity rate is the only rate which could reflect the change of the amount of funds. The increase in the equity fund would reduce the equilibrium equity rate and lower the fraction \( \alpha^{IM}_{t+1,DD} \) which the domestic investors would place in the equity market at the following period. Meanwhile, the changes on the volume of capital outflows and the composition in case ACIN is the same as the case NC. In terms of the ability to overcome liquidity shortfalls and to prevent banking crises, the cases with capital controls, except the case ACIN, require a lower loan rate to meet the demand deposits, and are more likely to overcome the liquidity shortfalls and to prevent banking crises than the case without controls. The case ACIN’s ability in overcoming the liquidity shortfalls is the same as the case without capital controls.
In Table 2, when the expected domestic return rate is more attractive, the domestic country would receive the deposits from the domestic and the foreign investors. Provided the maximum value of the loan rate, the amount of demand deposits which the domestic financial intermediaries could afford has its maximum. Therefore, if the domestic financial intermediaries accept deposits which are more than what they can afford, the bank runs will be the result. So in Table 2, it is the amount of accepted deposits compared to the affordable deposits.

\[ R_{t,FD}^{IM} = R_{t,FF}^{IM} + \xi_t / R_{t,FD}^{IM} > R_{t,FF}^{IM} + \xi_t \]

<table>
<thead>
<tr>
<th>Volumes</th>
<th>equity rate</th>
<th>deposit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case</td>
<td>COt</td>
<td>CI : RIF</td>
</tr>
<tr>
<td>CL</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>NC</td>
<td>0</td>
<td>( (1 - \gamma_t^{NC*)} / 1 )</td>
</tr>
<tr>
<td>SCCO</td>
<td>0</td>
<td>( (1 - \gamma_t^{SCCO*)} )</td>
</tr>
<tr>
<td>SCCI</td>
<td>0</td>
<td>( (1 - \gamma_t^{SCCI*)} )</td>
</tr>
<tr>
<td>ACON</td>
<td>0</td>
<td>( (1 - \gamma_t^{ACON*)} / 1 )</td>
</tr>
<tr>
<td>ACOI</td>
<td>0</td>
<td>( (1 - \gamma_t^{ACOI*)} / 1 )</td>
</tr>
<tr>
<td>ACIN</td>
<td>0</td>
<td>( (1 - \gamma_t^{ACIN*)} )</td>
</tr>
</tbody>
</table>

Table 2. When expected domestic return rate is relatively attractive: \[ R_{t,FD}^{IM} \geq R_{t,FF}^{IM} + \xi_t \]

As shown in Table 2, when the expected domestic rate is relatively attractive, both symmetric controls are effective in reducing the volume of inflows and in affecting the composition of inflows. Except for case ACIN, the other two cases of asymmetric controls have no effects on either the volume or the composition of capital flows. Among the three cases which are effective in affecting the volume and the composition of flows, the volume of capital inflows is restricted by domestic controls in cases SCCI and ACIN, but it is restricted by foreign controled in case SCCO. The decrease in the volume of inflows would lower the amount of equity fund and drive up the equity rate at the following period. This increase in the equity rate would, in turn, shift the composition of inflows towards to the equity market, and increase \( \alpha_{t+1,FD}^{IM} \).

Regarding to the ability to overcome liquidity shortfalls and to prevent banking crises, when the domestic deposit rate offered in the open economy is higher than that in the closed economy,
the amount of saving which the domestic financial intermediaries could afford in an open economy is less than that in a closed economy, \( \max S_{NC}^{D} < \max S_{CL}^{D} \). Compared to case NC, the cases in which the controls are effective in affecting the volume and the composition of flows, such as SCCO, SCCI, and ACIN, can overcome the liquidity shortfalls and prevent banking crises which case NC cannot achieve. The ability of the economy in the cases in which the controls have no effects on either the volume or the composition of flows perform the same as the case without controls in terms of overcoming liquidity shortfalls and preventing banking crises.

Coming both tables, symmetric controls, whether both countries have inflows or outflows, demonstrate the effectiveness on affecting both the volume and the composition of flows, regardless of which country has a relatively high expected return rate. However, whether the asymmetric controls are effective depends on which country has a higher expected return rate. Controls on outflows are effective when the foreign country has a more attractive expected return rate while controls on inflows are effective when the domestic country has a more attractive expected return rate.

Most interestingly, the majority of cases with controls could overcome liquidity shortfalls and prevent banking crises which the case without controls cannot achieve. To be more specific, when the expected foreign return rate is relatively attractive, four out of five cases with controls could perform better in terms of overcoming liquidity shortfalls. The only case which performs similarly to the case without controls is the one controlling on inflows asymmetrically. When the expected domestic return rate is relatively attractive, there are three out of five cases with controls which could perform better than the case without control in terms of overcoming the liquidity shortfalls and preventing banking crises. The two cases which perform similarly to the case without controls are the ones controlling output asymmetrically.

5 CONCLUSION & EXTENSIONS

Capital controls have been considered and adopted to protect the country from international finance crises and to change both the volume and the composition of capital flows. This paper examines capital controls on both outflows and inflows, and finds that capital controls on outflows and inflows
may or may not achieve the objectives on changing the volume and the composition of capital flows. Moreover, the ability of capital controls on protecting the economy from banking crises and sudden stops is limited.

To be more specific, controls on capital outflows and inflows could both change the volume of capital flows at the time when the controls are imposed. However, the ability of the controls on changing composition of capital flows and to protect the country from banking crises and sudden stops is limited. Taking controls on capital outflows as an example, composition of capital flows would change only when one of the return rates has gone below the lower limits. Sudden stops could be prevented when foreign banks run, but the domestic countries have remedied the liquidity problems. Therefore, sudden stops can be prevented due to the remedy on liquidity problems, not due to the controls on capital outflows. The controls on capital inflows could cause two opposite effects on the return rates of the credit markets. When the two effects offset each other and leave the return rates unchanged, the composition on capital flows may remain the same. Whether the controls on capital inflows could prevent immediate bank runs and sudden stops depend on whether the liquidity problems of the financial intermediaries can be overcome. To prevent future bank runs, the financial intermediaries must provide the affordable rates, rather than competitive rates. Therefore, it is not the capital controls on inflows that could protect the economy from sudden stops, but the remedies that could overcome the liquidity problems.

There are several limitation of this paper, which can be extended in the future. The direct extension is to discuss various types of capital controls together with different types of international shocks, as well as contagion effects. Moreover, introducing currencies would allow the discussion on currency risks, which capital controls could affect in some aspects.
References


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Figure 3: The decision making of the young individuals

Figure 4: The decision making of middle-aged individuals
Figure 5: The returns from production (the loan market)