A SCHEME FOR INCENTIVIZING INVESTMENTS IN TRANSMISSION ENHANCEMENTS

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Abstract – We present an incentive scheme to stimulate investment in the improvement/expansion of the transmission network in the competitive market environment. The formulation of these incentives is based on a decentralized transmission asset investment model and is derived from the value added to the social welfare by an asset investment. In the formulation, we view each potential investor as a player in a cooperative game and we use the unique solution provided by the Shapley value to allocate the payment to each successful investor commensurated with the increase in social welfare the investment brings to the system. The formulation brings valuable insights on the transmission investment topic. We apply the proposed methodology to the Garver 6-bus system to illustrate the capabilites and flexibility of the scheme and to gain insights into the development of network improvements through the formulation proposed.

Keywords: Transmission planning, social welfare, investment incentives, cooperative game theory, Shapley value.

1 INTRODUCTION

The restructuring of the electricity industry has resulted in the advent of many new players, brokers, marketers, independent power producers and the creation of new structures, most notably the Independent System Operator (*ISO*) and the Regional Transmission Operator (*RTO*). We refer to the latter by the generic term of Independent Grid Operator (*IGO*). The *IGO* is emblematic of the changes resulting from the separation of the ownership from the control and operation of the transmission network.

In the planning of new transmission asset additions, the objectives of market efficiency improvement and social welfare maximization compete with those of profit maximization of the individual players and investors. Typical situations requiring transmission asset investments stem from the need to efficiently address congestion relief requirements by making the necessary improvements to the transmission network. Such investments impact each market player differently, some faring better and some worse as a result of the provided congestion relief.

Network expansion is by nature a very complex multi-period and multi-objective optimization problem. Its nonlinear nature, the lumpiness of transmission resources and the inherent uncertainty of future developments constitute major complicating factors. Its solution is very difficult, even under central decision making. In the vertically integrated structure, the construction of new transmission facilities is typically associated with the addition of new generating resources to facilitate their integration into the existing network. Given the strong control exerted by the state regulators over virtually every aspect of the regulated utility's activities, transmission planning must meet the requirements for regulatory approval. For transmission asset investments, the planning objectives are typically simplified to the minimization of total costs involved.

A wide range of techniques has been applied to investigate transmission planning. They include mathematical optimization methods such as linear programming [1], mixed-integer linear programming [2], Benders decomposition [3], and dynamic programming [4]; intelligent systems, such as genetic algorithms [5] and simulated annealing; and others, such as game theory models [6]. In the competitive electricity market environment, the solution of the transmission improvement/expansion problem requires some important modifications, such as the introduction of a new objective function, e.g., social welfare maximization [7][8]. In addition, the problem requires the consideration of the interaction between Financial Transmission Rights (*FTR*) and market power, the analysis of merchant transmission investment, and the effect of lumpiness and imperfect competition.

The changes introduced and the consideration of the aforementioned issues are necessitated by the major changes emanating from the restructuring of the electricity industry. The open access regime entailed the breakup of the well entrenched vertically integrated structure in the electricity industry. As a result, centralized decision making has been replaced by decentralized decisions and the setting up of the new *IGO* structure has resulted in the separation of ownership from operational control. While the *IGO* has wide responsibilities for regional planning, including transmission, the implementation of the plans are in the hands of current transmission owners or new transmission investors. In this widely modified planning paradigm the transmission investments have, however, failed to keep up with the steadily increasing load demands and the ever more intense utilization of the grid by an increasing number of transmission customers. One way to overcome this sorry picture in transmission investment is through the provision of appropriate incentives for expansion/improvement of the grid. Such schemes must take into account the physical constraints such as loop flow and lumpiness issues. Moreover, there are the additional complications arising from the competing objectives of the *IGO* to maximize societal benefits with those of individual investors to maximize their expected profits.

Other than the lumpiness of transmission investments, the free rider problem arisen from the public goods property of transmission assets, lack of clarity in regulatory policy, lack of regional institutions and need for state approval are among the key reasons of transmission under-investment. The sluggishness of transmission construction is because mismatches between those benefiting from the new facilities and those paying for them are often such as to ensure the new facilities do not get built. Effective procedures must be set up to ensure the timely recovery of transmission investments so that the expansion costs will be paid by those who benefit – the so-called participant funding approach – in order to have sufficient incentives to site new facilities.

Incentives formulated as reimbursement schemes are well known in the economics literature given to the seminal work of Vickrey and the extensions to other economic problems. These schemes are based on the notion that the remuneration should be a function of the difference in the social welfare with and without the added investment. In transmission planning, the formulation of investment incentives needs to pay careful attention to the network effects of the existing transmission grid and the extensive interactions among individual investments. As such, incentive mechanisms which reward those investors whose investments lead to increased total social welfare are appropriate under these schemes. The thrust of this paper is to explore the development of such incentive mechanisms for transmission asset investment.

New transmission assets can produce improvements in the network, such as congestion relief, that are beneficial to some or, even, all transmission customers. Cooperative game theory allows participants to jointly create added value and to receive a compensation based upon their contribution to the welfare of the system. There are several cooperative value allocation methods, such as the core, the nucleolus, and the Shapley value [9]. The latter entails the attractive attribute of uniqueness, which serves as a basis for sharing benefits among all the investors.

This paper proposes an incentive mechanism design for transmission network investment where the problem is modeled as a cooperative game in order to allocate the new value created in the network expansion. In our game the players are investors in transmission assets and the Transmission Planner (*TP*) reimburses these investors by offering them all or part of the social welfare increase due to them. The investors receive these incentive offers and send their rate of return requirements to the *TP*. If their requirements are lower than the incentives, then they are invited to invest. The whole process is iterative until there are no more investors willing to build transmission assets.

The paper is structured as follows. Section 2 describes two formulations of the transmission investment problem: a centralized model and a decentralized model. In Section 3 we propose an incentive scheme that rewards the investors in the decentralized model based on the expected increase of social welfare that they can provide. To calculate the amount of reward we define a cooperative transmission expansion game to allocate the gains obtained by the expansion among the investors using the Shapley value allocation method. In Section 4 we illustrate the application of the proposed incentive scheme to the Garver 6-bus system. The results provide a good example of effective incentive formulation for this system. We conclude with suggestions for future work in Section 5. Appendix A compares the centralized and decentralized formulations showing their equivalence under several assumptions. Finally, Appendix B presents the necessary background on cooperative game theory.

2 CENTRALIZED AND DECENTRALIZED TRANSMISSION INVESTMENT FORMULATIONS

The market-based transmission planning models presented in this section are related to the control level over the new investments that the transmission planner (*TP*) has. The market is modeled as a pool-based system. The double auction pool-based market mechanism has the objective of maximizing the social welfare, so as to determine the maximum net benefits for society, measuring the overall impacts of both sellers and buyers.

In the first model presented, the *TP* invests in new transmission assets whose costs are publicly available. Although the generators and demands bid in the market, the investment in transmission is centrally planned.

The second model allows investors to build new transmission assets, provided that they want to recover their investments with a certain rate of return. In this case, the *TP* decides the amount of money given to the investors based on some measure of the overall improvement of the market: the social welfare.

The description of both models for transmission investment follows.

2.1 Transmission Planning Model with Centralized Transmission Investment

Without loss of generality, we assume a single seller and a single buyer at each node $n = 0,1,\cdots,N$ of the network, where $\mathcal{L} \triangleq \{ \ell_1, \ell_2, \cdots, \ell_L \}$ is the set of lines and transformers that connect the buses of the network and $\mathcal{L}^c \triangleq \{ \ell_1^c, \ell_2^c, \cdots, \ell_{L^c}^c \}$ is the set of candidate lines and transformers. We use the binary variable m_j^c $(j = 1,\cdots, L^c)$ to model the presence of new transmission assets: it takes the value of 1 if the investment in transmission asset ℓ_j^c $(j=1,\cdots,L^c)$ is made, and 0 otherwise. We define the set $\mathcal{K}^C \triangleq \{k_1^c, k_2^c, \cdots, k_{L^c}^c\}$ of investment costs in new transmission assets, where each individual cost of a new transmission asset ℓ_j^c is expressed as k_j^c. The node n selling entity's marginal offer in period t is integrated and denoted by $\beta_{n,t}^s(p_{n,t}^s)$, where $p_{n,t}^s$ is the power injected at node n in period t. Similarly, the node n buying entity's marginal bid in period t is integrated and denoted by $\beta_{n,t}^b(p_{n,t}^b)$, where $p_{n,t}^b$ is the power withdrawn at node n in period t. The *TP* has a budget constraint, B_C, that takes into account the amount of monetary resources that can be used to construct new transmission assets.

The process to determine the successful bids/offers of the pool players per period is based on the maximization of the social welfare, as shown in [10]. The *TP* needs the information per period to maximize the aggregate social welfare (*SW*) minus the investment costs (*IC*) subject to the network constraints over a predefined planning horizon $\mathcal{T} \triangleq \{t:1,2,\cdots,T\}$, where t represents one period of the planning horizon. This optimization problem can be expressed as

$$\max\,(SW - IC) = \sum_{t\in\mathcal{T}}\sum_{n=0}^{N}\left[\beta_{n,t}^b(p_{n,t}^b) - \beta_{n,t}^s(p_{n,t}^s)\right] - \sum_{j=1}^{L^c} m_j^c k_j^c; \quad (1)$$

s.t.

$$g_{n,t}\begin{pmatrix} p_{0,t}^s, p_{1,t}^s, \cdots, p_{N,t}^s;\ p_{0,t}^b, p_{1,t}^b, \cdots, p_{N,t}^b; \\ m_1^c, \cdots, m_{L^c}^c \end{pmatrix} = 0 \quad (2)$$
$$\leftrightarrow \lambda_{n,t}; \forall n = 0,1,\cdots,N; \forall t \in \mathcal{T}$$

$$-f_i^{\max} \le h_{i,t}\begin{pmatrix} p_{0,t}^s, p_{1,t}^s, \cdots, p_{N,t}^s;\ p_{0,t}^b, p_{1,t}^b, \cdots, p_{N,t}^b; \\ m_1^c, \cdots, m_{L^c}^c \end{pmatrix} \quad (3)$$
$$\le f_i^{\max} \leftrightarrow (\mu_{i,t}^{\min}, \mu_{i,t}^{\max}); \forall i = 1,\cdots, L; \forall t \in \mathcal{T}$$

$$-f_j^{\max} \le h_{j,t}\begin{pmatrix} p_{0,t}^s, p_{1,t}^s, \cdots, p_{N,t}^s;\ p_{0,t}^b, p_{1,t}^b, \cdots, p_{N,t}^b; \\ m_1^c, \cdots, m_{L^c}^c \end{pmatrix} \quad (4)$$
$$\le f_j^{\max} \leftrightarrow (\mu_{j,t}^{\min}, \mu_{j,t}^{\max});\ \forall j=1,\cdots, L^c;\ \forall t \in \mathcal{T}$$

$$\sum_{j=1}^{L^c} m_j^c k_j^c \le B_C \quad (5)$$

$$m_j^c \in \{0,1\}; \forall j = 1,\cdots,L^c \quad (6)$$

$$0 \le p_{n,t}^s \le p_n^{s,\max}; \forall n = 0,1,\cdots,N; \forall t \in \mathcal{T} \quad (7)$$

$$0 \le p_{n,t}^b \le p_n^{b,\max}; \forall n = 0,1,\cdots,N; \forall t \in \mathcal{T} \quad (8)$$

where $g_{n,t}(\bullet)$ is the nodal real power flow balance equation at node n in period t, $h_{i,t}(\bullet)$ is the expression of the real power flow in asset ℓ_i in period t, and $h_{j,t}(\bullet)$ is the expression of the real power flow in candidate asset ℓ_j^c in period t. Power flows are bounded by the capacities $f_i^{\max}$ and $f_j^{\max}$. Likewise, the powers injected and withdrawn at node n in period t are limited by their maximum respective values $p_n^{s,\max}$ and $p_n^{b,\max}$. For every constraint set there is a corresponding set of dual variables: $\{\lambda_{n,t} : \forall n = 0,1,\cdots,N; \forall t \in \mathcal{T}\}$ for the power flow balance equations, $\{(\mu_{i,t}^{\min}, \mu_{i,t}^{\max}); \forall i = 1,\cdots,L; \forall t \in \mathcal{T}\}$ and $\{(\mu_{j,t}^{\min}, \mu_{j,t}^{\max}); \forall j = 1,\cdots,L^c; \forall t \in \mathcal{T}\}$ for the real power flows in existing and candidate assets, respectively. Note that if we make the assumption of having a dc power flow, we can express (2) as:

$$p_{n,t}^s - p_{n,t}^b = \sum_{n \ne m} B_{nm}\cdot(\delta_{n,t} - \delta_{m,t})\cdot L_{nm}, \forall n = 0,1,\cdots,N;$$
$$\forall t \in \mathcal{T},$$

where $B_{nm} = 1/X_{nm}$, X_{nm} is the reactance of the transmission asset connecting nodes n and m, $(\delta_{n,t} - \delta_{m,t})$ is the difference between the angles of nodes n and m in period t, and L_{nm} is the number of both existing and new transmission assets connecting nodes n and m, assuming that all the assets connected in parallel between the nodes are identical. In addition, (3) and (4) can be set as $|F_{nm,t}| = |B_{nm}\cdot(\delta_{n,t} - \delta_{m,t})| \le F_{nm}^{\max}$, where $F_{nm,t}$ is the active power flow in the transmission asset connecting nodes n and m in period t and $F_{nm}^{\max}$ corresponds to the maximum limit of the active power flow in the asset connecting nodes n and m.

The optimal solution of (1)-(8) determines the amount sold and bought by the pool players. In addition, the dual variables $\lambda_{n,t}$, $(\mu_{i,t}^{\min}, \mu_{i,t}^{\max})$ and $(\mu_{j,t}^{\min}, \mu_{j,t}^{\max})$ provide the locational marginal prices at each node n in period t, and the marginal values of a change in the capacity for each existing asset ℓ_i and candidate asset ℓ_j^c in period t, respectively.

2.2 Transmission Planning Model with Decentralized Transmission Investment

We assume a single seller and a single buyer at each node $n = 0,1,\cdots,N$ of the network, where $\mathcal{L} \triangleq \{\ell_1, \ell_2, \cdots, \ell_L\}$ is the set of lines and transformers that connect the buses of the network. This model has three distinctive features: i) transmission asset costs are not publicly available, ii) investment is possible, and iii) the *TP* has a budget constraint that takes into account the amount of monetary resources that can be used to reward investors. To account for these new features of the problem we define a set of investors $\mathcal{Y} \triangleq \{y_1, y_2, \cdots, y_Y\}$, where each of them can build a set of new assets $\mathcal{L}_{\mathcal{Y}}^C \triangleq \{a_j^k; \forall j = 1,\cdots,Y; \forall k = 1,\cdots,K_j\}$, and a set of payments

$Q^C_{\mathcal{Y}} \triangleq \left\{ q^k_j; \forall j = 1, \cdots, Y; \forall k = 1, \cdots, K_j \right\}$ that the *TP* can initially offer to each individual investor. We use the binary variable m^k_j to model the presence of new investors: it takes the value of 1 if the investor y_j is paid for the new transmission asset a^k_j, and 0 otherwise. The value B_D represents the budget constraint of the *TP*, where the *TP* initially estimates the payments to the investors based on transmission asset costs. The decentralized planning model can be formulated as the following social welfare maximization problem:

$$\max \; SW = \sum_{t \in \mathcal{T}} \sum_{n=0}^{N} \left[\beta^b_{n,t}(p^b_{n,t}) - \beta^s_{n,t}(p^s_{n,t}) \right] \tag{9}$$

s.t.

$$g_{n,t}\begin{pmatrix} p^s_{0,t}, p^s_{1,t}, \cdots, p^s_{N,t};\; p^b_{0,t}, p^b_{1,t}, \cdots, p^b_{N,t}; \\ m^1_1, \cdots, m^{K_Y}_Y \end{pmatrix} = 0 \tag{10}$$
$$\leftrightarrow \lambda_{n,t}; \forall n = 0,1,\cdots,N; \forall t \in \mathcal{T}$$

$$-f^{\max}_i \le h_{i,t}\begin{pmatrix} p^s_{0,t}, p^s_{1,t}, \cdots, p^s_{N,t}; \\ p^b_{0,t}, p^b_{1,t}, \cdots, p^b_{N,t};\; m^1_1, \cdots, m^{K_Y}_Y \end{pmatrix} \tag{11}$$
$$\le \; f^{\max}_i \leftrightarrow (\mu^{\min}_{i,t},\; \mu^{\max}_{i,t}); \forall i = 1, \cdots, \; L; \forall t \in \mathcal{T}$$

$$-f^{k,\max}_j \le h^k_{j,t}\begin{pmatrix} p^s_{0,t}, p^s_{1,t}, \cdots, p^s_{N,t}; \\ p^b_{0,t}, p^b_{1,t}, \cdots, p^b_{N,t};\; m^1_1, \cdots, m^{K_Y}_Y \end{pmatrix}$$
$$\le \; f^{k,\max}_j \leftrightarrow (\mu^{k,\min}_{j,t},\; \mu^{k,\max}_{j,t});\; \forall j = 1, \cdots, L^c; \tag{12}$$
$$\forall k = 1, \cdots, K_j; \forall t \in \mathcal{T}$$

$$\sum_{j=1}^{Y} \sum_{k=1}^{K_j} m^k_j q^k_j \le B_D \tag{13}$$

$$m^k_j \in \{0,1\}, \forall j = 1, \cdots, Y; \forall k = 1, \cdots, K_Y \tag{14}$$

$$0 \le p^s_{n,t} \le p^{s,\max}_n; \forall n = 0,1,\cdots,N; \forall t \in \mathcal{T} \tag{15}$$

$$0 \le p^b_{n,t} \le p^{b,\max}_n; \forall n = 0,1,\cdots,N; \forall t \in \mathcal{T} \tag{16}$$

where $g_{n,t}(\bullet)$ is the nodal real power flow balance equation at node *n* in period *t*, $h_{i,t}(\bullet)$ is the expression of the real power flow in asset ℓ_i in period *t*, and $h^k_{j,t}(\bullet)$ is the expression of the real power flow in candidate asset a^k_j of investor y_j in period *t*. For every constraint set there is a corresponding set of dual variables: $\{\lambda_{n,t} : \forall n = 0,1,\cdots,N; \forall t \in \mathcal{T}\}$ for the power flow balance equations, $\{(\mu^{\min}_{i,t}, \mu^{\max}_{i,t}) : \forall i = 1, \cdots, L; \forall t \in \mathcal{T}\}$ and $\{(\mu^{k,\min}_{j,t}, \mu^{k,\max}_{j,t}) : \forall j = 1, \cdots, Y; \forall k = 1, \cdots, K_j; \forall t \in \mathcal{T}\}$ for the real power flows in existing transmission assets and investors candidate assets, respectively. Note that both objective functions in (1) and (9) do not incorporate the time value of money over the planning horizon for simplicity, but it should be added in a more realistic setting. Note also that we have not included transmission asset costs in the formulation, since it is not public information in the decentralized model. Instead, we assume that the investors want to obtain an adequate rate of return expressed as a percentage over their actual construction costs. We compare the centralized and decentralized formulations in Appendix A, showing the conditions that make them equivalent.

In the next section, we describe the bargaining process that coordinates both the investors' payment requirements and the *TP* offers to the investors that are initially selected in (9)-(16). In this case, the *TP* simply optimizes the social welfare and then receives the payment requirements of the investors, comparing these values with the Shapley value allocation. Since this is a two-step procedure, the results are not necessarily the same as in the centralized method. The budget constraint imposes a further restriction over the money paid to the investors, but cannot be directly used to compare the results with the ones from the centralized method, since the information set is different.

Note that both problem formulations, centralized and decentralized, allow sequential decomposition of the investment problem. The formulation also lends itself nicely for scenario analysis, thereby providing a consistent basis to compare the impacts of different investments.

3 INVESTMENT INCENTIVES IN DECENTRALIZED PLANNING: THE INVESTMENT GAME

The centralized transmission investment model presented in the previous section can provide the set of investments which result in the maximum increase in benefits to a network without an explicit formulation of the incentives. Since the costs of the new assets are known in advance and no investors are allowed, the *TP* can solve the planning problem in a centralized fashion. However, in the second model, a decentralized transmission investment needs to create incentives to the investors whose assets improve the network. Therefore, in order to make both the *TP* and the investors decide to build new assets, a simple and fair criterion must exist, based on the value that a new asset brings to the system.

We define the value of a transmission asset as the increase in social welfare that this new asset (or combination of assets) brings to the network over the planning horizon, as compared to the pre-investment scenario, where no new assets are considered.

Rewarding the investors based on the improvement that their new assets bring to the social welfare can be done in several ways. The simplest choice is to reward each individual investor with the increase in social welfare that its new assets produce alone. This approach, although simple, has the disadvantage of not considering the combined effects of multiple separate transmission investments in the network. For that reason, we use a method based on cooperative game theory: the Shapley value, which incorporates the efficiency and fairness principles [9]. By using the Shapley value we can analyze the combined effects of simultaneous investments and also remunerate only the investors that truly improve the social welfare.

We can treat the investment problem as a cooperative game, where the players are investors in transmission assets and the *TP* reimburses these investors by offering them all or part of the social welfare increase that they produce when they are selected. This can be seen as a cooperative value allocation game, where the players are rewarded as a function of the improvement that they can bring to the system. Using cooperative game theory standard notation, our transmission investment game is defined by a pair $(\mathcal{Y}, \Delta SW)$, where $\mathcal{Y} \triangleq \{y_1, y_2, \cdots, y_Y\}$ is the set of investors and ΔSW is the increase in social welfare of the network; in game theoretic terminology it is called the characteristic function (see Appendix B).

Using the notation from above, the Shapley value allocation per investor is given by

$$\phi_j[\Delta SW] = \sum_{\mathcal{S} \subseteq \mathcal{Y}} \frac{(Y-s)!(s-1)!}{Y!} \times [\Delta SW(\mathcal{S}) - \Delta SW(\mathcal{S} - \{y_j\})]; \forall j = 1, \cdots, Y \quad (17)$$

where

$\phi_j[\Delta SW]$: Shapley value allocation to investor y_j,

Y : total number of investors,

$\mathcal{S}$: coalition of investors,

$s = |\mathcal{S}|$: number of investors in coalition $\mathcal{S}$,

$\Delta SW(\mathcal{S})$: increase in social welfare brought by coalition $\mathcal{S}$.

Thus, the Shapley value of a player in a game can be interpreted as the increase of the coalition surplus brought by the player to a coalition.

We assume that the final values assigned to each investor as a result of the game can be expressed by a vector of payments $\bar{\phi}_{\mathcal{Y}} = \{\phi_1[\Delta SW], \phi_2[\Delta SW], \ldots, \phi_Y[\Delta SW]\}$, where $\phi_j[\Delta SW]$ is the payoff to investor y_j. The sum of all these values is equal to the increase in social welfare due to all the investors, as shown in Shapley value's axiom 1 of Appendix B. Note that investors do not really engage in actual coalitions. This is just an artifact used by the *TP* to account for all possible combinations of investors and their joint effect in the social welfare increase. In real-life transmission investment cases, where the number of coalitions is not too high, the proposed method can be applied without reaching an explosion of combinations of possible investors.

The following algorithm is proposed to represent the interactions between the *TP* and the investors for the decentralized investment model using the Shapley value allocation scheme:

Step 1: The *TP* selects the initial set of investors from those who have declared interest in building transmission assets. Then, the *TP* runs the decentralized investment model subject to budget constraints (9)-(16) to determine the best set of candidates.

Step 2: The *TP* calculates the increase in social welfare with respect to the pre-investment scenario for all the combinations of selected investors resulting from the decentralized investment model. Based on that, the *TP* calculates the Shapley values (17) and compares them to the investors' requirements. For a single asset investor, if the Shapley value is higher than the payment requested, the *TP* notifies the investor that he can build the transmission asset and that he will be paid what he requests. Otherwise, the *TP* tells the investor that he is not selected. In case of a non-selected investor with more than one transmission asset, the *TP* requests the investor to withdraw at least one of his transmission assets in the next iteration.

Step 3: The *TP* verifies how many investors have decided to build the assets and goes to step 2.

Step 4: The game ends when there are no more investors willing to build more transmission assets.

4 CASE STUDY

We show our proposed decentralized incentive scheme applied to the Garver 6-bus system [1]. We make the following assumptions:

1. Marginal offers and marginal bids by generators and demands are linear and remain unchanged for all the periods of study, such that $p_i = a_i + b_i PG_i$ and $p_j = c_j - d_j PD_j$, where p_i is the price offer of a generator *i* that produces PG_i MW, p_j is the price bid of a demand *j* that consumes PD_j MW, and a_i, b_i, c_j, d_j are the price intercepts and slope coefficients of the linear functions of the generators and demands, respectively. We consider that offers are at marginal cost, bids reflect actual demand utility functions, and therefore the cost function of generator *i* can be expressed as $C_i(PG_i) = a_i PG_i + 0.5 b_i PG_i^2$ and the demand benefit of demand *j* is defined as $B_j(PD_j) = c_j PD_j - 0.5 d_j PD_j^2$. Note that the fact that generators offer at their marginal costs is a good strategy when considering perfect competition.
2. We consider a time horizon of one year, that is, a "target year". For this "target year" we estimate the demand, the generation offers and the demand bids. Therefore, our model represents a "Static Transmission Expansion Planning" problem, since it considers a "target year" for which the net social welfare is maximized.
3. It is assumed that the new lines will be operative for at least 25 years, thus a 25-year investment return period has been considered. A 10% interest discount

rate is assumed as the cost of capital. Bearing these values in mind, the value of the capital recovery factor can be calculated so that, for the next 25 years, the investment cost in new transmission assets is yearly repaid at a rate of 11.02% of the total initial investment. This is also known as the annualized cost.

4. A dc model of the network is used and losses are not considered in the formulation.

The system considered comprises 5 nodes and 6 lines connecting them; moreover, a sixth node is considered, at which some generation is placed. This node is not connected to the other five nodes, but lines to connect it to the system could be built if necessary. Figure 1 shows this system where dashed lines indicate some possible lines. Table 1 lists the line data of the system. The first two columns provide the nodes of origin and destination of the lines, the third and fourth columns show the electric parameters of the lines, the fifth column shows, in pu, the capacity that the lines can transmit. The annualized line costs, proportional to the line reactances, are shown in the sixth column. Up to three parallel lines are accepted for every possible connection between the nodes. The last column shows the number of lines already built for every possible corridor.

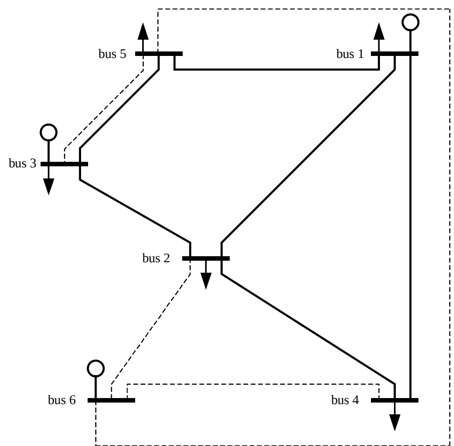

Figure 1: Garver 6-bus system.

From	To	R (pu)	X (pu)	Line flow limit (pu)	Annualized cost (M$)	Already built
1	2	0.10	0.40	1.00	4.0	1
1	3	0.09	0.38	1.00	3.8	0
1	4	0.15	0.60	0.80	6.0	1
1	5	0.05	0.20	1.00	2.0	1
1	6	0.17	0.68	0.70	6.8	0
2	3	0.05	0.20	1.00	2.0	1
2	4	0.10	0.40	1.00	4.0	1
2	5	0.08	0.31	1.00	3.1	0
2	6	0.08	0.30	1.00	3.0	0
3	4	0.15	0.59	0.82	5.9	0
3	5	0.05	0.20	1.00	2.0	1
3	6	0.12	0.48	1.00	4.8	0
4	5	0.16	0.63	0.75	6.3	0
4	6	0.08	0.30	1.00	3.0	0
5	6	0.15	0.61	0.78	6.1	0

Table 1: Garver 6-bus system line data.

Table 2 presents the location of generators and demands in the network and the offer and bid function coefficients. The time span of the study is one year and it is split into four seasons of equal duration (2190 hours per season out of 8,760 hours per year). Table 3 shows the maximum generation limits and Table 4 lists the demand limits for each season of the year. The rate of return required by the investors is 5% over actual costs and the budget constraint is $60M (a high value equivalent to no budget constraint).

	Generators			Demands		
Node	Name	a_i ($/MWh)	b_i ($/MW^{2}h)	Name	c_j ($/MWh)	d_j ($/MW^{2}h)
1	G_1	10	0.001	D_1	28	0.002
2	-	-	-	D_2	32	0.001
3	G_2	20	0.002	D_3	16	0.002
	G_3	22	0.003			
	G_4	25	0.003			
4	-	-	-	D_4	27	0.002
5	-	-	-	D_5	30	0.001
6	G_5	8	0.001	-	-	-
	G_6	12	0.001			
	G_7	15	0.002			
	G_8	17	0.002			
	G_9	19	0.002			
	G_{10}	21	0.003			

Table 2: Offer and bid function coefficients.

Generator	PG^{max} (MW)
G_1	150
G_2	120
G_3	120
G_4	120
G_5	100
G_6	100
G_7	100
G_8	100
G_9	100
G_{10}	100

Table 3: Upper generation limits.

Demand	PD^{max} (MW)			
	S_1	S_2	S_3	S_4
D_1	80	120	130	90
D_2	240	260	250	200
D_3	40	60	60	60
D_4	160	200	180	160
D_5	240	260	260	210

Table 4: Upper demand limits per season.

The expansion plans and the values of the social welfare achieved without expansion, with a centralized solution and with a decentralized solution are shown in Table 5.

Corridor	Pre-expansion	New lines	
		Centralized model	Decentralized model
1-2	1	-	-
1-3	0	-	-
1-4	1	-	-
1-5	1	-	-
1-6	0	-	-
2-3	1	-	-
2-4	1	-	-
2-5	0	-	-
2-6	0	2	3
3-4	0	-	-
3-5	1	1	-
3-6	0	-	-
4-5	0	-	-
4-6	0	2	2
5-6	0	-	1
# of lines	-	5	6
Annualized cost (1000$/year)	-	14,000	-
Required payment (1000$/year)	-	-	22,155
SW (1000$/year)	44,654	97,144	100,083
SW increase (%)	-	217.55	224.13

Table 5: Final centralized and decentralized solutions without a budget constraint.

Table 6 shows the evolution of the decentralized model. In the first iteration, the *TP* selects three investors and seven candidate lines by running the decentralized model (9)-(16). Investors in corridors 2-6 and 4-6 are accepted, since their required payments (based on a 5% rate of return over actual line costs) are smaller than the Shapley value allocations, but investor in corridor 5-6 is asked to withdraw one of his lines from the game, since the Shapley value allocation is not enough to reward his two lines. Note that only the investor in corridor 2-6 is allowed to build the maximum number of lines per corridor. In the second iteration, the investor in corridor 5-6 builds just one line and his payment request is accepted. Thus, the game ends in the second iteration and 6 lines are built.

It is also possible that the investors can ask for a higher rate of return to increase their profits. Table 7 shows the effect of a gradual increase of the required rate of return of all the investors. It can be observed that with a rate of return of 20% or higher for all investors, the investor in corridor 5-6 is no longer accepted and therefore, the final solution only has 3 lines from investor 2-6 and 2 lines from investor 4-6, as expected.

Iteration	Investor	Lines per investor	Required payments (1000$/year)	Shapley values (1000$/year)
1	2-6	3	9,450	27,271
	4-6	2	6,300	17,700
	5-6	2	12,810	11,527
2	2-6	3	9,450	29,511
	4-6	2	6,300	18,653
	5-6	1	6,405	7,265

Table 6: Decentralized model iterations without a budget constraint.

It.	Inv.	Lines	Required payments (1000$/year)			Shapley values (1000$/year)
			20%	25%	30%	
1	2-6	3	10,800	11,250	11,700	27,271
	4-6	2	7,200	7,500	7,800	17,700
	5-6	2	14,640	15,250	15,860	11,527
2	2-6	3	10,800	11,250	11,700	29,511
	4-6	2	7,200	7,500	7,800	18,653
	5-6	1	7,320	7,625	7,930	7,265
3	2-6	3	10,800	11,250	11,700	30,363
	4-6	2	7,200	7,500	7,800	19,882
	5-6	-	-	-	-	-

Table 7: Investors' rate of return effect on investments.

Table 8 shows the initial line proposals of the decentralized model when a range of budget constraints is imposed. Table 9 shows the corresponding centralized and decentralized solutions. It can be observed that the centralized and decentralized final solutions in Table 9 are not the same when the budget limit exceeds $15M. Note that this budget limit is sufficiently close to $14M, the optimal investment cost of the centralized problem in Table 5, for which the final solutions of both models are approximately the same if the rates of return are also sufficiently small, i.e., payments and costs are similar (see Appendix A).

	Corridor	Budget (M$/year)							
		0	5	10	15	20	25	30	60
Lines to install	1-2	-	-	-	-	-	-	-	-
	1-3	-	-	-	-	-	-	-	-
	1-4	-	-	-	-	-	-	-	-
	1-5	-	-	-	-	-	-	-	-
	1-6	-	-	-	-	-	1	-	-
	2-3	-	1	-	-	-	1	-	-
	2-4	-	-	-	-	-	-	-	-
	2-5	-	-	-	-	-	-	-	-
	2-6	-	-	1	2	3	3	3	3
	3-4	-	-	-	-	-	-	-	-
	3-5	-	-	-	1	1	-	-	-
	3-6	-	-	-	-	-	-	-	-
	4-5	-	-	-	-	-	-	-	-
	4-6	-	1	2	2	3	2	2	2
	5-6	-	-	-	-	-	-	2	2
# of proposed lines		0	2	3	5	7	7	7	7

Table 8: Decentralized model initial line proposals subject to budget constraints.

The software used to solve all the optimization models is the SBB solver under GAMS [11] through the web-based NEOS server [12]. The Shapley value allocations are obtained using the Cooperative Game Toolbox [13] in MATLAB [14] on an Intel Pentium 4 PC with 256 Mb of RAM at 2.8 GHz. Running times of GAMS and MATLAB models for all case studies are below 10 seconds.

5 CONCLUSIONS

We have presented two different models for transmission planning and investment in electricity markets. The first model is a centralized model, where the costs of expansion are publicly known and the investment is performed by the *TP*. The second model allows for a decentralized expansion of the network. In this model, the investors build new transmission assets according to the incentives provided by the *TP*. These incentives are calculated using the Shapley value formula and are based on the increase in social welfare produced by the combined effect of new transmission assets. To make this decentralized decision model a flexible tool, both a budget limit and a payment requirement are imposed by the *TP* and the investors, respectively. Further research will consider the combined effect of generation and transmission investments in more realistic scenarios and the development of a multi-year investment model.

APPENDIX A: EQUIVALENCE BETWEEN THE CENTRALIZED AND DECENTRALIZED FORMULATIONS

This Appendix presents the proof that the decentralized formulation of the investment problem (9)-(16) yields the same results as the centralized one (1)-(8) under the following assumptions:

1. Payments are made at the actual costs.
2. The overall decentralized investment payment offered by the *TP* is less than or equal to the optimal investment cost of the centralized problem.

The above conditions can be mathematically formulated as:

$$IP(x_d) = IC(x_d) \le IC(x_c^*) \tag{18}$$

where $IP(x_d)$ is the decentralized investment payment by the *TP*, $IC(x_d)$ is the actual decentralized investment cost, $IC(x_c^*)$ is the optimal centralized investment cost, and x_d and x_c are the decision vectors of the decentralized and centralized models, respectively. In other words, we claim that both models are equivalent when payments are equal to the actual costs and the decentralized budget limit, B_D, is equal to the optimal investment cost of the centralized problem.

Proof: The above claim will be proved by reductio ad absurdum. Let us assume that the optimal solution to the decentralized problem, x_d^*, yields a social welfare different from that obtained by the optimal solution to the centralized problem, x_c^*, i.e., $SW(x_d^*) \neq SW(x_c^*)$. Then, four cases must be analyzed:

1) $SW(x_d^*) > SW(x_c^*)$ and $IP(x_d^*) = IC(x_d^*) = IC(x_c^*)$.
In this case
$SW(x_d^*) - IC(x_d^*) = SW(x_d^*) - IC(x_c^*) > SW(x_c^*) - IC(x_c^*)$.
The maximum attainable value of $SW(x_d) - IC(x_d)$ is equal to $SW(x_c^*) - IC(x_c^*)$ as per the optimization of (1)-(8). Therefore we have a contradiction.

2) $SW(x_d^*) > SW(x_c^*)$ and $IP(x_d^*) = IC(x_d^*) < IC(x_c^*)$.
Similarly to case 1, $SW(x_d^*) - IC(x_d^*) > SW(x_c^*) - IC(x_c^*)$, and therefore, the same contradiction is found.

3) $SW(x_d^*) < SW(x_c^*)$ and $IP(x_d^*) = IC(x_d^*) = IC(x_c^*)$.
In this case we assume that the optimal solution to the decentralized problem, x_d^*, yields a level of social welfare lower than that corresponding to the optimal solution to the centralized problem. In addition, constraint (18) is binding at the optimal solution of the decentralized problem (9)-(16).
Under these assumptions:
a) $SW(x_d^*) - IC(x_d^*) = SW(x_d^*) - IC(x_c^*)$, and
b) $SW(x_d^*) - IC(x_c^*) < SW(x_c^*) - IC(x_c^*)$, which is the optimal value of the objective function of problem (1)-(8).
Since problem (9)-(16) maximizes the social welfare, x_d^* cannot be its optimal solution because a higher value of social welfare with the same level of investment cost is achieved by x_c^*.

4) $SW(x_d^*) < SW(x_c^*)$ and $IP(x_d^*) = IC(x_d^*) < IC(x_c^*)$.
In this case the constraint on investment payment (18) is not binding and consequently it can be removed from the optimization. Thus, $SW(x_d^*)$ is an upper bound of the solution to problem (9)-(16) with constraint (18) binding (case 3). We have already proved that the optimal solution to case 3 yields a social welfare equal to $SW(x_c^*)$. Therefore, $SW(x_d^*)$ has to be greater than or equal to $SW(x_c^*)$, which contradicts the initial assumption.

As a conclusion the four above situations are infeasible. The only feasible optimal solution is: $SW(x_d^*) = SW(x_c^*)$ and $IP(x_d^*) = IC(x_d^*) = IC(x_c^*)$. It should also be noted that this proof holds when the budget constraint (5) is binding. In this case the only feasible optimal solution is: $SW(x_d^*) = SW(x_c^*)$ and $IP(x_d^*) = IC(x_d^*) = IC(x_c^*) = B_C$. QED.

	Corridor	Budget (M$/year)															
		0		5		10		15		20		25		30		60	
		C	D	C	D	C	D	C	D	C	D	C	D	C	D	C	D
Lines to install	1-2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	1-3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	1-4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	1-5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	1-6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	2-3	-	-	1	1	-	-	-	-	-	-	-	1	-	-	-	-
	2-4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	2-5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	2-6	-	-	-	-	1	1	2	2	2	3	2	3	2	3	2	3
	3-4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	3-5	-	-	-	-	-	-	1	1	1	1	1	-	1	-	1	-
	3-6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	4-5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	4-6	-	-	1	1	2	2	2	2	2	3	2	2	2	2	2	2
	5-6	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	1
Total number of new lines		0	0	2	2	3	3	5	5	5	7	5	6	5	6	5	6

Table 9: Centralized (C) and decentralized (D) solutions subject to budget constraints.

APPENDIX B: COOPERATIVE GAME THEORY BACKGROUND

A cooperative game is defined by a real-valued function u called the *characteristic function* [9]. The function u assigns to each subset $\mathcal{C}$ of $\mathcal{P}$ (the set of all players) the maximum value of a game played between $\mathcal{C}$ and $\mathcal{P} - \mathcal{C}$, i.e., $u(\mathcal{C})$ is the best total utility that the coalition $\mathcal{C}$ can obtain under the worst scenario induced by the actions of the remaining players. The players can form coalitions in many different ways; the way in which players can group in m mutually exclusive and excluding coalitions $\mathcal{S}$ is given by $\delta = \{\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_m\}$, where δ is a partition of $\mathcal{P}$ that satisfies these three conditions:

$$\begin{aligned} &\mathcal{S}_j \neq \varnothing; \qquad \forall j = 1, \ldots, m \\ &\mathcal{S}_i \bigcap \mathcal{S}_j = \varnothing; \quad \forall i \neq j \\ &\bigcup \mathcal{S}_j = \mathcal{P}, \end{aligned} \tag{19}$$

where $\varnothing$ is the empty set.

The Shapley value of a game (defined by a characteristic function u) for player i, $\phi_i[u]$, is given by (17), and is the unique value vector that satisfies these four axioms:

Axiom 1: the set of players receives all the resources available: $\sum_{i \in \mathcal{P}} \phi_i [u] = u(\mathcal{P})$.

Axiom 2: if $\mathcal{S}$ is a dummy, i.e., $u(\mathcal{C}) - u(\mathcal{C} - \{i\}) = u(\{i\})$ for each coalition $\mathcal{C}$ in $\mathcal{P}$, then $\phi_i [u] = u(\{i\})$.

Axiom 3: the value assigned to player i does not depend on the position of the player in the set of players.

Axiom 4: if u and v are the characteristic functions of two games, then $\phi_i [u + v] = \phi_i [u] + \phi_i [v]$.

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