Modelling Secular Variation in the Southwest Pacific for the last 400 years

by

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Modelling Secular Variation in the Southwest Pacific for the last 400 year
DEDICATION

To wonderful Parents and great husband, who set me on my path, I am profoundly grateful to them for their love, prayers and constant support and encouragements. Also I dedicate this thesis to my lovely daughter Maryam and lovely son Laith, wishing them a life full of success and happiness.
A spherical cap harmonic analysis (SCHA) model has been used to derive a high-resolution regional model of the geomagnetic field in the southwest Pacific region over the past 400 years. Two different methods, a self-consistent and the gufm1 dipole method, have been used to fill in gaps in the available data.

The data used in the analysis were largely measurements of the magnetic field recorded in ships logs on voyages of exploration in the region. The method chosen for the investigation used a spherical cap of radius $\theta_0 = 50^\circ$ centered at co-latitude and longitude of $(115^\circ, 160^\circ)$. The results of each method used for SCHA are presented as contour plots of magnetic field declination, inclination and intensity and are compared with similar plots for a global model, gufm1. The root mean square misfit of the self-consistent and gufm1 dipole model to the actual data were around 2900 nT and 23000 nT respectively.

Overall, the results suggest that the self-consistent model produces a more reliable model of the geomagnetic field within the area of interest than does the gufm1 dipole model. With more data included the self-consistent model could be further improved and used to develop a high-resolution mathematical model of the geomagnetic field in the southwest Pacific region.
ACKNOWLEDGEMENTS

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Maha Ali Alfheid
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<td><strong>H</strong></td>
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<td><strong>J</strong></td>
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CHAPTER 1

INTRODUCTION

1.1 Background to the study

William Gilbert was the first person who identified, in his book De Magnete, that the Earth’s magnetic field is a property of the Earth itself. It originates from an active, self-sustaining dynamo operating in the liquid outer core of metallic composition (Mattis, 1965). As explained further in Merrill, McElhinny and McFadden (1998), the Earth’s magnetic field can be approximated by a magnetic dipole tilted at an angle of about 11 degrees from the Earth’s rotational axis. Such a dipole accounts for roughly 90% of the present day geomagnetic field at any point on the Earth’s surface. The measured field shows that the dipole is oriented towards the south rotation pole, so that the field has an upward component in the southern hemisphere and a downward component in the northern hemisphere. The remaining approximately 10% of the field is termed the non-dipole field. Both the dipole and the non-dipole fields contribute to the overall pattern of the Earth’s magnetic field and they both vary with time (Merrill, McElhinny, & McFadden, 1998).
The Earth’s magnetic field at any location on the surface of the Earth is a vector which can be represented in terms of three parameters: Declination “D”, Inclination “I” and Intensity “F” as shown in Fig 1.1, which comes from Merrill, McElhinny and McFadden (1998,p20).

The declination is the angle between the horizontal component of the magnetic field and true north, the inclination is the angle the field makes with the horizontal and “F” is the intensity or magnitude of the field. Inclination is $90^\circ$ at the north magnetic Pole and $-90^\circ$ at the south magnetic pole. Due to the changing non-dipole field, the north and south magnetic poles move around independently of each other and they are not directly opposite each other.

Maxwell’s equations of electromagnetism can be used to obtain a mathematical model to describe the geomagnetic field. Two reasonable assumptions are that the atmosphere is an insulator and that it is non-magnetic. As a result the geomagnetic field is normally measured in a region,
between the surface of the Earth and the ionosphere, in which there are no electric currents and no magnetic sources. In this region, where there are no currents or sources of the magnetic field, the field can be expressed as the gradient of a scalar magnetic potential “U”, such that

\[ B = -\nabla U \]  

(1.1)

The Maxwell equation, stating that

\[ \text{div } B = 0 \]  

(1.2)

therefore implies that the scalar magnetic potential obeys Laplace’s equation at the surface of the Earth.

\[ \nabla^2 U = 0 \]  

(1.3)

Solving Laplace’s equation in spherical polar coordinates \((r, \theta, \phi)\) gives a method of modeling the magnetic scalar potential of the Earth’s magnetic field. Although this is only strictly applicable in the region between the surface and the ionosphere, it is often also applied within the Earth, for example to look at the field on the core-mantle boundary. This involves the assumption that the mantle can be treated as an insulator. Solving Laplace’s equation can also, provide a way of modeling the vector field on a restricted area on the surface of the earth.

The main field of the Earth is entirely of internal origin and the solution of Laplace’s equation in spherical polar coordinates, known as a Spherical Harmonic Analysis (SHA) is in the form:
where $r$ is the distance of the observational point from the center of the Earth and $\theta, \phi$ are the colatitude and longitude of the observational point, respectively, $a$ is the radius of the Earth, and $g_l^m$ and $h_l^m$ have the same dimensions as $B$ (i.e. SI units are Tesla) and are referred to as Gauss coefficients. The $p_l^m(cos\theta)$ are Schmidt-normalized associated Legendre polynomials, and divide the meridian, or longitudinal line, into $l - m + 1$ zones of alternate signs. The $\cos m\phi$ or $\sin m\phi$ terms divide the longitudinal line into $2m$ longitudinal sectors of alternate signs at equal intervals $\pi/m$. The product of the Legendre polynomials with the $\cos m\phi$ or $\sin m\phi$ terms divide up the surface of the sphere into regions created by the latitude zones and longitude sectors. It also gives surface spherical harmonics which vary with $\theta$ and $\phi$ with degree $l$ and order $m$ and show the symmetry of the various contributions to the geomagnetic field at the surface of the Earth (Parkinson, 1983).

Each harmonic is equivalent to a particular arrangement of magnetic poles at the center of the Earth. The lowest degree of Gauss coefficient is $g_0^0$, which would correspond to $l=0$ and $m=0$, i.e. a monopole. As $\text{div } B = 0$ implies that isolated magnetic monopoles do not exist, this term does not appear in equation 1.4 in which the summation starts at $l=1$. The next three coefficients $g_1^0, g_1^1$ and $h_1^1$ define the direction and magnitude of the geocentric dipole, which is equivalent to two opposing charges brought close together (Parkinson, 1983). These terms give the first approximation to the observed geomagnetic field. The spherical harmonic terms with $l=2$ describes the best fitting geocentric quadrupole, which is equivalent to two dipoles brought together, and the terms with $l=3$ describe the best fitting geocentric octupole and so on for higher degree terms(Merrill et al., 1998).

Gradual changes of the Earth’s magnetic field on a time scale of a year or more are referred to as geomagnetic secular variation. The variations of the magnetic field on a time scale shorter than
about a year are caused by sources of external origin, largely due to the changing intensity of the solar wind. The longer time scale variations are of internal origin due to the continual motion of the Earth’s fluid outer core (Bloxham & Gubbins, 1985). Secular variation can be described by continuous changes of small amplitude with periodicities ranging from a year to 100,000 years. However, the polarity of the main dipole reverses on time scales from hundreds of thousands to a million years, changing the north pole to the south pole and the south pole to the north pole (MacMillan, 1958; Parkinson, 1983). Henry Gellibrand was the first to note the fact that the geomagnetic field is not constant. He found that the declination in London had decreased from $11.3^\circ$ E to $4.1^\circ$ E between 1580 and 1634.

The historical dataset describes secular variation in terms of three phenomena. First, a steady decay in the magnetic moment of the dipole, which is described by the derivative $\frac{\partial g_0^1}{\partial t}$, $\frac{\partial g_1^1}{\partial t}$, and $\frac{\partial h_1^1}{\partial t}$. Second, an overall westward drift of the non-dipole field with a drift rate estimated by Bullard (1950) of about $0.18^\circ / \text{yr}$. Third, a slow westward movement of the geomagnetic poles, which is described by $\frac{\partial g_0^2}{\partial t}$. The magnitude of the secular variation is also observed to be generally smaller over the Pacific hemisphere and the non-dipole field weaker (Parkinson, 1983).

The standard mathematical model of the geomagnetic field, called the International Geomagnetic Reference Field (IGRF), is revised every five years and is based on Spherical Harmonic Analysis (SHA), which is described in more detail in chapter two.

This thesis concerns the southwest Pacific region and deals with developing a mathematical model of the geomagnetic field specific to this area. Models of the Earth’s magnetic field from SHA using only observations in the southwest Pacific are of low accuracy, while models determined using SHA with a full global data are of low resolution in the southwest Pacific(Haines, 1985a). Therefore, a Spherical Cap Harmonic Analysis (SCHA), which is a technique specifically designed to produce a model only for a local region will be used. Over
small or large areas from a few to many millions of square kilometer, SCHA is preferable to SHA for modeling the magnetic field in a restricted region (Leroy R. Alldredge, 1983). The spherical cap corresponds to an area defined around a central selected point by a predetermined angle $\alpha$ subtended at the center of Earth. The spherical cap in this study chosen covers the southwest Pacific region has a radius of 50° and is centered on latitude 25°S, longitude 160°E. This cap covers New Zealand, Australia and a significant portion of the Antarctica region. This technique is described in detail below.

1.2 The aim and overview of the study

The regional model to be developed is based largely on historical data measured during voyages of exploration in the Pacific region over the last four centuries.

This study aims to extend our knowledge of the geomagnetic field variations backward in time and map a picture of the Earth’s magnetic field from the 17th century up to present day in the southwest Pacific region. Part of the data used in determining the regional model of the magnetic field, which is applicable only for the southwest Pacific region, was also used in the gufm1 model which is based on a large amount of historical magnetic field observational data from 1590 to 1990 (Jackson, Jonkers, & Walker, 2000).

Chapter 2 outlines solutions of Laplace’s equation to give a global magnetic field model using the SHA technique, and a regional magnetic field model using SCHA. Chapter 3 reviews the data used in this study and which sources have been used. Chapter 4 illustrates two different methods of analyzing the data that were used to derive smooth magnetic field models. Chapter 5 presents and discusses the results, and Chapter 6 summarizes the thesis and suggests ways in which the analysis could be improved.
CHAPTER 2

GEOMAGNETIC FIELD ANALYSIS AND DESCRIPTION

This chapter outlines the common methods of modeling the earth’s magnetic field. The following discussion illustrates deriving a global model of the magnetic field by using spherical harmonic analysis and a regional model of the magnetic field in the southwest Pacific region by using spherical cap harmonic analysis. A discussion of some of the relevant literature is also presented, with a focus on spherical cap harmonic analysis studies.
2.1 Spherical Harmonic Analysis (SHA)

Spherical Harmonic Analysis is a method used to study any quantity that varies upon the surface of the sphere (Blakely, 1996). Spherical Harmonic Analysis has been applied to various types of data. For example, it is useful in analysis and synthesis of the gravitational field of the earth and to model variations of the earth or moon surface height (Brett, 1988).

In the region just above the Earth’s surface, where there are no electric currents and no other sources of magnetic fields, between the earth’s surface and the ionosphere (Barraclough, 1976), the Maxwell equation relating to the curl of the magnetic field is

\[ \nabla \times \mathbf{H} = \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \tag{2.1} \]

where \( \mathbf{H} \) is the magnetic field intensity; \( \mathbf{J} \) is the electric current density; \( \varepsilon_0 \) is the permittivity of free space and \( \mathbf{E} \) is the electric field and in this region

\[ \nabla \times \mathbf{H} = 0 \]
\[ \nabla \times \mathbf{B} = 0 \tag{2.2} \]

where \( \mathbf{B} \) is the magnetic induction. So from (2.2) if the curl of the vector is zero then it means that \( \mathbf{H} \) is a conservative vector and can be represented by the gradient of a scalar potential. Therefore \( \mathbf{B} \) may be expressed as
\[ B = -\text{grad} \, U \]  

(2.3)

where \( U \) is such a potential. The divergence of \( B \) is equal to zero because of the non-existence of single magnetic poles, therefore

\[ \text{div} \, B = \text{div} \, (-\text{grad} \, U) = - \nabla^2 U = 0 \]  

(2.4)

at the surface of the Earth, the magnetic induction may be described in terms of scalar potential function that obeys Laplace’s equation:

\[ \nabla^2 U = 0 \]  

(2.5)

Any geomagnetic field description at the Earth’s surface should be a solution of equation 2.5 (Merrill et al., 1998). The solution of this equation in spherical coordinates, \((r, \theta, \phi)\), can be obtained by separation of variables. The most complete and general solution is the sum of Spherical Harmonic functions(Chapman & Bartels, 1962), such that

\[
U(r, \theta, \phi) = a \sum_{l=1}^{\infty} \sum_{m=0}^{l} \left( \frac{R}{a} \right)^l (b_l^m \cos m\phi + c_l^m \sin m\phi) p_l^m(\cos \theta) \\
+ a \sum_{l=1}^{\infty} \sum_{m=0}^{l} \left( \frac{a}{R} \right)^{l+1} (g_l^m \cos m\phi + h_l^m \sin m\phi) p_l^m(\cos \theta)
\]  

(2.6)

in which \( b_l^m, c_l^m, g_l^m \) and \( h_l^m \) are the Gauss coefficients and are all measured in Tesla. The complete expression is known as the spherical harmonic representation of the geomagnetic potential. It shows that the geomagnetic potential is made up of two parts with different origins.
The first summation is the potential of the field which originates from outside the earth (external) and the second summation is the potential of the field that originates within the surface of the earth (internal) (Parkinson, 1983).

Calculating Gauss coefficients from observations of $B_x, B_y, B_z$ indicates the relative importance of sources of the magnetic field, whether internal or external. When field values are averaged over time scale of about a year the coefficients indicate that the external field is significantly small over a long time scale compared to the internal field (Hurwitz, Knapp, Nelson, & Watson, 1966). Therefore, the main earth’s magnetic field is of internal origin and equation (2.6) becomes

$$U(r, \theta, \phi) = a \sum_{l=1}^{\infty} \sum_{m=0}^{l} \left( \frac{a}{r} \right)^{l+1} (g_l^m \cos m\phi + h_l^m \sin m\phi) p_l^m(\cos\theta)$$

(2.7)

As already specified in the introduction, the coefficients $g_l^m$ and $h_l^m$ are obtained by using many sets of observational data of $X(B_x), Y(B_y)$ and $Z(B_z)$. As any term in, $g_0^0$ is equal to zero because magnetic monopoles do not exist, the degree of $l$ will start with $l = 1, 2$ and $3$ and for that there are $2, 3$ and $4$ terms of $g_l^m$ and $1, 2$ and $3$ terms of $h_l^m$ respectively (MacMillan, 1958). To satisfy boundary conditions $m$ has to be integral and less than or equal to $l$. To obtain estimates of the Gauss coefficients up to some degree $l$ requires a minimum of $(l + 1)^2 - 1$ measurements of $X, Y$ or $Z$, data on the surface of the Earth (Merrill et al., 1998).

The scalar potential ($U$) at the surface of the Earth can be introduced as a series of the spherical harmonics function with the Schmidt-normalized associated Legendre Polynomials, which are given by Rodrigues’ formula (Blakely, 1996; Cain, Hendricks, Langel, & Hudson, 1967):
\[ p_l^m (\cos \theta) = N \sin^m \theta \frac{d^m}{d(\cos \theta)^m} p_l(\cos \theta) \]  

(2.8)

Such that \( p_l(\cos \theta) \) are the Legendre Polynomials defined by

\[ p_l(\cos \theta) = \frac{1}{2^l l!} \left( \frac{\partial}{\partial (\cos \theta)} \right)^l (\cos^2 \theta - 1)^l \]  

(2.9)

\( N \) is a normalizing factor in which the product of two functions of \( \theta \) must be zero unless they both have the same values of \( l \) and \( m \). The \( N \) factor is defined by

\[ N = \left\{ \begin{array}{ll} 1, & m = 0 \\ \frac{2(l - m)!}{(l + m)!} & m > 0 \end{array} \right. \]  

(2.10)

Once these calculations are performed, the geomagnetic field components at the surface of the Earth can be represented from the derivatives of the scalar magnetic potential \( (U) \). The north, east and downward components of the field \( (B_x, B_y, \text{and } B_z \text{ respectively}) \), are given by (Blakely, 1996)

\[ B_x = \frac{1}{r} \frac{\partial U}{\partial \theta} \]

\[ B_y = -\frac{1}{r \sin \theta} \frac{\partial U}{\partial \phi} \]  

(2.11)

\[ B_z = \frac{\partial U}{\partial r} \]
In 1839 Gauss used measurements of the geomagnetic field to calculate the coefficients up to \( l = 4 \) (Merrill et al., 1998). Since then, a lot of effort has been made to calculate more terms even up to \( l=25 \), although there is increasing complexity in interpretation of the terms (L.R. Alldredge & Stearns, 1974; Harrison & Carle, 1981; Kolesova & Kropachev, 1973). The first spherical harmonic term \( (g_1^0 p_1^0 (\cos \theta) \frac{a^3}{r^2}) \) represents the potential of geocentric dipole oriented along the \(+z\) direction, the \( (g_1^1 p_1^1 (\cos \theta) \cos \phi \frac{a^3}{r^2}) \) and the \( (h_1^1 p_1^1 (\cos \theta) \sin \phi \frac{a^3}{r^2}) \) terms represent dipoles in the equatorial plane lying respectively along \( \phi = 0^\circ \) and \( \phi = 90^\circ \) (Merrill et al., 1998).

The magnitude \( (m) \) of the geocentric dipole moment calculated from these three terms together is

\[
m = \frac{4\pi a^3}{\mu_0} \sqrt{(g_1^0)^2 + (g_1^1)^2 + (h_1^1)^2} \quad (2.12)
\]

This dipole describes nearly 90% of the observed magnetic field at the surface of the earth and it is inclined to the rotation axis at an angle \( \alpha \), hence

\[
\tan \alpha = \frac{\sqrt{(g_1^1)^2 + (h_1^1)^2}}{g_1^0} \quad (2.13)
\]

The SHA technique is normally used to determine the magnetic potential when data are available over the whole earth. The least squares method is usually used to determine the Gauss coefficients from the data on the surface of the earth and the results become highly reliable if the data are well distributed over the globe. If the data distribution is biased to one particular region then the results are not necessarily very accurate in other regions (Malin, 1983). So, if the analysis is required to be done over a restricted area on the surface of the earth, SHA is no longer a particularly helpful representation of the potential. In such a situation the technique of Spherical Cap Harmonic Analysis (SCHA) is preferable to solve Laplace’s equation in spherical coordinates over only a small portion of the Earth.
2.2 Spherical Cap Harmonic Analysis (SCHA)

SCHA gives a regional model designed to represent the magnetic field in a particular portion of the earth’s surface – either when a high proportion of the observations are in a particular region, or because of special interest to study the field over a certain area (J. Miquel Torta, Gaya-PiquÉ, & De Santis, 2006; J. M. Torta, Gaya-Pique’, & De Santis, 2006). SCHA is also used in modeling other applications like regional secular variation (Korte & Haak, 2000; J. M. Torta, Garcia, Curto, & De Santis, 1992), the crustal magnetic anomaly field (Santis, Kerridge, & Barraclough, 1989) and in modeling sea level data (Hwang & Chen, 1997).

Both regional (SCHA) and the global (SHA) models have basis functions which encompass associated Legendre functions in colatitude, trigonometric functions in longitude, and power of radial distance (Haines, 1988). However, a SCHA model differs from a SHA model in the Legendre polynomials functions, such that the Legendre functions require a non-integral degree in SCHA but integral degree in SHA. This difference arises from the boundary conditions that are applied at the edge of the spherical cap, whereas the spherical harmonics functions are mutually orthogonal within the cap (J. M. Torta et al., 1992). This orthogonality of the functions allows the potential and its derivatives to be expressed as uniform infinite convergent series.

A spherical cap of radius $\theta_0 = 50^\circ$ centered at $\theta_c = 115^\circ$ and $\phi_c = 160^\circ$ has been chosen for SCHA of geomagnetic data concentrated in this region of the southwest Pacific (figure 2.1, adapted from (Ingham, 2009))
Figure 2.1: A spherical cap of radius $\theta_0 = 50^\circ$, centered at geographic coordinates $(\theta_c, \phi_c) = (115^\circ, 160^\circ)$ in the southwest Pacific.

Appropriate boundary conditions for the SCHA model (suggested by Haines, 1985) are that the scalar potential at $\theta_0$ (the edge of the spherical cap) as shown in the figure 2.1and its derivative with respect to $\theta$ must satisfy equations (2.14) and (2.15) below, where $f$ and $g$ are arbitrary functions:

$$U(r, \theta_0, \phi_\tau) = f(r, \phi_\tau)$$  \hspace{1cm} (2.14)

$$\frac{\partial U(r, \theta_0, \phi_\tau)}{\partial \theta_\tau} = g(r, \phi_\tau)$$  \hspace{1cm} (2.15)

$\theta_\tau$ and $\phi_\tau$ are the colatitude and longitude with respect to the center of the cap (taken as origin).
A solution of Laplace’s equation over a spherical cap for internal sources with (2.14) and (2.15) as boundary conditions is given by

\[
U(r, \theta_T, \phi_T) = a \sum_{m=0}^{\infty} \sum_{l=m}^{\infty} \left( \frac{a}{r} \right)^{n_l(m)+1} (g_l^m \cos m\phi_T + h_l^m \sin m\phi_T) p_{n_l(m)}(\cos \theta) \]  

(2.16)

where \( \theta_T \) and \( \phi_T \) give the location of a point relative to the centre of the spherical cap.

The Gauss coefficients \( g_l^m \) and \( h_l^m \) of this regional model no longer define a best fitting geocentric dipole, quadrupole and octupole etc. In fact, \( g_0^0 \) has a non-zero value in the solution of (2.16) (Haines, 1988). The Schmidt normalized associated Legendre polynomials can be expressed as:

\[
P_l^m(\cos \theta) = \sum_{l=0}^{\infty} A_l(m, n) \left\{ \frac{1 - \cos \theta}{2} \right\}^l \]  

(2.17)

with

\[
A_0(m, n) = K(m, n) \sin^m \theta_c \]  

(2.18)

and

\[
A_l(m, n) = \frac{(l + m - 1)(l + m) - n(n + 1)}{l(l + m)} A_{l-1}(m, n) \]  

(2.19)
where $K(m,n)$ in (2.18) is a normalizing factor, $K(m,n)$ is given by

$$
K(m,n) = \begin{cases} 
1 & \text{if } m = 0 \\
\frac{2^{-m} n^{m+1} m^m}{\sqrt{m\pi} (n-m)^{m+1}} \exp(e_1 + e_2 + \cdots) & \text{if } m > 0 
\end{cases}
$$

In these expressions $e_1$, $e_2$ and $p$ can be found as:

$$
P = \left(\frac{n}{m}\right)^2 - 1
$$

$$
e_1 = -\frac{1}{12} \left(1 + \frac{1}{p}\right)
$$

$$
e_2 = \frac{1}{360} \left(1 + \frac{3}{p^2} + \frac{4}{p^3}\right)
$$

From the mathematical formulation above, it is found that for any values of $l$ and $m$, the value of $n_l(m)$ has to mean that $U$ satisfies one (or both) of the two boundary conditions (2.14) and (2.15).

These analytical boundaries conditions lead to two infinite sets of spherical harmonic functions which are mutually orthogonal within themselves, but not orthogonal with each other. They can be solved numerically for the purpose of calculating values of real $n_l(m)$ that satisfy one or both of the boundary conditions. As the values of $n$ depend on $m$, the $n_l(m)$ are the roots of equation (2.14) if $l-m$ is even and the roots of equation (2.15) if $l-m$ is odd (J. M. Torta et al., 2006), where $l$ is the integer index that represent the order of the different roots $n$ for a given $m$. 
Haines (1988) lists a table [2.1] of \( n_l(m) \) values corresponding to the \( m \) values, which are found by solving (2.14) and (2.15) using the summation (2.16) for some values of \( l \) as shown below, for a spherical cap of radius 50°.

<table>
<thead>
<tr>
<th>( m )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2.24</td>
<td>1.78</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3.92</td>
<td>3.92</td>
<td>3.27</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>5.82</td>
<td>5.66</td>
<td>5.50</td>
<td>4.71</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>7.56</td>
<td>7.56</td>
<td>7.30</td>
<td>7.02</td>
<td>6.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>9.41</td>
<td>9.31</td>
<td>9.21</td>
<td>8.88</td>
<td>8.52</td>
<td>7.51</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>11.17</td>
<td>11.17</td>
<td>11.00</td>
<td>10.82</td>
<td>10.42</td>
<td>9.99</td>
<td>8.90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>13.01</td>
<td>12.94</td>
<td>12.86</td>
<td>12.63</td>
<td>12.40</td>
<td>11.94</td>
<td>11.44</td>
<td>10.27</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.1: Values of \( n_l(m) \) that satisfy the boundary conditions (2.14) and (2.15) for a spherical cap of radius 50 degree. This table comes from Haines (1985).

Just as for SHA the magnetic field components of the Earth \( B_x \), \( B_y \) and \( B_z \) can be found from (2.16) by using equations (2.11). As all field measurements are taken at the surface of the Earth, \( r = a \), and so the \( a/r \) dependence cancels out from \( B_x \), \( B_y \) and \( B_z \) (Hurwitz et al., 1966; Langel, Estes, Mead, Fabiano, & Lancaster, 1980).
Geomagnetic Field Analysis and Description

\[ B_x = - \sum_{l=m}^{\infty} \sum_{m=0}^{l} \frac{\partial P_n^m (\cos \theta_T)}{\partial \theta_T} \left[ g_i^m \cos m\phi_T + h_i^m \sin m\phi_T \right] \]

\[ B_y = \sum_{l=m}^{\infty} \sum_{m=0}^{l} \frac{P_n^m (\cos \theta_T)}{\sin \theta_T} \left[ g_i^m \sin m\phi_T + h_i^m \cos m\phi_T \right] \]

\[ B_z = \sum_{l=m}^{\infty} \sum_{m=0}^{l} (n + 1) P_n^m (\cos \theta_T) \left[ g_i^m \cos m\phi_T + h_i^m \sin m\phi_T \right] \]

In order to perform a SCHA it is necessary to convert data site locations from geographic coordinate to new coordinates with respect to the center of the cap. We must also rotate the measured magnetic field components \( B_x, B_y \) and \( B_z \) to be in the spherical cap reference frame\((\text{Pavón–Carrasco, Osete, Torta, & Gaya–Piqué, 2009})\). From the spherical cap geometry, the new longitude and colatitude for any data site can be found by considering a triangle on the surface of the earth with the North geographic pole, the centre of the spherical cap and the desired data point as shown in figure 2.2.
Figure 2.2: Geometry of the spherical cap showing the angles $\theta_T$, $\phi_T$ the co-latitude and longitude of a point on the surface of the Earth relative to the cap centre. By setting up a triangle between the geographic North Pole, the centre of the spherical cap and the observational point it is possible to calculate $\theta_T$, $\phi_T$ by using a spherical version of the sine and cosine rules.
The two length sides of the spherical triangle ($\theta_c$ and $\theta_p$) are the geographic longitudes passing through the centre of the cap and data site. The angle between these two sides is the difference in the longitudes of the two sites, $\varphi_c - \varphi_p$.

We can determine the colatitude $\theta_T$ of the data point with respect to the centre of the cap from the spherical version of the cosine rule.

$$\theta_T = \cos^{-1}(\cos \theta_c \cos \theta_p + \sin \theta_c \sin \theta_p \cos(\varphi_c - \varphi_p))$$  \hspace{1cm} (2.21)

The spherical sine rule also can be used to find the longitude $\varphi_T$ of the data point with respect to the cap pole

$$\varphi_T = \sin^{-1}\left[\frac{\sin \theta_p \sin(\varphi_c - \varphi_p)}{\sin \theta_T}\right]$$  \hspace{1cm} (2.22)

As $\sin^{-1}$ produces angles from -90 to 90, Ingham (2009), as detailed in table 2.2, states which quadrant $\varphi_T$ should be in according to the relative values of $\theta_p$, $\theta_c$, $\varphi_p$, and $\varphi_c$. 

In terms of finding the north and east components of the magnetic field \((B_x \text{ and } B_y)\) with respect to the center of the cap, the rotation angle of these components should be determined for every data point. This angle \(\gamma\) as shown in figure [2.3] can be determined using the spherical sine rule as:

<table>
<thead>
<tr>
<th>The cap</th>
<th>(\theta_p, \theta_c \text{ and } \phi_p, \phi_c)</th>
<th>(\phi_T) Quadrant</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Diagram 1](Image 1)</td>
<td>(\theta_c &gt; \theta_p) (\phi_c \geq \phi_p)</td>
<td>0-90</td>
</tr>
<tr>
<td>![Diagram 2](Image 2)</td>
<td>(\theta_c \leq \theta_p) (\phi_c &gt; \phi_p)</td>
<td>90-180</td>
</tr>
<tr>
<td>![Diagram 3](Image 3)</td>
<td>(\theta_c \leq \theta_p) (\phi_c &lt; \phi_p)</td>
<td>180-270</td>
</tr>
<tr>
<td>![Diagram 4](Image 4)</td>
<td>(\theta_c &gt; \theta_p) (\phi_c &lt; \phi_p)</td>
<td>270-360</td>
</tr>
</tbody>
</table>

Table 2.2: Quadrants for \(\phi_T\)
Figure 2.3: Geometry of the spherical cap on the surface of the earth, \((\theta_c, \varphi_c)\) are the geographic coordinates of the cap centre and \((\theta_p, \varphi_p)\) are the geographic coordinates of observational data site. However, \((\theta_T, \varphi_T)\) is the observational data with respect of the centre of the cap and \(\gamma\) is the required rotating angle of the \(B_X\) and \(B_Y\) components of the magnetic field.

\[
\gamma = \sin^{-1} \left( \frac{\sin \theta_c \sin (\varphi_c - \varphi_p)}{\sin \theta_T} \right) \tag{2.23}
\]

Again the \(\sin^{-1}\) in the equation (2.23) above requires the right quadrant for the rotation angle to be determined. Ingham (2009) lists a table of angle \(\gamma\) as below in Table 2.3.
The $B_x$ and the $B_y$ components of the magnetic field with respect of the centre of the cap are then:

\[
B_x = B_x \cos \gamma + B_y \sin \gamma
\]

\[
B_y = -B_x \sin \gamma + B_y \cos \gamma
\]

The downward magnetic field component $B_z$ is not affected by coordinate transformation.

### 2.3 Method of Solution

To calculate the Gauss coefficients of the SCHA models, the magnetic field components must be fit over the spherical cap. Expressions for $B_x$, $B_y$ and $B_z$ at each site location are dependent on $\theta_T$ and $\phi_T$, the latitude and longitude respectively, of the site, the Gauss coefficients, and the ratio of $a$ to $r$. As all data had been measured on the surface of the Earth then $a=r$, and so the radial dependence cancels out.
A MATLAB computer program has been used to calculate a least squares best fit to the input data. This is done by setting up a matrix of the spatial terms in (2.20) and using a matrix inversion to determine values of the Gauss coefficients (Ingham, 2009).

\[
\begin{bmatrix}
B_{x0} & B_{x1} & B_{x2} & \cdots \\
B_{y0} & B_{y1} & B_{y2} & \cdots \\
B_{z0} & B_{z1} & B_{z2} & \cdots \\
\vdots & \vdots & \vdots & \ddots
\end{bmatrix}
\begin{bmatrix}
g_0 \\
g_1 \\
h_1 \\
\vdots
\end{bmatrix}
= 
\begin{bmatrix}
B_{x} \\
B_{y} \\
B_{z}
\end{bmatrix}
\]

**Cap reference frame:** where the \(B_{x}^{l} \sin \theta \cos \phi \), \(B_{y}^{l} \sin \theta \sin \phi \), \(B_{z}^{l} \cos \theta \) and \(B_{z}^{m} \sin \phi \), with \(l\) degree and \(m\) order. The \(g\)’s and \(h\)’s are the calculated Gauss coefficients at the measurement site \((\theta, \phi)\). The \(B_{x}^{l} \), \(B_{y}^{l} \), \(B_{z}^{l} \) are the magnetic field measurements.

**Geographic reference frame:** To calculate the Gauss coefficients from this matrix inversion, all historical data recorded in terms of declination, inclination and intensity should be converted to \(B_{x} \), \(B_{y} \) and \(B_{z} \) first. From Figure 1.1 above we can find that:

\[
\begin{align*}
B_{x} &= F \cos l \cos D \\
B_{y} &= F \cos l \sin D \\
B_{z} &= F \sin l
\end{align*}
\] (2.26)
The matrix inversion was carried out using MATLAB a program sc2ver2.m written by Ingham (2009) which takes as input the latitude, longitude of each site and $B_x$, $B_y$ and $B_z$ at the site in geographic coordinates, does the coordinate and field transformations, calculates the SCHA and then converts back into geographic coordinates.

2.4 The geomagnetic field model “gufm1”

A brief description of the gufm1 model (Jackson et al., 2000) is included as part of the analysis in this thesis involves comparisons with this model. The gufm1 model provides the most complete picture of the geomagnetic field evaluation from 16th century onwards. Jackson et al. (2000) constructed a continuous time-space magnetic field model with a global SHA model based on a large number, about 91000, of historical observations. From the magnetic potential equation

$$ U(r, \theta, t) = a \sum_{l=1}^{1} \sum_{m=0}^{l} \left( \frac{a}{r} \right)^{l+1} (g_l^m(t) \cos m\phi + h_l^m(t) \sin m\phi) p_l^m(\cos \theta) $$

(2.27)

where $g_l^m(t)$ and $h_l^m(t)$ are the Gauss coefficients as a function of time expanded to fourth order using B-spline basis functions $B_n(t)$ such that

$$ g_l^m(t) = \sum_n g_n^{nm} B_n(t) $$

(2.28)
where the $B_n(t) > 0$ if the $t \in |t_n, t_{n+1}|$ and zero otherwise. The magnetic field components therefore will be

$$B_r = \sum_{l=1}^{\infty} \sum_{m=0}^{l} (l + 1) \left( \frac{a}{r} \right)^{l+2} (g_l^m(t) \cos m\phi + h_l^m(t) \sin m\phi) p_l^m(\cos\theta)$$

$$B_\theta = -\sum_{l=1}^{\infty} \sum_{m=0}^{l} \left( \frac{a}{r} \right)^{l+2} (g_l^m(t) \cos m\phi + h_l^m(t) \sin m\phi) \frac{dp_l^m(\cos\theta)}{d\theta}$$

$$B_\phi = \frac{1}{\sin\theta} \sum_{l=1}^{\infty} \sum_{m=0}^{l} m \left( \frac{a}{r} \right)^{l+2} (g_l^m(t) \sin m\phi - h_l^m(t) \cos m\phi) p_l^m(\cos\theta)$$

(2.29)

Jackson et al. (2000) constructed this model to fit the input data smoothly in both time and space. Given the large number of declination observation before 1800, Jackson et al. used the decay of the dipole coefficients $g_0^1(t)$ with time to estimate intensity values before 1800. Spherical harmonic expansions, equation 2.22, were truncated at $l = 14$ to obtain the smoothest model and give the best representation of the field. Two model norms are used in the gufm1 model, one measuring the roughness in the spatial domain, based on (Gubbins, 1975) and the other roughness in the time domain. For further discussion on gufm1 model see (Jackson et al., 2000)
CHAPTER 3

BACKGROUND TO THE SPATIAL AND TEMPORAL DISTRIBUTION OF THE DATA

The previous chapter presented the mathematical techniques normally used in describing the earth’s magnetic field. Before investigating different methods of developing a mathematical model of the field in the southwest Pacific, observational data have to be collated. Data recorded in the ships’ logs of various explorers and traders over the last 400 years will be outlined in this chapter.
3.1 Input Data

The data used to calculate a mathematical model of the field are normally made up of measurements of declination, inclination and intensity at different latitude and longitude. The data used in this project are those recorded by the explorers and traders who sailed in the southwest Pacific. These cover a period of time extending from Abel Tasman’s discovery of Van Diemen's Land and New Zealand in the 17th century up to the Antarctic Explorations in the 19th century (G. Turner, 2010).

Many of the earlier ships’ logs from various explorers and traders contained numbers of declination measurements. The declination at sea was normally observed by the ship’s standard compass taking into account any correction for this compass (Moseley, 1879). This was determined by making a comparison of the declination values observed by the compass on land with those determined by another trusted instrument on land, e.g. a unifilar magnetometer (Malin, 1983; Moseley, 1944). Declinations measurements were made from the 16th century because it was believed that these measurements might help to determine longitude (Jackson et al., 2000). This was the reason why Edmond Halley charted magnetic declinations on his map of the Atlantic (Halley, 1710). This map is presented in Appendix A. After secular variation was recognized this idea was abandoned but magnetic charts remained a crucial navigational tool.

Even though inclination was recognized around 1600 by Robert Norman (Norman, 1720), few inclination data were recorded in the ships’ logs and most of these data were in the late 18th century and few before that as shown in Figure 3.1. The reason for this was partly the difficulty of measuring inclination whilst on a ship, also, inclination was not used in navigation as declination was at that time (Ingham, 2009).

Similar to inclination, intensities were only routinely recorded in the ship’s logs from the 18th century (Thompson & Barraclough, 1982). Von Humboldt made relative intensity measurements in South America in 1798 and intensity data start to appear in the southwest Pacific about the
same time as shown in Figure 3.1. The method of measurement was by timing the oscillations of the ship’s dip needle (Lilley & Day, 1993). Elisabeth Paul Edourd De Rossel made the first magnetic intensity measurements recorded in the southwest Pacific during 1791-1794 (E. Sabine, 1838). He measured the magnetic field intensity as a relative intensity referenced to a set location. For instance all intensities data measured on the D’Entrecasteaux expeditions in 1792 while sailing throughout the Pacific were referenced to the intensity measured in Paris (De Labillardière, 1800). In this study, all intensities have been converted to absolute intensity in nano-tesla.

There were several problems that cause errors and uncertainties in taking measurements of magnetic data. These problems include the disturbance of the ship’s iron on the magnetic instruments. This was accounted for by taking measurements in different places upon the ship with different magnetic instruments while in a port with a land-based measurement for comparison. By the time of 1800 there was a concerted effort to design the structure of the ships so magnetic compasses could be placed on ships where there was minimum effect from the ship’s structure (E. Sabine, 1838). Most of the voyages at this time also carried supplementary dip needles for inclination and intensity measurements in order to avoid error arising from any damage to an instrument during the voyage. The difficulty of determining the absolute position of a ship in the open ocean had been one of the major problems since the 16th century. Longitude was estimated by dead reckoning, that is, by averaging the speed of the ship and considering the direction of sailing from a previously known position (G. Turner, 2010). However, chronometers started to be used in the 1800s to determine longitude, which decreased the error significantly (Bemmelen, 1898).
Figure 3.1 shows the temporal distribution of the observed data (declination, inclination and intensity) used for the time interval 1600 to 1910. Most of the data in the earliest epoch were declination-only data and were collected by Abel Tasman on his journey to discover the Van Diemen's Land and New Zealand in 1642 (Van Bemmelen, 1989), by William Dampier 1699, in his voyage to New-Holland (Dampier, 1709), Captain Rogers on his global voyage of discovery (Rogers), Captain Cook’s observations while he sought for evidence of the Australian coast (Cook, 1768) and D’Entrecasteaux on his exploration of the Australian coast in 1792 while searching for the La Pérouse expedition (Lilley & Day, 1993).
Modelling Secular Variation in the Southwest Pacific for the last 400 years

- 1606-1643
- 1685-1710
- 1765-1780
- 1786-1793
Figure 3.2: Spatial distributions of the input data in the interval 1600 to 1910.
+ are data with declinations only,  + are data with declinations and inclinations,  • are data with declinations and intensities,  * are data with inclinations only,  • are data with inclinations and intensities,  • are data with intensities only, and  + are data with declinations, inclinations and intensities. Projection is stereographic.
A much higher data density is found from the late 1700’s onward, where there were declination, inclinations and intensity data as is shown in Figure 3.1. Data from the late 18 and early 19 centuries were collated from the voyages of HMS Dolphin of Captain Wallis (Hawkesworth et al., 1785), de La Perouse (Galaup, 1798), Cook on his second and third voyages (Cook, 1768); (Cook, Beaglehole, & Skelton, 1967), and the voyages of Captain Sir Edward Belcher (Huxley & Weeds, 1966). Other data used in this project were from General Sir Edward Sabine (Edward Sabine, 1876), D’Entrecasteaux (Lilley & Day, 1993), the records of HMS Beagle (Darwin, King, & Fitzroy, 1839), records of The Challenger expedition (Moseley, 1879), the observatory of Royal Greenwich.

In the first decade of the 20th century, an extensive survey of New Zealand was conducted by the British admiralty to detect any local magnetic features (Farr, 1916). Data from this magnetic survey have been included in the compilation of data for 1900. The databases of both 1950 and 2000 have been constructed using the IGRF coefficients to calculate field values at 92 locations in the southwest Pacific region (Mandea & Macmillan, 2000).

The distribution of the data in southwest Pacific is inhomogeneous, so to visualize the number of the data centred on various times in the interval 1600 to 1910, the spatial distribution of the data in each period of time has been mapped as shown in Figure 3.2. The data have been divided up into different time intervals centred on the dates shown in the figure. Thus (a) in the figure is centred on 1633 and contains 113 observational data from 1606 to 1643. All these data are declination only and most of them are concentrated south and east of Australia, north of New Zealand, and some north of Papua New Guinea. Figure 3.2(b) is centred on 1700 and contains 88 data from 1685 to 1710. Again all are declination. These data are confined southwest of Australia and few in the Pacific Ocean. Figure 3.2(c) is centred on 1773 and contains 226 declination and 3 inclination data from 1765 to 1780. These data are mostly in the region north and east of Australia and around New Zealand. Figure 3.2(d) is for 1790 and it contains 491 declination, 3 inclinations and one intensity data from 1786 to 1793 from all around the Pacific Ocean in the study area. Figure 3.2(e) is centred on 1825 and contains 254 declination, 286 inclinations and 177 intensity data from 1805 to 1849. These data are concentrated mostly south of Australia and north of New Zealand.
Zealand. Figure 3.2(f) is for 1875 and it contains 365 declination, 148 inclinations and 138 intensity data from 1850 to 1890. These data are concentrated between Australia and New Zealand and around the Papua New Guinea. Figure 3.2(g) is for 1900 and contains 44 declination and 29 inclinations data from 1900 to 1949, but mainly from 1900 to 1910. The majority of these data come from the magnetic survey of New Zealand.

The database for the 19th century contains more directional data than intensity, figure 3.1(d,e,f) and most of the distribution is concentrated in the south and east of Australia and north of New Zealand. The amount of data in 1825 to 1875 represents about 49% of the total data in figure 3.1.

Many of the original data from the voyages documented only one or two of the magnetic field components. Where data in the same year and same position but with different components existed, they were considered to complement each other. Also data points of the same component from the same place in the same year were averaged.

It can be see that the distribution of the data is uneven both spatially and in terms of declination, inclination and intensity values. The data sets centred on 1633, 1700 and 1733 are particularly incomplete as they do not contain all of declination, inclination and intensity measurements. To apply a SCHA, gaps where not all magnetic components are available need to be filled. Methods of estimating values to fill the data gaps are described in the next chapter.
CHAPTER 4

METHODS OF ANALYSIS

This chapter presents an overview of the methods considered in order to complete the dataset and so derive a model of secular variation in the southwest Pacific. Two different possible techniques are discussed; one of them uses the gufm 1 model while the other is based only on the observational data.
4.1 Division of the Observational Data into Epochs

The observational data are greatly biased towards declination measurements (Fig. 3.1) since before Gauss deduced his method of determining intensity in 1832 (Jackson et al., 2000) there are few intensity data in ships’ records. Although there are a few inclination and intensity measurements, they are not as common. A full SCHA needs $B_x$, $B_y$ and $B_z$ data therefore where there are gaps in the record, a sensible method of mathematical estimation is required to fill the gaps.

There are several ways to approximate the magnetic field at any required epoch such as representing the Gauss coefficients as being time dependent (Haines & Coles, 1982). In this study, due to the large temporal range of data, in order to obtain a smooth regional model that fits the input data it was decided to divide the observational data into seven epochs. As the data distribution is not very even, this division has been chosen in a way that gives some consistency between amount of data in each epoch of SCH models. Taking into account the number of input data for each epoch, the epochs are labeled as follows:

1. 1633, which includes 113 observation data from 1606 to 1643. Figure 3.2(a).
2. 1700, which includes 88 observation data from 1685 to 1710. Figure 3.2(b).
3. 1773, which includes 284 observation data from 1765 to 1780. Figure 3.2(c).
4. 1790, which includes 446 observation data from 1786 to 1793. Figure 3.2(d).
5. 1825, which includes 717 observation data from 1805 to 1849. Figure 3.2(e).
6. 1875, which includes 655 observation data from 1850 to 1890. Figure 3.2(f).
7. 1900, which includes 44 observation data from 1900 to 1949. Figure 3.2(g) with additional simulated measurement points added from the 1900 IGRF.

Simulated measurements from the IGRF have also been used for 1950 and 2000 to calculate SCHA models.
This dataset has been compiled using two methods. One method uses the dipole coefficients of the gufm1 model to estimate the unrecorded magnetic field components and it will be referred to as gufm1 dipole coefficients method. The other method depends only on the observational data and will be referred to as the self-consistent analysis method. Each of these techniques is used to produce a SCHA model of each epoch and the results of this method of SCHA are compared with what the SHA based gufm1 model itself gives for the field; further explanation of each method has been outlined later.

4.2 Filling the data gaps

4.2.1 The gufm1 dipole coefficients method

The gufm1 model as described in Chapter 2 appears to be the best historical model to use as a reference field to estimate missing magnetic elements (Jackson et al., 2000). This was done by creating linear equations for each of the gufm1 model dipole terms \( (g^0_1(t), g^1_1(t) \text{ and } h^1_1(t)) \) as a function of time, and using these to give the values of the dipole coefficients for each of the seven epochs mentioned earlier. The dipole coefficients were then used to estimate the field components, \( B_x, B_y \text{ and } B_z \), \[\text{equation 4.2}\] for a site at \((\theta, \varphi)\).

From the truncated spherical harmonic expansion \( (u) \) up to \( l=1 \), which includes only the dipole terms \( (g^0_1, g^1_1 \text{ and } h^1_1) \)
the estimation of $B_x$, $B_y$ and $B_z$ as predicted by the dipole coefficients from \textit{gufm1} on the surface of the earth ($r=a$) at $(\theta, \phi)$ are

\begin{align}
U &= \frac{a^3}{r^2} g_1^0 \cos \theta + \frac{a^3}{r^2} g_1^1 \cos \phi \sin \theta + \frac{a^3}{r^2} h_1^1 \sin \phi \sin \theta \\
B_x &= \frac{1}{r} \frac{\partial U}{\partial \theta} = -g_0^0 \sin \theta + g_1^1 \cos \phi \cos \theta + h_1^1 \sin \phi \cos \theta \\
B_y &= -\frac{1}{r \sin \theta} \frac{\partial U}{\partial \phi} = g_1^1 \sin \phi + h_1^1 \cos \phi \\
B_z &= \frac{\partial U}{\partial r} = -2 g_1^0 \cos \theta - 2 g_1^1 \cos \phi \sin \theta - 2 h_1^1 \sin \phi \sin \theta
\end{align}

(4.1)

(4.2)

For any missing declination and/or inclination and/or intensity data, gaps can therefore be filled using the \textit{gufm1} model. If intensity ($F$), declination ($D$), or inclination ($I$) are missing they can be estimated from the values of $B_x$, $B_y$ and $B_z$ calculated from the \textit{gufm1} dipole coefficients by

\begin{align}
F &= \left[ B_x^2 + B_y^2 + B_z^2 \right]^{1/2} \\
D &= \tan^{-1} \left[ \frac{B_y}{B_x} \right] \\
I &= \tan^{-1} \left[ \frac{B_z}{\left( B_x^2 + B_y^2 \right)^{1/2}} \right]
\end{align}

(4.3)
There are seven different possibilities for filling in the data gaps depending on what combination
of $F$, $D$ and $I$ is missing. Each possibility while filling in the missing $F$, $D$ or $I$ must make sure that
any actual measured values of declination, inclination or intensity are kept. Hence if measured
value of declination ($D$) is observed already, then the $B_x$ and $B_y$ values used in the SCHA should
satisfy $D = \tan^{-1} \left( \frac{B_y}{B_x} \right)$, and not the declination value predicted by \textit{gufm1}. The seven different
situations are:

1. If the all magnetic field elements are available at site $(\theta, \phi)$, then magnetic field
   components can easily calculate from

   \[
   B_x = F \cos I \cos D \\
   B_y = F \cos I \sin D \\
   B_z = F \sin I
   \]  \hspace{1cm} (4.4)

   Using the measured values of $F$, $D$ and $I$.

2. If the declination and inclination data are both available for a site at $(\theta, \phi)$ but not
   intensity, then the intensity needs to be estimated from the $g_l^0, g_l^1$ and $h_l^1$ coefficients for
   the relevant year from \textit{gufm1} as:

   \[
   F_e = \left[ B_x^2 + B_y^2 + B_z^2 \right]^{1/2}
   \]

   where $B_x$, $B_y$ and $B_z$ are calculated from equation 4.2. From measured declination and
   inclination and estimated intensity ($F_e$), the magnetic field components are then:
3. If the declination data is the only data available from the observational data at site \((\theta, \phi)\), then both inclination and intensity need to be estimated from \textit{gufm1} coefficients such that

\[
F_e = \left[ B_x^2 + B_y^2 + B_z^2 \right]^{1/2}
\]

as before and inclination equal to

\[
l_e = \tan^{-1} \left[ \frac{B_z}{\left( B_x^2 + B_y^2 \right)^{1/2}} \right]
\]

where \(B_x, B_y\) and \(B_z\) come from equation 4.2. Now estimates of \(B_{xe}, B_{ye}\) and \(B_{ze}\) are given by

\[
\begin{align*}
B_{xe} &= F_e \cos l_e \cos D \\
B_{ye} &= F_e \cos l_e \sin D \\
B_{ze} &= F_e \sin l_e
\end{align*}
\]  

where \(D\) is the measured declination value.
4. If the inclination and intensity are available at site \((\theta, \phi)\) but not declination, then declination needs to be estimated from the \textit{gufm1} coefficients hence

\[
D_e = \tan^{-1}\left(\frac{B_y}{B_x}\right)
\]

Again \(B_x\) and \(B_y\) are calculated from equation 4.2 and the estimated \(B_{xe}, B_{ye}\) and \(B_{ze}\) become

\[
\begin{align*}
B_{xe} &= F \cos I \cos D_e \\
B_{ye} &= F \cos I \sin D_e \\
B_{ze} &= F \sin D_e
\end{align*}
\] (4.7)

Equations used for other possibilities, such as if the declination and intensity are available but not inclination; if the inclination is available but not declination and intensity; or if the intensity is available but not declination and inclination, can be found in Appendix B.

The complete dataset, that now includes both the observed and estimated data at each epoch, now can be used to produce a SCH model for that epoch.
4.2.2 The self-consistent analysis method

The analysis using a self-consistent method included the following steps.

1. A spherical cap harmonic analysis was applied separately to observational data from epochs 2000, 1950 and 1900. Simulated data generated from IGRF values were used for 2000 and 1950, and for 1900 the actual observed data were supplemented by a small number of simulated data generated from IGRF values.

2. Use the spherical cap harmonic analysis results for 2000, 1950 and 1900 to estimate the spherical cap Gauss coefficients for 1875 by linear extrapolation as shown for $g_0$ in Figure 4.1(a).
Repeat the same linear extrapolation method for all Gauss coefficients up to $l = 2$.

3. The predicted SCHA coefficients for the 1875 epoch were then used to estimate values of $Bx$, $By$ and $Bz$ at those sites with data gaps. These values were then used to fill in gaps of unrecorded magnetic fields components (either declination, inclination and/or intensity).

4. The estimated missing value of intensity, declination and inclination were then used with the measured values, in the same way as in the *gufm1* dipole coefficient method section, to give estimates of $Bx$, $By$ and $Bz$.

5. A final spherical cap harmonic model for 1875 was found from this complete dataset.

6. Plots of the spherical cap harmonic coefficients for 2000, 1950, 1900 and 1875 were then used to estimate the coefficients for 1825, and the process repeated to get final 1825 coefficients (e.g. Figure 4.1(b)).
7. The previous steps were then repeated for the 1790, 1773, 1700 and 1633 epoch’s i.e. deriving initial Gauss coefficient for an epoch from the extrapolation of the coefficients for the preceding epochs, and using these to fill in data gaps before calculating final coefficients.

Plots of the final results for all Gauss coefficients are shown in Figure 4.2.
Modelling Secular Variation in the Southwest Pacific for the last 400 year

\[ g_1^1 \]

\[ y = 2.31635E+01x - 6.72744E+04 \]

\[ 1600 \quad 1650 \quad 1700 \quad 1750 \quad 1800 \quad 1850 \quad 1900 \quad 1950 \quad 2000 \quad 2050 \]

\[ -35000 \quad -33000 \quad -31000 \quad -29000 \quad -27000 \quad -25000 \quad -23000 \quad -21000 \quad -19000 \quad -17000 \]

\[ \text{years} \]

\[ h_1^1 \]

\[ y = 3.52871E+00x - 3.39879E+03 \]

\[ 1600 \quad 1650 \quad 1700 \quad 1750 \quad 1800 \quad 1850 \quad 1900 \quad 1950 \quad 2000 \quad 2050 \]

\[ 0 \quad 1000 \quad 2000 \quad 3000 \quad 4000 \quad 5000 \quad 6000 \quad 7000 \quad 8000 \]

\[ \text{extrapolated value} \]

\[ \text{year} \]

\[ g_2^0 \]

\[ y = -8.06163E-01x + 2.73499E+01 \]

\[ 1600 \quad 1650 \quad 1700 \quad 1750 \quad 1800 \quad 1850 \quad 1900 \quad 1950 \quad 2000 \quad 2050 \]

\[ 0 \quad -500 \quad -1000 \quad -1500 \quad -2000 \quad -2500 \quad -3000 \]

\[ \text{extrapolated value} \]

\[ \text{year} \]
Methods of Analysis

\begin{align*}
g_2^1 &= y = -4.45389E+00x + 1.13543E+04 \\
h_2^1 &= y = -3.68518E+00x + 6.74594E+03 \\
g_2^2 &= y = -5.86293E+00x + 1.05245E+04
\end{align*}
In this method, linear extrapolation was used rather than quadratic or higher order because for the period 1590-1990. Although, most gufm1 coefficients up to \( l = 2 \) are reasonably linear (Figure 4.3), and it has been assumed that a reasonable first order approximation of SCHA coefficients will also have a linear time variation, the expanded scale does show that \( g_0^2 \) and \( h_2^2 \) are in fact not very linear.
Figure 4.3: Plot of the *gufm1* model Gauss coefficients up to $l=2$ splits out into (A) dipole and (B) non-dipole coefficients.
It should be noted that the data sites used in the self–consistent analysis are the same as in the \textit{gufm1} dipole coefficient model. A direct comparison can thus be made between the self-consistent technique and filling the un-recorded field components by using the \textit{gufm1} dipole coefficient technique. These two models can also be compared with contour plots derived from the entire Gauss coefficients of the \textit{gufm1} model.
CHAPTER 5

RESULTS OF MODELLING SECULAR VARIATION IN THE SOUTHWEST PACIFIC

The first part of this chapter outlines the results of the SCHA modeling in two sections. The first section presents the results obtained from the *gufm1* dipole coefficients method. The second section presents the results from the self-consistent method. In the second part of this chapter, these results are discussed after interpolation to 25-year intervals.
5.1 Epochs Results

5.1.1 Results of gufm1 dipole model

As the gufm1 model is the accepted global model representing the magnetic field since 17th century, it makes sense to compare results of the gufm1 dipole model with it. In this study, nearly 1484 data points with data gaps filled using gufm1 dipole data have been used with 343 actual observational data, giving a total of 1827 data in all. Results of the SCH models are shown as a series of contour plots of declination, inclination and intensity for each period of time, mentioned in Section 4.1. Hence, the left hand side of the figure shows the gufm1 dipole model results and the right hand side shows the field as given by gufm1 model.

5.1.1.1 Declination Plots

The declination plots for the gufm1 dipole and gufm1 model have been presented side by side to test the reliability of gufm1 dipole technique for the time from 1633 to 2000. The major points of comparison can be broken down into four main points, From Figure 5.1:

1. There is quite a good agreement between the gufm1 dipole and gufm1 models for the time 2000 to 1950, figure 5.1 [A,J and B,K]. This was expected because both models are based largely on the same, quite well distributed data- complete (IGRF).

2. This agreement gets worse further back in time. An example of this is indeed the declination for 1700, figure 5.1 [H,R], and the reason for this is not only the small amount of data but also the complete lack of data around New Zealand for this epoch. For all the epochs prior to 1825 the declination plots tend to be similar where there is a
reasonable distribution of data (e.g. in 1790 [F,O]) and dissimilar for 1773,1633 [S,P-I,Q] respectively, where there is a poor distribution of data.

3. The *gufm1* gives much larger negative declinations south of Australia than does the *gufm1* dipole model.

4. The change in declination across Australia is very similar in both and the values vary between -5 to +15. These values are in agreement with values of radiocarbon age recorded in lake sediments at Keilambete lake, south east Australia, (Barton & McElhinny, 1981).

As mentioned before in section 2.4 that the *gufm1* is based on calculating time dependence of Gauss coefficients which results in quite smoothly changing models from epoch to epoch. The way of modeling used in the *gufm1* dipole model does not guarantee the same smoothness, which is why results for 1700, 1773 and 1633 are so affected by a poor data distribution.
<table>
<thead>
<tr>
<th>Year</th>
<th>Dipole</th>
<th>gufm1</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td><img src="Image" alt="Map A" /></td>
<td><img src="Image" alt="Map J" /></td>
</tr>
<tr>
<td>1950</td>
<td><img src="Image" alt="Map B" /></td>
<td><img src="Image" alt="Map K" /></td>
</tr>
<tr>
<td>1900</td>
<td><img src="Image" alt="Map C" /></td>
<td><img src="Image" alt="Map L" /></td>
</tr>
</tbody>
</table>
Results of Modelling Secular variation in the Southwe...

<table>
<thead>
<tr>
<th>Year</th>
<th>Dipole $gufm1$</th>
<th>$gufm1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1875</td>
<td><img src="image" alt="D" /></td>
<td><img src="image" alt="M" /></td>
</tr>
<tr>
<td>1825</td>
<td><img src="image" alt="E" /></td>
<td><img src="image" alt="N" /></td>
</tr>
<tr>
<td>1790</td>
<td><img src="image" alt="F" /></td>
<td><img src="image" alt="O" /></td>
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</tbody>
</table>
Modelling Secular Variation in the Southwest Pacific for the last 400 year

<table>
<thead>
<tr>
<th>Year</th>
<th>Dipole gufm1</th>
<th>gufm1</th>
</tr>
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<td><img src="image" alt="G" /></td>
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<tr>
<td>1700</td>
<td><img src="image" alt="H" /></td>
<td><img src="image" alt="R" /></td>
</tr>
<tr>
<td>1633</td>
<td><img src="image" alt="I" /></td>
<td><img src="image" alt="Q" /></td>
</tr>
</tbody>
</table>

Figure 5.1: Declination maps for the southwest Pacific region from 1633 to 2000. Comparing the results of the gufm1 dipole model (A-I) with the gufm1 model (J-R). Contour interval is 5°.
5.1.1.2 Inclination Plots

As for the declination plots, inclination has been presented for both models in the same epochs. Inclination values in Figure 5.2 are negative in New Zealand and Australia. Inclination values given by the *gufm1* dipole model are compatible with those of *gufm1*. Inclinations in New Zealand vary from -72° to -63°. The *gufm1* dipole model gives more rapid change of inclination north of Australia with values 20° different from the *gufm1* model. The *gufm1* dipole model gives inclination contours, which tend to be flatter than in *gufm1*, especially around New Zealand. The exception is for 1700 where most of the data are in the far north of the spherical cap, figure 5.2 [H,Q].
<table>
<thead>
<tr>
<th>Year</th>
<th>gufm1 dipole</th>
<th>gufm1</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td><img src="image" alt="Image A" /></td>
<td><img src="image" alt="Image J" /></td>
</tr>
<tr>
<td>1950</td>
<td><img src="image" alt="Image B" /></td>
<td><img src="image" alt="Image K" /></td>
</tr>
<tr>
<td>1900</td>
<td><img src="image" alt="Image C" /></td>
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## Results of Modelling Secular Variation in the Southwest Pacific

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<th>Year</th>
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<th>gufm1</th>
</tr>
</thead>
<tbody>
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<td>1875</td>
<td><img src="image1.png" alt="1875 D" /></td>
<td><img src="image2.png" alt="1875 M" /></td>
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<tr>
<td>1825</td>
<td><img src="image3.png" alt="1825 E" /></td>
<td><img src="image4.png" alt="1825 N" /></td>
</tr>
<tr>
<td>1790</td>
<td><img src="image5.png" alt="1790 F" /></td>
<td><img src="image6.png" alt="1790 O" /></td>
</tr>
</tbody>
</table>
Figure 5.2: Inclination maps for the southwest Pacific region from 1633 to 2000. Comparing the results of the *gufm1* dipole model (A-I) with the *gufm1* model (J-R). Contour interval is 10°.
5.1.1.3 Intensity plots

Again, intensity plots in the same period of time for both models are shown (Figure 5.3). In general, the gufm1 dipole model gives lower intensity around the equator, but higher intensity towards the south magnetic pole – in Antarctica - than the gufm1 model. The disagreement gets bigger further back in time. For example, in the gufm1 dipole model there is very low intensity northwest of Papua New Guinea in 1700, and very high intensity across New Zealand, (Figure 5.3 [H]). In 1875 and 1825 (D, M and E, N respectively), the shape of the intensity contours of both model are not compatible although there is a good distribution of data in these epochs. This difference could relate to gufm1 being a smooth model with Gauss coefficients derived as a function of time while, the gufm1 dipole model, calculated separately for each epoch, is more affected by roughness in the data distribution.

The differences of declination, inclination and intensity plots suggest that using the gufm1 dipole coefficients to fill the data gaps does not appear to be a very good method to represent the magnetic field in the southwest Pacific region.
<table>
<thead>
<tr>
<th>Year</th>
<th>gufm1 dipole</th>
<th>gufm1</th>
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</thead>
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<tr>
<td>1875</td>
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<td>1825</td>
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</tr>
<tr>
<td>1790</td>
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<td><img src="image" alt="O" /></td>
</tr>
<tr>
<td>Year</td>
<td>gufm1 dipole</td>
<td>gufm1</td>
</tr>
<tr>
<td>------</td>
<td>-------------</td>
<td>-------</td>
</tr>
<tr>
<td>1773</td>
<td><img src="image" alt="Intensity map for 1773" /></td>
<td><img src="image" alt="Intensity map for 1773" /></td>
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<td>1700</td>
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<td>1633</td>
<td><img src="image" alt="Intensity map for 1633" /></td>
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Figure 5.3: Intensity maps for the southwest Pacific region from 1633 to 2000. Comparing the results of the gufm1 dipole model (A-I) with the gufm1 model (J-R). Contour interval is 5000 nT.
5.1.2 Results of the self-consistent model

As for the *gufm1* dipole model, results for the self-consistent model are presented as a sequence of contours plots of declination, inclination and intensity for the periods of time between 1675 and 1875.

5.1.2.1 Declination Plots

Four main points can be made from the comparison between the self-consistent and *gufm1* models for the time between 1900 to 1633 in the southwest Pacific region, Figure 5.4.

1. Both models show a good degree of similarity for all epochs except for 1700. Again, this difference arises probably from the effect of the data distribution on the SCHA [F].
2. A particular similarity is the gradient in declination across Australia.
3. The differences include that the self-consistent model gives slightly lower declinations for New Zealand than are given by the *gufm1* model.
4. The self-consistent model shows that the declination has a more gradual change in the Antarctica region close to the magnetic pole than the *gufm1* model. This may be related to this declination contours for the *gufm1* model being based on global data, while the SCHA of self-consistent model has very little data in the deep south.
<table>
<thead>
<tr>
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<td><img src="image" alt="I" /></td>
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<tr>
<td>1825</td>
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Results of Modelling Secular variation in the Southwest Pacific

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Modelling Secular Variation in the Southwest Pacific for the last 400 year

Figure 5.4: Declination maps for the southwest Pacific region from 1900 to 1633. Comparing the results of the self-consistent model (A-G) with the gufm1 model (H-N). Contour interval is 5°.
5.1.2.2 Inclination Plots

The inclination plots for both the self-consistent and gufm1 models are shown in Figure 5.5 below for the time from 1633 to 1900. No major differences are observed between the self-consistent and global gufm1 models for inclination in the time period considered. The inclination plots from the self-consistent model show generally slightly lower values over New Zealand and adjacent areas than the gufm1 models, especially between 1633 and 1790. All the self-consistent plots show a maximum inclination value reaching close to $+50^\circ$ in the north Pacific region where the maximum values in the gufm1 model reach only to $+30^\circ \sim +35^\circ$. In New Zealand, the inclination given by both models increases slowly going backwards in time. This increase agrees with the inclination variation found by (G. M. Turner & Lillis, 1994) from palaeomagnetic lake sediment data.
<table>
<thead>
<tr>
<th>Year</th>
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<th>$gufm1$</th>
</tr>
</thead>
<tbody>
<tr>
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<td><img src="image" alt="A" /></td>
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<tr>
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</tr>
<tr>
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<td><img src="image" alt="C" /></td>
<td><img src="image" alt="J" /></td>
</tr>
<tr>
<td>Year</td>
<td>Self-consistent</td>
<td>gufm1</td>
</tr>
<tr>
<td>------</td>
<td>-----------------</td>
<td>-------</td>
</tr>
<tr>
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<tr>
<td>1773</td>
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<td>1700</td>
<td><img src="image" alt="F Diagram" /></td>
<td><img src="image" alt="M Diagram" /></td>
</tr>
</tbody>
</table>
Figure 5.5: Inclination maps for the southwest Pacific region from 1900 to 1633. Comparing the results of the self-consistent model (A-G) with the gufm1 model (H-N). Contour interval is 10°.
5.1.2.3 Intensity Plots

Figure 5.6 shows intensity plots from the regional and global models for the southwest Pacific region. The *gufm1* model predicts lower intensity values than does the self-consistent model in the Antarctica region. Hence the maximum intensity value presented in the self-consistent model is 75000 nT in 1875 and 1825 increasing backwards in time. However, the maximum intensity value of the *gufm1* model is only 65000 nT since 1775 increasing slowly back in time. The clearest disagreement between the two models is observed in the north of the spherical cap where there are some low intensity areas (≈30000 nT “orange spaces”) in the self-consistent model which do not appear in the *gufm1* model.
<table>
<thead>
<tr>
<th>Year</th>
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<tr>
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<td>1825</td>
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</table>
## Results of Modelling Secular variation in the Southwest Pacific

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<thead>
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<th>gutm1</th>
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</thead>
<tbody>
<tr>
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<tr>
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<td><img src="image" alt="E" /></td>
<td><img src="image" alt="L" /></td>
</tr>
<tr>
<td>1700</td>
<td><img src="image" alt="F" /></td>
<td><img src="image" alt="M" /></td>
</tr>
</tbody>
</table>
Figure 5.6: Intensity maps for the southwest Pacific region from 1900 to 1633. Comparing the results of the self-consistent model (A-G) with the \textit{gufm1} model (H-N). Contour interval is 8000 nT.
5.1.3 Comparison of the two models

To investigate and assess the usefulness of the, self-consistent and \textit{gufm1} dipole models, a comparison between these models has been made. The declination plots for the \textit{gufm1} dipole model show poor field prediction in the study area in period between 1650-1800. The declinations from both models are reasonably similar from 1825 onwards although the declinations for the self-consistent model are about $\sim 5^0$ higher than for the \textit{gufm1} dipole model. The self-consistent model declinations are also more like \textit{gufm1} over most of the area.

The inclination plots for both the \textit{gufm1} dipole and self-consistent models are similar although the inclination values for the \textit{gufm1} dipole model are about $\sim 5^0$ steeper than for the self-consistent model before about 1775. Intensity contours for the \textit{gufm1} dipole model do not appear realistic from 1633 until around 1775. Contours after this epoch are of similar shape to those from the self-consistent model. Overall, the intensity values at equatorial and high latitudes for the self-consistent model are higher by about $\sim 25000$ nT than for the \textit{gufm1} dipole model. It appears that the declination, inclination and intensity values from the self-consistent model have a closer similarity to \textit{gufm1} global SHA although there are some differences.

Although contour plots of declination, inclination and intensity of both the \textit{gufm1} dipole and self-consistent models are interesting and visual, they do not clearly show which is the better model to describe the field in our area of interest. Therefore, in order to investigate the reliability of both models, the misfit to the data has been calculated for each period of time from 1633 to 1875, Table 5.1. The values are the root mean squared difference between the magnetic field values $(B_X, B_Y, B_Z)$ which have been used to calculate the SCHA model,( see 4.2.1), and the predicted magnetic field value calculated from the final SCHA model coefficients at the latitude and longitude of each data point.
<table>
<thead>
<tr>
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<th>Self-consistent model</th>
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</thead>
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</tr>
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<td>1700</td>
<td>5473 nT</td>
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<tr>
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<td>12155 nT</td>
<td>1562 nT</td>
</tr>
<tr>
<td>1790</td>
<td>10838 nT</td>
<td>1406 nT</td>
</tr>
<tr>
<td>1825</td>
<td>8515 nT</td>
<td>6560 nT</td>
</tr>
<tr>
<td>1875</td>
<td>13309 nT</td>
<td>3279 nT</td>
</tr>
</tbody>
</table>

Table 5.1: The root mean square difference between estimated values (Xe, Ye, Ze) and modeled values at each latitude and longitude of data point for each period of time from 1633 to 1875.

It is clearly apparent from the results in the table 5.1 that the calculated root mean square value for the self-consistent model gives significantly smaller values than the gufm1 dipole model. A comparison of root mean square misfits between magnetic field values in the X, Y and Z direction for both models, Table C.1 in Appendix C, shows that the $B_x$ and $B_z$ magnetic field values for the gufm1 dipole model are the most significant contributions to the magnetic field misfits especially in the historical period from 1633 to 1790. This is clearly visible in the gufm1 dipole declination contours plots, Figure 5.1, as these plots are poor for these epochs and differ from those for the gufm1 model. As the total magnetic field strength is of the order of 50000 nT, the gufm1 dipole average root mean square difference is up to nearly 40% of the total field, whilst the self-consistent root mean square difference is about 6% only. Therefore, the self-consistent model appears to provide a more reliable model field for our region of interest than dipole gufm1.

There are a number of important reasons of why the gufm1 dipole model does not give as good a fit to the data values as the self-consistent model. As explained previously, for the gufm1 dipole model for each epoch the first three coefficients, $g_0^1$, $g_1^1$ and $h_1^1$ of the gufm1 model were used to compute the missing declination and/or inclination and/or intensity values. The estimated values therefore are based only on the dipolar part of the gufm1 model and do not consider the non-
dipole part of the geomagnetic field in the southwest Pacific region. The self-consistent model was built from a distribution of data within the cap with the aid of a linear extrapolation of SCH Gauss coefficients up to degree 2 that include both the dipolar and non-dipolar parts of the field. The extrapolation also includes reliable IGRF observed data for 2000, 1950 and 1900.

Another reason of why one should not put much weight on the *gufm1* dipole model is the uneven distribution of the data sites through the spherical cap in some epochs. An example is the contours of declination for 1700, Figure 5.1, where that data align linearly across the top of the cap. In contrast with this the use, through extrapolation, of the previous values of SCH coefficients to help fill in data gaps in the self-consistent model means that the effect of poor data distribution on the results is reduced. The only exception to this is the declination plot for 1700, Figure 5.4, although this still has some of the features seen for the other epochs.
5.2 The 25 years intervals estimations

This section contains a summary of the declination, inclination and intensity predicted from the self-consistent model for the southwest Pacific region for the last 400 years for a timescale of 25 years intervals. These semi-continuous geomagnetic field variations were obtained from the final SCHA Gauss coefficients as follows:

- First, the SCH coefficients for each epoch [1900, 1875, 1825, 1790, 1773, 1700 and 1600] were taken. See section 4.2.2, figure 4.2.
- Second, for consistency with the way in which the model was developed, linear functions were fitted to these coefficients \([g_0^0, g_1^0, g_1^1, h_1^1, g_0^1, g_2^1, h_2^1, g_2^2, \text{ and } h_2^2]\).
- Third, the coefficients were interpolated to 25-year intervals.
- Fourth, the interpolated values were used to produce contour plots of declination, inclination and intensity based on the same set of latitudes and longitudes for each plot, as given in Table 5.2. These locations give a relatively uniform distribution of points over the spherical cap.
Table 5.2: lists the latitudes and longitudes that have been used to produce a smooth magnetic field model for last 400 years in the Southwest Pacific region.
5.3 **Continuous geomagnetic field models from the self-consistent model**

Figure 5.7 shows the directional and magnitude plots for 25-year intervals from 1600 to 2000 derived from the self-consistent model. A region of $>10^\circ$ eastward declination occurs in the southwest Pacific in 1600’s and 1700’s until it disappears thereafter. From about 1825 there is a gradual change to higher declinations moving up over New Zealand. Declinations in New Zealand provided by the model lie, in general, between $5^\circ$ to $10^\circ$ from 1700 to 1900 then declination increase gradually with time and reach up to $15^\circ$ to $20^\circ$ in 2000. This declination change agrees with secular variation curves for New Zealand for the last 2500 year proposed by (G. M. Turner & Lillis, 1994)) except for the 1600’s. This difference could be due to the lack of input data in the earlier epochs.

All the inclination curves are shallower from 1600 to 1750; they then steepen slowly in the south. The inclination in New Zealand varies from $-55^\circ$ to $-65^\circ$ from 1600 up to 1725 then it steepens gradually to be $-65^\circ$ to $-73^\circ$ in 2000. Generally, inclination curves provide consistent values for the time span considered. A band of low intensity $\sim 32000$ nT appears north of Papua New Guinea and expands gradually in the north of the spherical cap from 1700 to 2000.

Although, there is a big gradient in intensity in the region close to Antarctica from 1600-1725, the intensity is almost steady $\sim 40000$ nT over most of the southwest Pacific region. The general behavior of the intensity is a steady decrease through the time span considered. An animation with declination, inclination and intensity maps is available as auxiliary material.
### Results of Modelling Secular variation in the Southwest Pacific

<table>
<thead>
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<td><img src="image9" alt="Image" /></td>
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<tr>
<td></td>
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<td>Inclination</td>
<td>Intensity</td>
</tr>
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<td>-------------</td>
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### Results of Modelling Secular variation in the Southwest Pacific

<table>
<thead>
<tr>
<th>Year</th>
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<th>Intensity</th>
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<tr>
<td>Year</td>
<td>Declination</td>
<td>Inclination</td>
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</tr>
<tr>
<td>------</td>
<td>-------------</td>
<td>-------------</td>
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</tr>
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<td><img src="image5" alt="Map of Inclination" /></td>
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Modelling Secular Variation in the Southwest Pacific for the last 400 years.
### Results of Modelling Secular variation in the Southwest Pacific

<table>
<thead>
<tr>
<th>Year</th>
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</table>
Figure 5.7: Declination, inclination and intensity maps for the self-consistent model for southwest Pacific region interpolated to every 25 years from 1600 to 2000. An animation is available as auxiliary material. Contour intervals are 5° for declination, 10° for inclination and 5000 nT for intensity.
CHAPTER 6

Summary and Suggestions for Future work

A summary of the key results from this project is presented, followed by suggestions for further work to extend these results.
6.1 SUMMARY

In this study, two regional model of the geomagnetic field for the southwest Pacific region have been developed covering the time span from 1600 to 2000. Both models have been calculated by using the SCHA regional technique. These models have been calculated using observational and estimated declination, inclination and intensity for different epochs.

The aim of this research was determining the best geomagnetic field model in the southwest Pacific region for the last 400 years. Observations of all three magnetic field components are required in order to produce a model for the field. Where data gaps occur unrecorded values of inclination, intensity and declination at a site have been calculated using the geocentric dipole components from the gufm1 global model. Analysis of these data gives the gufm1 dipole model. Maps for the geomagnetic field for this model suggest that this model does not give a good representation of the geomagnetic field in southwest Pacific region.

A self-consistent model has been obtained from linear extrapolation of SCHA Gauss coefficients up to $l=2$ derived initially from 2000, 1950 and 1900 IGRF observed data. The extrapolation has then been applied further back in time to seven different epochs. The magnetic field plots show that this model gives a much closer fit to the gufm1 model which is the best historical global model since 1600.

The results suggest that the self-consistent model provides a more reliable model field for the southwest Pacific region than the gufm1 dipole model. The root mean squared misfit of the self-consistent model to the field values averages 2900 nT, compared to 23000 nT for the gufm1 dipole model.
6.2 **FUTURE WORK**

The SCHA used for the southwest Pacific region was the method presented by (Haines, 1985a). Several revised SCHA methods have been published since that, for example the revised-SCHA method (R-SCHA) of (Thébault, Schott, & Mandea, 2006). It would be useful to use the revised-SCHA to represent the field in the region of interest.

Moreover, more data could be added to the observational data. Paleomagnetic and archeomagnetic data acquisition from either heated artifacts/rocks or sedimentary sequences have good magnetic records. More available data would help to give a better model of the geomagnetic field. For example, including a-palaeomagnetic data such as (G. M. Turner & Lillis, 1994) would be a useful step.
APPENDIX A

Figure A. 1: The first Atlantic Ocean map of Edmond Halley in 1701 declaring only declination lines. Circles and triangles on the map represent latitude and longitude west of London associated with magnetic declination from two different voyages (Halley, 1710).
APPENDIX B

FILLING IN THE DATA GAPS USING GUFM1 MODEL

Other possibilities for filling in gaps in the available data other than those covered in section 4.2 are:

5. If the Declination and intensity both are available for a site at \((\theta, \varphi)\) but not inclination, then inclination need to be estimated from the \textit{gufm1} coefficients hence

\[
B_{xe} = F \cos I_e \cos D
\]

\[
B_{ye} = F \cos I_e \sin D
\]

\[
B_{ze} = F \sin I_e
\]

6. If the inclination data is the only data available from the observational data at site \((\theta, \varphi)\), then both declination and intensity need to be estimated from \textit{gufm1} coefficients hence

\[
D_e = \tan^{-1} \left( \frac{B_y}{B_x} \right)
\]
the $B_x$ and $B_y$ are calculated from equation 4.2 and the estimated $B_{xe}$, $B_{ye}$ and $B_{ze}$ become

\begin{align*}
B_{xe} &= F_e \cos I \cos D_e \\
B_{ye} &= F_e \cos I \sin D_e \\
B_{ze} &= F_e \sin I
\end{align*}

7. If the intensity data is the only data available from the observational data at site $(\theta, \phi)$, then both declination and inclination need to be estimated from $gufm1$ coefficients hence

\begin{align*}
B_{xe} &= F \cos l_e \cos D_e \\
B_{ye} &= F \cos l_e \sin D_e \\
B_{ze} &= F \sin l_e
\end{align*}
APPENDIX C

A table of the calculated root mean square of both *gufm1* dipole and self-consistent model of the magnetic field value in the X, Y and Z direction

<table>
<thead>
<tr>
<th>Year</th>
<th>$\varepsilon_X$ nT</th>
<th>$\varepsilon_Y$ nT</th>
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</tr>
</thead>
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<td><em>gufm1</em> dipole</td>
<td>Self consistent</td>
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<td>1875</td>
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<td>1988</td>
<td>2096</td>
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</table>

Table C.1: The X, Y and Z root mean squares of *gufm1* dipole and self-consistent model from 1633 to 1875.
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