
José Félix Sanz-Sanz

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Further enquiries to:
The Administrator
Chair in Public Finance
Victoria University of Wellington
PO Box 600
Wellington 6041
New Zealand

Phone: +64-4-463-9656
Email: cpf-info@vuw.ac.nz

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REVENUE-MAXIMISING TAX RATES IN PERSONAL INCOME TAXATION
IN THE PRESENCE OF CONSUMPTION TAXES: A NOTE

José Félix Sanz-Sanz*
Universidad Complutense de Madrid

Abstract:

This note computes revenue-maximising tax rates in personal income taxes in the presence of consumption taxes. It finds that the traditional Laffer analysis, which neglects the effects of marginal tax rates on consumption, overestimates the magnitude of revenue-maximising tax rates. The bias caused by this oversight is computed.

JEL codes: D11, D61, H21, H24, H2, H31

Key words: marginal tax rates, Laffer curve, consumption taxes, tax revenue, tax behaviour.

* E-mail address: jfelizs@ccce.ucm.es, Fax 00 34 913 942 431

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1. Introduction

The so-called Laffer curve embodies the well-established proposition that maximum tax revenue is reached somewhere between 0 and 100 per cent of marginal tax rates. As the literature has shown, this result stems from the interaction of two opposing effects: the mechanical and the behavioural effects caused by marginal tax rate alterations\(^1\). The former captures, in the absence of any behavioural reaction, the pure arithmetical change in tax revenue, whereas the latter quantifies the revenue change due to the impact of marginal tax rates on economic activity. Although the notion of the Laffer curve is applicable to any tax as well as to the tax system as a whole, most of the empirical and theoretical work on the Laffer curve has focused on personal income taxation (PIT). However, the traditional formulation of the Laffer curve in PIT has omitted the impact of marginal tax rates on consumption\(^2\). As long as PIT marginal tax rates modify household disposable income and therefore taxable consumption, changes in PIT marginal tax rates also alter the tax revenue from consumption. This oversight has important implications for the graphical profile of the Laffer curve and on the magnitude of the revenue-maximising marginal tax rates. In this note the computation of revenue-maximising tax rates is remodelled in order to take into consideration the impact of PIT marginal tax rates on consumption tax revenue.

2. Model and results

Let us assume an economy that levies taxes on personal income and on consumption according to the following setting:

1. \(T_Y = T_Y(y, \zeta)\) is the personal income tax function, where \(y\) is taxable income and \(\zeta\) is the applicable tax schedule, defined in terms of a vector of increasing marginal tax rates \(\tau = (\tau_0, \tau_1, ..., \tau_k)\) and a set of sequential thresholds \(A = (a_0, a_1, ..., a_k)\).

2. \(T_C = T_C(V, \zeta)\) represents the consumption tax function, defined on disposable income, \(V = y-T_Y\), according to a set of tax-inclusive rates, including VAT and excises, levied on the \(Q\) categories of goods and services exchanged in the economy, \(\zeta = (t_1, t_2, ..., t_q)\).

Given this general outline, if an individual taxpayer, \(i\), earns taxable income, \(y_i\), has an average propensity to consume \(\chi_i\), and his total consumption, \(C_i\), is distributed among the \(Q\) existing goods according to the weights \(w_{qi}\), such that \(w_{qi} = \frac{c_{qi}}{C_i}\) and \(\sum w_{qi} = 1\), then, replicating Creedy and Gemmell (2006), his or her total tax liability as a consumer as well as an income generator will be \(T_{T_i} = T_{Y_i} + T_{C_i}\), where:

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\(^1\) Using different approaches, Saez (2004), Giertz (2009), Creedy (2011, chapter 13) and Saez et al. (2012) present detailed discussions on this issue.

\(^2\) In a recent paper by Carey et al. (2015) the implications of changes in the consumption tax rate on the elasticity of taxable income (ETI) is explored. For the case of a quasi-linear utility function, these authors find that ETI is unaffected by the level of the consumption tax rate. This, however, does not imply that the PIT Laffer curve is also unaffected.
\[ T_{y_i} = \tau_{k_i}(y_i - a'_{k_i}) \]  

and  
\[ T_{c_i} = \alpha_i \cdot [y_i - T_{y_i}] \]  

in which \( \tau_{k_i} \) and \( a'_{k_i} \) denote, respectively, the marginal tax rate and the effective threshold of the tax bracket into which the taxpayer’s taxable income falls and \( \alpha_i \) represents the taxpayer’s effective tax rate on consumption - i.e. \( \alpha_i = \chi_i \cdot \sum_{q=1}^{Q} w_{qi} t_q \). The analytical expression for the effective threshold is  
\[ a'_{k_i} = \sum_{j=0}^{k-1} a_j \cdot \left( \frac{\tau_{j+1} - \tau_{j}}{\tau_{k_i}} \right)^3 \]

2.1 The Laffer curve

The so-called Laffer curve can be characterised by identifying its peak. This locus of maximum revenue meets the following elasticity condition \( \eta_{T_c, \tau} = 0 \). Therefore, when consumption and income taxes are simultaneously taken into account, the maximum tax bill paid by an individual taxpayer when his or marginal tax rate, \( \tau_{k_i} \), changes will be reached when:

\[ \eta_{T_y, \tau_k} = -\frac{\tau_{c_i} \cdot \eta_{T_c, \tau_k}}{\tau_{y_i}} \]

where \( \eta_{T_y, \tau_k} \) and \( \eta_{T_c, \tau_k} \) are, respectively, the elasticity of PIT and consumption tax revenue with respect to the PIT marginal tax rate. The former, as obtained by Creedy (2011, page 277) and Creedy and Gemmell (2014), can be expressed as:

\[ \eta_{T_y, \tau_k} = \frac{y_i - a_k}{y_i - a'_{k_i}} - \left( y_i - a'_{k_i} \right) \cdot \left( 1 - \tau_{k_i} \right) \ast \eta_y(1-\tau_{k_i}) \]

where the latter is derived from [2] after some mathematical manipulation and rearrangement of terms - see Appendix:-

\[ \eta_{T_c, \tau_k} = -\left( 1 + \eta_{\tau_i, V_i} \right) \cdot \tau_{k_i} \ast y_i \ast \eta_y(1-\tau_{k_i}) + (y_i - a_k) \]

As a consequence, when income and consumption taxes are considered, the relevant (Laffer) revenue-maximising marginal tax rate for taxpayer \( i, \tau_{k_i}^{L+i+c} \), will be given by:

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3 As highlighted by Creedy and Gemmell (2006), equation [1] and the analytical expression of the effective threshold result directly from rearranging the expression:

\[ T_{y_i} = \tau_{k_i} \cdot y_i - \sum_{j=1}^{k} a_j \cdot (\tau_j - \tau_{j-1}) + \tau_{k_i} \cdot (y_i - a_k) \]

which represents formally the application of the tax schedule to taxable income, \( y_i \).

4 This derives directly from the fact that \( \eta_{T_y, \tau_k} = \frac{\tau_{c_i} \cdot \eta_{T_c, \tau_k}}{\tau_{y_i}} \). For the explicit form of the elasticity functions \( \eta_{T_y, \tau_k} \) and \( \eta_{T_c, \tau_k} \), see the Appendix.
Practice computes revenue overestimates the peak of the PIT Laffer curve is wider than normally assumed. 

As a result, dismissing the behavioural effect of PIT marginal tax rates on consumption generates biased revenue-maximising tax rates. To be specific, the next condition holds:

\[
\tau^L_{k+i} = \frac{\left(1-\frac{a_k}{y_{i}}\right)\eta^{-1}_{y_{i}(1-\tau_{k})} - \alpha_i (1+\eta_{\alpha_i V_i})}{1+\left(1-\frac{a_k}{y_{i}}\right)\eta^{-1}_{y_{i}(1-\tau_{k})}}
\]

and consequently the following proposition can be stated:

**Proposition 1:** if \(\alpha_i > 0\) the neglect of the effects of income taxation on consumption tax revenue overestimates the peak of the PIT Laffer curve, \(\tau^L_{k} > \tau^L_{k+i}\), as long as \(\alpha_i < \frac{1}{1+\eta_{\alpha_i V_i}}\).

Note that \(\alpha_i < \frac{1}{1+\eta_{\alpha_i V_i}}\) is a mild condition, since the regular range of actual effective tax rates on consumption are well below that boundary. Thus, according to proposition 1, commonly computed revenue-maximising tax rates, \(\tau^L_{k}\), are misleading as they are higher than the actual revenue-maximising tax rates, \(\tau^L_{k+i}\). In other words, if consumption tax revenue is taken into account, then maximum tax revenue is reached at lower PIT marginal tax rates than otherwise. As a consequence, revenue forecasting is overestimated and the “prohibitive range” of the Laffer curve is wider than normally assumed.

By way of illustration for the case of Spain, the left-hand side of the graph in Figure 1 exhibits the Laffer curves for the median Spanish taxpayer in 2008\(^5\). Both Laffer curves, regarding and neglecting consumption tax revenue (VAT+Excises), are depicted. As shown, taking account of consumption tax revenue reduces PIT revenue-maximising tax rates from 0.598 to 0.556,

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\(^5\)This median Spanish taxpayer in 2008 is characterised by the following parameter values \(\eta_{\alpha_i V_i} = 0.1150\), \(\alpha_i = 0.0863\), \(\eta_{y_{i}(1-\tau_{k})} = 0.67\).
shrinks the available tax revenue set and expands the “prohibitive” area of the relevant Laffer curve. As a result, misguided revenue forecasts follow. As depicted in the right-hand side of the graph in Figure 1, for an equiproportional increase in 2008 marginal tax rates, revenue miscalculation rises with the level of income. Table 1 displays the median difference generated in the computation of Laffer marginal tax rates on 578,157 Spanish taxpaying units. These data contain information regarding the magnitude and nature of the incomes of 578,157 taxing units, as well as the composition of their consumption with regard to 262 distinct goods. The Appendix briefly describes the data used. As Table 1 displays for the whole country, the median difference reached 6.7 percentage points which represents over 20% in relative terms. This median bias varies according to the background peculiarities of the taxpayer, such as the level of income, the region of residence or the size of the tax unit. These figures indicate that Laffer marginal tax rates are significantly overestimated in Spain when the impact of PIT marginal tax rates on consumption tax revenue is omitted.

Figure 1. Impact of consumption tax revenue on PIT Laffer curves and total tax revenue (simulation)
Table 1. Median values of the distribution of maximising-tax rate difference due to neglecting the influence of PIT marginal rates on consumption tax revenue (2008).

<table>
<thead>
<tr>
<th></th>
<th>$(\bar{\tau}<em>{k_i}^{c_t} - \bar{\tau}</em>{k_i}^L)$</th>
<th>$(\bar{\tau}<em>{k_i}^{L+c_t} - \bar{\tau}</em>{k_i}^{c_t})$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Whole tax population</strong></td>
<td>-6.7</td>
<td>20.78</td>
</tr>
<tr>
<td><strong>By tax brackets (2008) in €</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 - 17,707.20</td>
<td>-4.8</td>
<td>8.95</td>
</tr>
<tr>
<td>17,707.20 – 33,007.20</td>
<td>-7.9</td>
<td>36.34</td>
</tr>
<tr>
<td>33,007.20 – 53,407.20</td>
<td>-8.6</td>
<td>55.96</td>
</tr>
<tr>
<td>&gt; 53,407.20</td>
<td>-6.5</td>
<td>18.99</td>
</tr>
<tr>
<td><strong>By region of residence</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Andalusia</td>
<td>-7.3</td>
<td>22.77</td>
</tr>
<tr>
<td>Aragon</td>
<td>-6.5</td>
<td>21.13</td>
</tr>
<tr>
<td>Asturias</td>
<td>-7.3</td>
<td>26.54</td>
</tr>
<tr>
<td>Balearic Islands</td>
<td>-6.5</td>
<td>18.66</td>
</tr>
<tr>
<td>Canary Islands</td>
<td>-6.7</td>
<td>20.14</td>
</tr>
<tr>
<td>Cantabria</td>
<td>-7.1</td>
<td>23.44</td>
</tr>
<tr>
<td>Castile and León</td>
<td>-6.6</td>
<td>20.44</td>
</tr>
<tr>
<td>Castile-La Mancha</td>
<td>-6.7</td>
<td>19.36</td>
</tr>
<tr>
<td>Catalonia</td>
<td>-6.2</td>
<td>19.90</td>
</tr>
<tr>
<td>Valencian Community</td>
<td>-6.7</td>
<td>18.86</td>
</tr>
<tr>
<td>Extremadura</td>
<td>-7.4</td>
<td>20.71</td>
</tr>
<tr>
<td>Galicia</td>
<td>-7.0</td>
<td>21.23</td>
</tr>
<tr>
<td>Madrid</td>
<td>-6.5</td>
<td>20.93</td>
</tr>
<tr>
<td>Murcia</td>
<td>-7.3</td>
<td>20.36</td>
</tr>
<tr>
<td>La Rioja</td>
<td>-6.5</td>
<td>18.78</td>
</tr>
<tr>
<td>Ceuta and Melilla</td>
<td>-7.8</td>
<td>29.13</td>
</tr>
<tr>
<td><strong>By size of tax unit</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One family member</td>
<td>-6.5</td>
<td>21.05</td>
</tr>
<tr>
<td>Two family members</td>
<td>-6.8</td>
<td>22.11</td>
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<tr>
<td>Three family members</td>
<td>-7.0</td>
<td>21.06</td>
</tr>
<tr>
<td>Four family members</td>
<td>-6.9</td>
<td>20.14</td>
</tr>
<tr>
<td>Five or more family members</td>
<td>-7.0</td>
<td>18.66</td>
</tr>
</tbody>
</table>

3. Conclusion

This note models how consumption taxes may affect the revenue power of the tax system when PIT marginal tax rates change. It is found that not taking into consideration the effect of PIT marginal tax rates on consumption overestimates the revenue impact of marginal tax rates. This implies that the reported standard revenue-maximising tax rates, the so-called Laffer tax rates, are systematically higher than the actual Laffer tax rates, that is to say they are higher than those which take account of consumption tax revenue. This result has significant implications for personal income tax design.
References


Appendix

This Appendix derives the analytical expressions for \( \eta_{T_{yi}T_{ki}} \) and \( \eta_{T_{ci}T_{ki}} \) and succinctly describes the data used.

- **Derivation of analytical expressions for \( \eta_{T_{yi}T_{ki}} \) and \( \eta_{T_{ci}T_{ki}} \).**

  As shown in Creedy (2011, page 277) and Creedy and Gemmell (2014), given the income tax function \( T_Y = \tau_k(y_i - a_k) \) and \( a_k = \sum_{j=0}^{k-1} a_j \cdot \left( \frac{\tau_j - \tau_i}{\tau_{ki}} \right) \), the elasticity of the taxpayer’s tax bill with respect to his/her maximum marginal tax rate, \( \tau_{ki} \), will be expressed as -the prime indicates partial derivative-:

  \[
  \eta_{T_{yi}T_{ki}} = \eta'_{T_{yi}T_{ki}} + \eta'_{T_{yi}y_i} \cdot \eta_{y_iT_{ki}} \tag{A.1}
  \]

  where \( \eta'_{T_{yi}T_{ki}} \) is the direct effect on revenue caused by the change in \( \tau_{ki} \), which equals \( \frac{y_i - a_k}{y_i - a_{ki}} \) and \( \eta'_{T_{yi}y_i} \) is the (partial) revenue elasticity with respect to income that takes on \( \frac{y_i}{y_i - a_{ki}} \) and \( \eta_{y_iT_{ki}} \) is the taxable income elasticity that can be rewritten in terms of the more popular net-of-tax rate as \( \eta_{y_iT_{ki}} = \frac{\tau_{ki}}{\tau_{ki} - 1} \cdot \eta_{y_i(1-\tau_{ki})} \). Bearing all these considerations in mind, the result is:

  \[
  \eta_{T_{yi}T_{ki}} = \frac{y_i - a_k}{y_i - a_{ki}} \cdot \left( \frac{y_i}{y_i - a_{ki}} \right) \cdot \frac{\tau_{ki}}{1 - \tau_{ki}} \cdot \eta_{y_i(1-\tau_{ki})} \tag{A.2}
  \]

  In relation to \( \eta_{T_{ci}T_{ki}} \), if we depart from the tax function \( T_c = \alpha_i \cdot [y_i - T_Y] \), then

  \[
  \eta_{T_{ci}T_{ki}} = \left[ \eta'_{T_{ci}a_i} \cdot \eta_{a_i\nu_i} + \eta'_{T_{ci}\nu_i} \right] \cdot \eta_{\nu_iT_{ki}} \tag{A.3}
  \]

  This can be rewritten as:

  \[
  \eta_{T_{ci}T_{ki}} = \left[ 1 + \eta_{a_i\nu_i} \right] \cdot \left[ \eta'_{\nu_i\nu_i} \cdot \eta_{\nu_iT_{ki}} + \eta'_{\nu_i\tau_{ki}} \right] \tag{A.4}
  \]

  as \( \eta'_{T_{ci}a_i} = \eta'_{T_{ci}\nu_i} = 1 \) and \( \eta_{\nu_i\nu_i} = \eta'_{\nu_i\nu_i} = \eta'_{\nu_i\tau_{ki}} = 0 \). Furthermore, if we take into account that \( \eta'_{\nu_iy_i} = \frac{(1-\tau_{ki})y_i}{\nu_i} \), \( \eta'_{\nu_i\nu_i} = \frac{(1-a_k)\tau_{ki} y_i}{\nu_i} \) and, as above, that \( \eta_{y_iT_{ki}} = \frac{\tau_{ki}}{\tau_{ki} - 1} \cdot \eta_{y_i(1-\tau_{ki})} \), the final equation is

  \[
  \eta_{T_{ci}T_{ki}} = -\frac{(1+\eta_{a_i\nu_i})\tau_{ki}}{\nu_i} \cdot \left[ y_i \cdot \eta_{y_i(1-\tau_{ki})} + (y_i - a_k) \right] \tag{A.5}
  \]
Data

The individualised calculation of the expressions generated in this paper requires having available a microdatabase with individualised information regarding income and consumption expenditure. In the case of Spain there exist two separate bases of microdata with individualised information on incomes and consumption: the PIT tax returns panel (PIT panel) and the Family Expenditure Survey (FES). The first is elaborated annually by the Tax Administration on the basis of tax returns filed and includes highly detailed information on declared incomes and other socioeconomic variables. In turn, the FES, elaborated annually by the Spanish Statistical Office, incorporates the details of household consumption structure, disaggregated into 262 categories of goods. The microdata used in this paper are the result of the statistical matching between the two databases. The procedure followed has consisted of imputing to each PIT tax return its structure of consumption on the basis of the information contained in the FES. The statistical fusion was performed for 2008. This database is available on request from the author.
About the Authors

José Félix Sanz Sanz is a Professor in the Department of Applied Economics at Universidad Complutense de Madrid. His academic interest is mainly focused on the economic analysis of tax reforms and the impact of taxation on the behaviour of economic agents. In the period 2000-2004 he was the Deputy Director of Tax Studies at the Institute for Fiscal Studies (Ministry of Finance) and since 2005 he has run the tax research Department at FUNCAS, a Foundation of the Spanish Saving Banks.

Email: jfelizs@ccee.ucm.es