Waiting and Weighting:
Public Transportation Model
Sensitivity to Waiting Time and Schedule Deviation

by

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Abstract

Public transport (PT) is not famed for its dependability, yet current PT accessibility models presume that the schedule is an adequate reflection of reality. Consequently, it is implicit that PT vehicles always operate according to their schedule, and that all passengers arrive at random rather than plan their arrival. In reality, such assumptions do not always hold. The sensitivity of model results to such assumptions is largely untested.

This thesis seeks to determine to what extent PT accessibility models based on the schedule are robust to substitution by automatic vehicle location (AVL) data on the real arrival times of PT vehicles. The inclusion of this high resolution data on punctuality enables the relaxation of assumptions about the random arrival of passengers, and the enumeration of expected waiting time in the presence of uncertain PT arrival time.

Using an open standard for the publication of public transit information (the General Transit Feed Specification (GTFS)), this thesis develops a number of models of Wellington’s transit system, assuming that passengers arrive at random, and vehicles arrive punctually. Equivalent models are then built with AVL data from the Greater Wellington Regional Council’s real-time information (RTI) system. This information is used to determine actual service headways, headway variability, probability of service arrival over an interval, and realistic vehicle travel times.

Results of pairwise comparisons of accessibility model outcomes indicate that for every definition of accessibility model considered, GTFS and AVL sources of information do not predict similar travel times. The magnitude and the direction of differences varied between model types, but in all cases were highly statistically significant.

The results of the research are of use to transportation and accessibility modellers, who previously could not estimate travel time error associated with making assumptions about PT reliability and passenger arrival behaviour (implied by the use of PT schedules in models).
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Maintaining Headway

‘The truth is, they’d rather you were late than early,’ said Edward.

‘But that’s preposterous!’ said Jeff.

‘Preposterous or not,’ Edward replied. ‘Lateness is something they know how to deal with. They can quantify it, label it and apportion the blame accordingly. In some circumstances they can write it off altogether. There’s no excuse for being early but there are plenty for being late. Look at your log cards: each one is pre-printed with about ten different causes of delay.’

With a flourish he then produced a log card from his inside pocket and read out a list of examples:

‘“Traffic delay; no serviceable bus; ticket machine failure; extra mileage; road traffic accident; mechanical fault; road closure; staff shortage; other operating causes (unspecified).”’ He put the card away again. ‘It all proves that they’re quite prepared to accept lateness without question. What they don’t like is wilful earliness.’

‘But what about the maintenance of headway?’ I asked. ‘I thought that was supposed to be paramount.’

‘The answer is fiendishly simple,’ said Edward. ‘They make sure every bus is late by exactly the same degree.’

‘In other words it’s a conspiracy,’ remarked Jeff.

‘Correct.’

‘So there’s no point in trying to run on time.’

‘None at all. The timetables are a complete sham. You’ve probably seen the notices at bus stops: “Buses depart at these
“minutes past each hour.” It’s all meaningless: a line of dots and a set of random numbers; no more than a sleight of hand to fool the people.’

‘They’re not fooled,’ said Jeff.

‘Of course they’re not,’ said Edward. ‘Neither are they ever satisfied. If the bus happens to arrive on schedule it’s good for the public record but little else. Nobody believes the timetables. Waiting for buses is therefore pradoxical; hence the refrain:

The people expect the bus to be late,
yet they go to the bus stop early and wait.’

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Acronyms and initialisms

ANZSIC  Australian and New Zealand Standard Industrial Classification
API    application programming interface
AVL    automatic vehicle location
CBD    central business district
CoV    coefficient of variation
DTD    deterministic and time-dependent
DTI    deterministic and time-invariant
ECDF   empirical cumulative distribution function
ESP    expected shortest path
GDAL   Geospatial Data Abstraction Library
GeoJSON Geographical JavaScript Object Notation
GTF    General Transit Feed
GTFS   General Transit Feed Specification
GWRC   Greater Wellington Regional Council
HFT    headway function and transfer
HHNT   half-headway, no transfer
HHT    half-headway and transfer
IVT    in-vehicle time
ACRONYMS AND INITIALISMS

LOS  level of service
NZISCR  New Zealand Institute for the Study of Competition and Regulation
OSM  OpenStreetMaps
OTP  OpenTripPlanner
PCT  passenger choice and transfer
PDF  probability density function
PL/pgSQL  Procedural Language extensions to PostgreSQL
PT  public transport
rANOVA  repeated measures analysis of variance
RDBMS  relational database management system
REST  Representational State Transfer
RMSE  root-mean square error
RTI  real-time information
SATURN  Simulation and Assignment of Traffic to Urban Road Networks
SEP  shortest expected path
SQL  Structured Query Language
STD  stochastic and time-dependent
STI  stochastic and time-invariant
SWT  static waiting and transfer
TCQSM  Transit Capacity and Quality of Supply Manual
WPTM  Wellington Public Transport Model
WTSM  Wellington Transport Strategy Model
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Chapter 1

Introduction

Computational models of public transport (PT) accessibility are constructed to estimate how long it takes to travel within a region using PT. This estimate of travel time includes all aspects of the journey: walking, waiting, in-vehicle time (IVT), and transferring. These models enable estimates of travel time or disutility, which in turn enable the identification of relative accessibility to particular amenities across a city, such as access to employment. Such information is particularly useful when planning transportation networks as the places and people that may or may not benefit can be identified if a particular change is made to the underlying network.

1.1 Problem

PT accessibility models are typically constructed on the basis of the timetable. Vehicles are assumed to be punctual, and passengers are modelled as using the PT system as though this were true (e.g. Curtis & Scheurer, 2010; Joyce & Dunn, 2010; Liao, Arentze & Timmermans, 2011; Hadas & Ranjitkar, 2012; Mavoa, Witten, McCreanor & O’Sullivan, 2012; Szeto, Jiang, Wong & Solayappan, 2013; Mattingly & Morrissey, 2014). These punctuality and passenger arrival assumptions may be very strong, but it is not clear how much accessibility model outcomes may change if these assumptions are relaxed based on real data (Chen, Yu, Zhang & Guo, 2009; Chen, Li, Wang, Shaw, Lam, Yuan & Fang, 2013).

Solely using the schedule to form a PT accessibility model limits the
model’s realism. In particular, waiting time is generally reduced to a function of scheduled service frequency, without consideration of how passengers may account for expected punctuality and allow safety margins so as not to miss the service.

Increasingly, automatic vehicle location (AVL) data is being collected and archived by PT system operators as a way of monitoring performance and providing customer-facing real-time information (RTI) (e.g. Mazloumi, Currie & Rose, 2010; Li-Jun, Yan, Li-Nan & Xu, 2011; Nassir, Khani, Lee, Noh & Hickman, 2011; Yetiskul & Senbil, 2012; Firmani, Italiano, Laura & Santaroni, 2013). This provides an opportunity to substitute schedules in accessibility models with data that describes how the real system operates. To date, few models have included AVL (Firmani et al., 2013; Allulli, Italiano & Santaroni, 2014; Delling, Italiano, Pajor & Santaroni, 2014). Our understanding of how influential AVL is on an accessibility model is limited. Because the schedule is by definition the set of instructions that drivers attempt to replicate, it is plausible to suggest that the difference in modelled outputs may be minor. The lack of models with AVL data may be due to the difficulty of accessing this data (if it even exists), and of manipulating such a large, un-standardised database.

Moving away from a reliance on the schedule in accessibility models requires that these models account for how passengers react to deviations from schedule when passengers decide how early to arrive to wait for PT (Knight, 1974; Jolliffe & Hutchinson, 1975; Bowman & Turnquist, 1981). At present, most models estimate waiting time purely as a function of frequency of service. Including waiting time theory has not been a consideration because by itself a schedule does not include any information useful to the estimation of punctuality at the stop- and route-level.

1.2 Research question

This thesis poses the question:

Does the inclusion of observed data on the reliability of a public transport system in models of that system significantly alter accessibility model outputs relative to models that do not incorporate reliability information?
1.2. RESEARCH QUESTION

1.2.1 Objectives

To answer this question, the following objectives have been identified:

1. To develop PT and walking accessibility models of Wellington using standardised transit data representing the schedule. Four broad model types are considered: deterministic and time-invariant (DTI), stochastic and time-invariant (STI), deterministic and time-dependent (DTD), and stochastic and time-dependent (STD).

2. To produce parallel versions of the above models using archived AVL data in place of the schedule. This step includes considering theoretical passenger responses to uncertainty and the consequences in terms of expected waiting time.

3. Following from 1. and 2., to perform accessibility model simulations for statistical geographical units (meshblocks)\(^1\) in Wellington: using AVL data to represent the real system operation, and the General Transit Feed Specification (GTFS) for the schedule.

4. Finally, to compare whether the accessibility model outcomes significantly vary, using tests for pairwise differences at the meshblock level. The individual analyses are intended as demonstrations of results, and are not the object of interest in themselves. For most models, the results take the form of estimated travel time to the Wellington central business district (CBD). An estimate of the number of jobs that are deemed accessible at the meshblock level is also considered.

This research will improve understanding of the realism of current schedule-based PT accessibility models, by quantifying the error associated with this assumption over a region. As AVL data becomes more widespread, it may become less acceptable for an accessibility model to be built without reference to AVL. If AVL data affects accessibility model outcomes relative to scheduled models, the validity of the outputs of scheduled models will be called into question.

\(^1\)The meshblock is the smallest geographic unit at which Statistics New Zealand publishes information from the Census. A meshblock is equivalent to census tracts in the United States of America.
This study has access to a large database of time-at-location AVL data for a recent and continuous period in the Wellington region of New Zealand (the Greater Wellington Regional Council (GWRC) RTI system). The method of constructing different forms of PT accessibility models from archived AVL data will be useful to PT accessibility modellers as this data increasingly becomes available. Further, as AVL data is likely to be much less standardised than scheduled data, automated tools to construct accessibility PT models may be less applicable with AVL data than at present.

1.3 Outline

Chapter 2 is a literature review primarily considering computational PT accessibility models. A model typology is introduced to frame the discussion on PT modelling. Each of the four models identified in the typology is then considered in turn, with discussion of methodological issues and evidence from studies that have included reliability data.

Models falling into three of the four categories in the typology are considered in this study. Chapter 3 details the methodology behind the specific models within the three model categories, particularly the scheduled and observational treatments in each case.

Chapter 4 reports the results of the pairwise statistical analysis for each model. These are used to determine whether the schedule and the AVL data truly predict similar accessibility model outcomes across a population of meshblocks. The magnitude of any differences is noted, as well as the corresponding level of statistical significance. These values differ according to the definitions of particular models.

Chapter 5 discusses these results and their implications for PT accessibility analysis. Chapter 6 concludes with the main findings and recommendations for future research.
Chapter 2

Literature Review

2.1 Model typology

A geographical network model, or graph, represents travel time through space. Real-world pedestrian and public transport (PT) networks are commonly represented by a graph composed of ‘edges’ (or ‘arcs’) representing movement in space-time, and ‘nodes’, which represent intersections and decision points, such as boarding a bus. Each edge holds a ‘cost’ or penalty associated with movement along that edge between two nodes. To represent PT movement, models at a minimum attempt to represent in-vehicle time (IVT). Consideration of waiting time, transfer time, reliability and other penalties are additional details that particular models often consider in addition to IVT for realism.

PT system models take a variety of forms. A typology is therefore useful to discuss these throughout this thesis. The typology shown in Table 2.1 distinguishes between the types of values edge costs can take (i.e. to represent time spent walking, waiting, or in-vehicle), and whether these costs may vary with the time of day. These four model types are different means to measure hypothetical travel impedance (time or generalised time) between a pair of locations separated in space (Handy & Niemeier, 1997).

It is expected that models in different categories in the typology will differ in pairwise accessibility estimates when constructed with timetabled data or with observed data. Determining the difference reliability data makes on modelled travel time outcomes under multiple model types is
2.2 Deterministic and time-independent models

Networks where each edge in a network holds a single, constant, cost value for each direction of movement are termed deterministic and time-invariant (DTI) networks. As the edges are time-invariant, the model either represents a narrow time period where travel times can validly be considered static, or a wider period where different travel times are aggregated (e.g. into an average travel time).

DTI networks are the simplest form of graph on which shortest paths may be determined. However, the definition of DTI cost variables can be complex, including time, monetary cost, behavioural weights and other costs in a generalised univariate cost or a multi-cost structure requiring more complicated algorithms (Modesti & Sciomachen, 1998).

The topology of a DTI network may need to be complex in order to correctly model real-world systems and human behaviour, such as modelling hundreds of possible transfer combinations at a transit interchange, each with a different expected cost. Figure 2.1 demonstrates a walking and PT DTI topology.

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<table>
<thead>
<tr>
<th>Time-independent</th>
<th>Deterministic</th>
<th>Stochastic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DTI: Single constant, for each edge</td>
<td>STI: Single random variable, for each edge</td>
</tr>
<tr>
<td>Time-dependent</td>
<td>DTD: Constant for each $t$ in $T$, for each edge</td>
<td>STD: Random variable for each $t$ in $T$, for each edge</td>
</tr>
</tbody>
</table>

Table 2.1: A typology of transportation networks and the form of edge costs they validly model. Adapted from Yang et al. (2014, p.2). The third dimension of the full typology (multi-cost edges) is outside of the scope of this research.
2.2. DETERMINISTIC AND TIME-INDEPENDENT MODELS

Using a DTI model to represent PT is a considerable abstraction. As schedules are aggregated rather than modelled directly, travel times may significantly differ from time-dependent representations. This is particularly acute for route transfers. The benefit of using a DTI model is that behavioural weights can be introduced to affect optimal route decisions simply by multiplying the estimated travel time for each edge according to its properties. This makes DTI models popular in traffic assignment models as they can be calibrated to fit real measures of traffic volume (Hall, 1986).

Two crucial elements of developing an estimate of overall travel time with a stochastic and time-invariant (STI) network are estimates of IVT and waiting time, which will now be discussed in turn.

2.2.1 In-vehicle time

To represent IVT, edge costs are usually the average of all scheduled travel times between two places over a time interval (Choosumrong, Raghavan & Bozon, 2012). Hadas & Ranjitkar (2012) determined such a model for Auckland using General Transit Feed Specification (GTFS) data. Scheduled travel times may be considerably different from observed travel times.
(Mazloumi et al., 2010), and the choice of modelled time interval affects estimated IVT as it determines exactly which services are included in the aggregation.

The estimation of IVT is made difficult by the presence of multiple routes providing service between the same pair of stops; known as the ‘common lines’ problem. The presence of both express and non-express services between the same stop pairs is a difficult edge case for DTI models. In addition, estimates of waiting time should be influenced by the common lines problem. For both waiting and IVT, a general approach to developing a robust estimate of travel time involves considering both an edge’s origin and destination nodes and a subset of ‘attractive’ arriving services to consider to determine appropriate estimates of IVT and waiting time (Chriqui & Robillard, 1975; Marguier & Ceder, 1984; De Cea & Fernández, 1993; Lam, Gao, Chan & Yang, 1999; Sumalee, Uchida & Lam, 2011; Szeto et al., 2013). The existence of the common lines problem is not often acknowledged (see for example Yigitcanlar, Sipe, Evans & Pitot, 2007; Hadas & Ranjitkar, 2012; Abley & Halden, 2013).

IVT is not the only aspect of mobility that can be included in the cost attribute. Generalised cost functions exist to determine the cost of IVT to reflect how people perceive this time. It is possible that in certain situations people even perceive travel time positively (Ettema, Friman, Gärling, Olsson & Fujii, 2012). In some studies, frequency of arrivals has been included in a generalised cost function by dividing the estimate of IVT by the average frequency (Burns & Inglis, 2007; Curtis & Scheurer, 2010). This method allows a reduced travel impediment with increased speed, increased frequency, or both (Curtis & Scheurer, 2010). Using generalised costs is useful for calibrating traffic assignment models, but makes accessibility analyses more difficult to interpret.

Including reliability in route choice

Carrion & Levinson’s (2012) meta-analysis of the value of IVT variability identified three theoretical frameworks for including PT travel variability and unreliability within network accessibility modelling. Of these three frameworks, centrality-dispersion best applies to DTI models.

Based around the principles of financial risk-return models, centrality-
2.2. Deterministic and Time-Independent Models

dispersion considers the mean of travel time as the ‘return’ and the standard deviation of travel time as the associated ‘risk’ (Jackson & Jucker, 1982; Carrion & Levinson, 2012). Over a route, a traveller attempts to minimise their disutility represented by the sum of the return and the risk multiplied by $\gamma$ (risk aversion) (Equation 2.1). This is also known as the mean-variance approach, and is based on expected utility theory. It considers risk not uncertainty as it requires complete information of both the centrality and the variance and therefore expected ‘return’.

$$U = \gamma_1 \mu_T + \gamma_2 \sigma_T$$  \hspace{1cm} (2.1)

The $\gamma$ coefficient represents risk aversion; $\mu_T$ is the mean travel time for the edge; and $\sigma_T$ is the standard deviation of travel time around the mean (which can be zero when using the schedule). Given $\gamma$, the mean-variance model is straightforward to consider for IVT because it simply requires that the cost function minimised by the shortest path algorithm be adjusted to also consider the variability of IVT and individual risk preferences.

2.2.2 Waiting time

A penalty to represent passengers waiting at stops is regularly used in PT accessibility models. This can take a variety of forms and may be additionally weighted for calibration (AECOM, 2013). Generally for DTI models, waiting is either:

1. A constant penalty (e.g. ten minutes) for accessing any PT route (Liao et al., 2011; Mavoa et al., 2012). This penalty may differ by mode (Peipins, Graham, Young, Lewis, Foster, Flanagan & Dent, 2011).


Constant waiting time penalties in DTI models are generally in the range of 7.5–10 minutes (Liao et al., 2011; Mavoa et al., 2012). Mavoa et al. (2012) acknowledged that the global use of a 10 minute penalty in their
Auckland model is unrealistic, but justified this approach due to ease of implementation and that the penalty reflects the observational research of Lester & Walton (2009), who considered waiting times of New Zealand bus and train passengers.

Using a fraction of the headway to represent waiting time allows service frequency to affect waiting time (Hadas & Ranjitkar, 2012; AECOM, 2013). In particular, half of the headway does represent a good estimate of waiting time in DTI models, but only if the following conditions are met:

1. Passengers arrive randomly to PT.
2. Headways are constant.¹

**Passenger arrival behaviour**

Passengers may choose to arrive at a PT stop at random, or consult (or already know) the schedule and plan to arrive at their stop at a particular time in an attempt to minimise their waiting time (Jolliffe & Hutchinson, 1975; Marguier & Ceder, 1984; Tisato, 1998; Vincent, 2008). It is generally considered that passengers are more likely to arrive at random when services are very frequent (a 10–12 minute cutoff is often cited) (Jolliffe & Hutchinson, 1975; Okrent, 1974; Hall, 2001; van Oort, 2011). However, Csikos & Currie (2007) disagree, finding random arrivals to be more common in the off-peak when many passengers are making non-routine trips and do not know the schedule. Csikos & Currie’s (2007) result contradicts most other pieces of research.

As frequency increases, both the passenger’s ‘penalty’ for missing a service diminishes, as well as their ability to minimise waiting time by planning their arrival time at the stop to coordinate with the vehicle’s scheduled arrival.² This is why passengers tend to arrive at random more often when headways are small. This random arrival principle accords with Vincent’s (2008) examination of the arrival decisions of 732 New Zealand bus and train passengers. For services with a five minute headway, Vincent (2008) noted that 83 percent of passengers arrived to wait at random.

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¹This is not necessarily synonymous with punctual services.
²For many cities, schedules only consist of a stated headway and do not refer to particular times at all. An example of this is a schedule reading, “buses arrive every ten minutes”. Wellington does not have this form of schedule.
For ten minute headways, only 49 percent did so. For headways fifteen minutes and up to more than an hour, only 20–29 percent of passengers arrived at random (Vincent, 2008, Table 7.2, p.52).

**Schedule punctuality**

When passengers do not arrive at their stop at random, the punctuality of services is more relevant for determining passenger waiting time than headway alone (Jolliffe & Hutchinson, 1975).\(^3\) In a DTI model, expected waiting time should not be defined as half of the headway for infrequent services, because passengers catching such services are typically aware of the schedule. In particular, planning passengers are particularly sensitive to the occurrence of early-running services, whereas randomly arriving passengers do not even perceive the difference between early- and late-running services (Knight, 1974; Jolliffe & Hutchinson, 1975).

However, imperfect service punctuality leads to uncertainty regarding how early a planning passenger should arrive. Therefore it is difficult for a modeller to estimate expected waiting time. Techniques exist that represent a compromise between random and planned arrivals, by allowing random arrivals within a constrained window of passenger arrival time (van Oort, 2011). Bowman & Turnquist (1981) proposed an estimation technique that uses an entirely non-random behavioural mechanism between the observed probability of a service arriving early or late and a passenger’s arrival time decision (Bowman & Turnquist, 1981). The latter technique accords very well with the data-rich situation we now have with automatic vehicle location (AVL) data.

While Bowman & Turnquist’s (1981) method relies on passengers being perfectly aware of the pattern of punctuality in order to choose the best arrival time, there is evidence that passengers are poor learners of typical punctuality (Hall, 2001; Lester & Walton, 2009). Indeed, passengers in Hall’s (2001) survey poorly estimated the lateness of buses they had just caught, with almost no correlation found between perceived and measured bus lateness—even when the model was limited to those who stated that they knew the schedule. Perceived lateness was overwhelmingly lar-

\(^3\)Although the two issues are related, as long headways creates an incentive for passengers to plan arrival.
ger than actual lateness. However, most passengers evidently consider that PT services can come considerably early even if they are bad at developing an accurate perception of punctuality. Vincent (2008), for example, found that an average train service in Auckland or Wellington arriving five minutes early is still able to pick up 87% of train passengers who do not arrive at random.

Compared to a model of uniformly random passenger arrivals, Bowman & Turnquist’s (1981) model allowing for complete passenger choice resulted in much smaller waiting times than than a half-headway assumption across the full range of headways and service punctualities, often substantially so. Figures 2.2 and 2.3 demonstrate the magnitude of this difference, which is largest when services are infrequent, or very reliable. The effects of frequency and reliability compound.

Waiting time estimation in DTI models

Abley & Halden’s (2013) New Zealand accessibility methodology uses a half-headway waiting time assumption, with a cap at 7.5 minutes’ waiting. Internationally, the half-headway assumption is very common (see O’Sullivan et al., 2000; Asensio, 2002; Hess, 2005; Salon, 2006, 2009; Cheng & Agrawal, 2010; Tribby & Zandbergen, 2012; Salonen & Toivonen, 2013).

Although absolute waiting time may be over-estimated with a half-headway assumption, time spent waiting is also generally perceived as relatively more costly than other aspects of PT mobility, particularly due to its uncertainty (Small, Winston & Yan, 2005; Abrantes & Wardman, 2011; Ettema et al., 2012). Both of AECOM’s (2013) Wellington PT models estimate waiting time as capped multiples of the headway, but then weight according to the transport mode (with buses considered worse than light rail). The NZ Transport Agency (2013, p.4-82) recommends weighting waiting time at twice the cost of IVT; although Abrantes & Wardman’s (2011) meta-analysis of value of time studies suggests that this is high.

An estimate of waiting time is also required to model transfers between different routes. The half headway waiting time penalty is often incorrectly applied to transfer times (Modesti & Sciomachen, 1998; Hadas & Ranjitkar, 2012; Mavoa et al., 2012; Tribby & Zandbergen, 2012; Abley & Halden, 2013). This is incorrect because transfers are not subject to ran-
Figure 2.2: Expected average waiting time for a scheduled service with a ten minute headway under different vehicle punctuality scenarios. The random arrival model, which predicts a half-headway waiting time influenced slightly by the coefficient of variation in arrivals, predicts considerably longer waiting time for these services, especially when punctuality is good. Adapted from Bowman & Turnquist 1981, p.470, Fig. 4.
Figure 2.3: The relationship between expected average waiting time and service frequency, with the random passenger arrival model compared to Bowman & Turnquist’s (1981) passenger choice model. Adapted from Bowman & Turnquist 1981, p.470, Fig. 5.
2.3 STOCHASTIC AND TIME-INDEPENDENT MODELS

Unlike the DTI network representation, a STI representation of a PT network models travel times as random variables. These random variables are presumed to follow a constant distribution with respect to time. This representation of travel time is the sole difference from the DTI, and introduces additional modelling issues. Topologically, exactly the same edges are included, so common lines should also be present (Spiess & Florian, 1989; Sumalee et al., 2011). The result is a network that is capable of returning several paths that each have a non-zero probability of being the optimal path for a given origin-destination pair. In one simulation, there is a single optimal path; but the optimal path may follow a different routes and/or have a different cost in a subsequent simulation. The overall estimated trip travel time itself is therefore a random variable in its own right (Rasouli & Timmermans, 2014).

Passenger preferences can also be modelled as random variables drawn from population distributions, leading to different travel time estimates even if time itself is deterministic (Tong & Wong, 1999; Hollander & Liu, 2008; Sumalee et al., 2011). These preferences can be made conditional (such as a preference for using the underground train when it is raining—a stochastic event), and subject to perception error (Recker, Chung, Park,
Wang, Chen, Ji, Liu, Horrocks & Oh, 2005; Sumalee et al., 2011). Preferences may be used to influence a shortest path algorithm to avoid unreliable PT services. (Recker et al., 2005; Ettema & Timmermans, 2006).

The concepts of IVT and waiting time also need attention for STI models as they do within DTI models.

### 2.3.1 In-vehicle time

A shortest path algorithm for a STI PT network must consider that each edge has a probability distribution for the travel time. Peak hour IVT can defensibly be modelled with a normal distribution, although log-normal distributions better model off-peak travel (Mazloumi et al., 2010; Rasouli & Timmermans, 2014).

Different assumptions can be made about when the randomly-drawn edge costs are made known to the algorithm. These include:

- The shortest path with recourse.
- The expected shortest path (ESP).
- The shortest expected path (SEP).

**Shortest path with recourse**

The shortest path with recourse considers an individual to ‘discover’ the cost of travelling along an edge only upon arrival at either of its tail nodes and is used in most stochastic road network models (Waller & Ziliaskopoulos, 2002). Depending on the randomly-drawn cost of an edge (discovered upon the routing algorithm’s arrival at the tail node), the algorithm may prefer the use of a different edge (recourse) or to opt for travelling over the edge that has just been arrived at (Polychronopoulos & Tsitsiklis, 1996; Cheung, 1998; Provan, 2003; Gao & Chabini, 2006). Each time the algorithm reaches a node (even previously visited ones), a new value is drawn (Waller & Ziliaskopoulos, 2002; Provan, 2003).

The goal of the shortest path with recourse typically is not to identify the least cost path, because such a path does not exist. Rather, many different paths simultaneously hold a non-zero probability of being the shortest path, and these probabilities can be determined (Waller & Ziliaskopoulos,
2.3. STOCHASTIC AND TIME-INDEPENDENT MODELS

This is a useful intermediate output for traffic assignment models that may assign traffic demand to different routes according to these probabilities.

**Expected shortest path**

The ESP imagines that an individual knows all of the edge costs in the network before beginning their trip, and therefore will always choose the shortest path for that particular realisation of the stochastic network (Polychronopoulos & Tsitsiklis, 1996; Miller-Hooks & Mahmassani, 1998). The implicit assumption of the method is that passengers are pre-informed about disruption affecting their trips, and have perfect knowledge about alternative routes (Polychronopoulos & Tsitsiklis, 1996). In each simulation, the network is realised as a DTI network (Miller-Hooks & Mahmassani, 1998). Each simulation of the network and shortest path decision returns a different shortest path: travel time itself is a random variable (Frank, 1969; Hassin & Zemel, 1985; Hollander & Liu, 2008). In short, the ESP makes routing decisions conditional on stochastic travel times.

**Shortest expected path**

The SEP is a variant of the information-revelation assumption in which the traveller never becomes aware of edge costs for a particular simulation. In this case, the optimal path is simply determined from the expected costs (as a DTI model), and the overall travel time is then determined from random draws from the cost distributions for each edge of this path (Polychronopoulos & Tsitsiklis, 1996). Re-simulation of the shortest path decision is therefore not conducted. The result can be considered equivalent to the travel time distribution of an individual who travels by the same typically optimal route, and does not or cannot react to disruptions affecting this route (Nie & Wu, 2009). In other terms, the SEP makes an unconditional routing decision.

**2.3.2 Waiting time**

Waiting time in an STI model should also be drawn from a probability distribution, however defined. The preceding discussion on information
revelation is equally applicable to waiting time. To be consistent with the half-headway assumption of DTI models, in the absence of anything but the schedule, waiting time would be represented by a uniform distribution from zero seconds to the value of the headway (Sumalee et al., 2011). There is little guidance in the applied literature on what form waiting time costs should take in stochastic models of PT when there is access to information beyond scheduled arrival times, such as the likelihood of delay. Indeed certain STI models reduce the waiting time estimate to a deterministic parameter (e.g. Spiess & Florian, 1989).

2.4 Deterministic and time-dependent models

The third type of transport network model is a deterministic and time-dependent (DTD) model. A deterministic network models edge costs as a single constant value per direction. When the model is also time-dependent, each edge holds multiple edge costs, one for each moment or interval of time in the model period. Between the same origin and destination, there may be different shortest paths when travelling at different times of the day, both in terms of which edges comprise a path, and the time cost of a path. For example, it may be faster to choose the train during the peak period, but the bus in the inter-peak period.

Waiting time is no longer a deterministic or probabilistic function of headway or punctuality, but a deterministic function of the scheduled vehicle arrival and departure at a stop, and the passenger’s arrival at that stop (Tong, Wong, Poon & Tan, 2001). Time-dependence therefore allows for an accurate accounting of transfer waiting time if the schedule is accurate (or if AVL data is being used), but an unfairly low (potentially zero) waiting time for an initial service.

2.4.1 Topology

A DTD graph generally requires fewer edges than a DTI graph of the same system because complicated aspects of the use of PT (such as transfers) can be captured directly through time-dependent edge costs. However, whether fewer edges are actually required in practice strongly depends
on the choice of topological data model. The two most common representations of time-dependent networks are:

- Time-expanded graphs.
- Time-aggregated graphs.

**Time-expanded graphs**

Time expanded graphs replicate each edge and node in a graph for each time instant within a period of time \( t \in T \) (Schulz, Wagner & Weihe, 2000; Dean, 2004). The time parameter \( t \) is required when querying a DTD graph: time governs the availability (topological existence, connectivity) and cost (travel time) of each edge. Any edges with an internal \( t \) value that is less than the ‘current time’ \( t \) can no longer be accessed. Since each edge only exists for a single moment, the size of the graph (number of edges and nodes) is close to being a multiplicative function of the temporal resolution. The array of values for \( t \) could be very large, such as each second in a day. Therefore, time expansion may lead to significant storage overhead and computationally expensive search algorithms (Cooke & Halsey, 1966; Dean, 2004; George, 2008). However, the biggest increases in graph size is due to the modelling of pedestrian edges, which need to be replicated for each moment without change in their attributes.\(^4\)

The advantage of time-expanded graphs over time-aggregated representations (discussed next) is that standard DTI network algorithms (e.g. Dijkstra’s algorithm) can be used (Dreyfus, 1969). However, in addition to a much larger graph, time-expanded graphs do not benefit from most of the algorithmic speed-up techniques that can be applied to time-independent graphs, such as bi-directional search (Delling, 2009).

**Time-aggregated graphs**

Time-aggregated graphs replicate neither nodes nor edges with respect to moments of time in an interval, \( t \in T \) (Cooke & Halsey, 1966; Orda & Rom, 1990). Rather, deterministic, time-aggregated graphs model the properties

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\(^4\)Walking time is generally considered constant with respect to time, but this is not a requirement.
of nodes and edges as a time series with discrete values for all $t \in T$. That is, as a vector or parametricised function with $t$ as input (Brodal & Jacob, 2004).

Efficient algorithms have been developed to determine the shortest path through a time-aggregated graph given a starting time (Tong et al., 2001; Brodal & Jacob, 2004; Poon, Wong & Tong, 2004; George, 2008). Algorithms also exist to find the best start time given the objective of minimising travel time (George, 2008; Brodal & Jacob, 2004). However, few implementations of these are available. At the time of writing PostGIS/pgRouting only has a proof of concept time-aggregated shortest path algorithm. Figure 2.4 visually compares time-expanded and time-aggregated networks.

### 2.4.2 Punctuality

Modelling PT service variability and punctuality in a DTD graph is difficult under certain model specifications. In the time-expanded case, modelling different travel times requires adjusting the topological relationships to most edges in the graph, which is considerably resource-intensive (Delling, Giannakopoulou, Wagner & Zaroliagis, 2008). In contrast, adjusting the costs of a time-aggregated graph simply requires updating the self-contained cost function for each row in the database with no effect on topology.

It is possible to define a series of DTD networks and to issue repeated shortest path queries to determine the likelihood of on-time arrival at a destination, given a departure time (see the ‘reliable space-time prism’ of Chen et al., 2013, for the driving case). This is the same approach as the STI shortest expected path. Neutens, Van de Weghe, Witlox & De Maeyer (2008) used an alternative binary classification that compares the size of the areas that are always and never accessible within a travel time threshold after multiple days have been considered. While these methods describe variation in travel time, there is no mechanism to intro-

---

5The extension is available, but the master branch has not been developed since August 2011 and it is not compatible with recent versions of PostgreSQL: [https://github.com/pgRouting/pgrouting/tree/gsoc-tdsp](https://github.com/pgRouting/pgrouting/tree/gsoc-tdsp). Testing indicated that it could not cope with sufficiently large networks to be used in this study.
Figure 2.4: Schematic time-expanded and time-aggregated graphs of the same network. A single route travels from A to B, and another from C to B, with several trips scheduled in the time interval \((t \in [1, 14])\). Time is represented on the time-expanded graph’s vertical axis, with space shown on the horizontal axis. The time-expanded graph requires a greater number of edges and nodes to represent the same situation; particularly for the full range of pedestrian movement. In the time-aggregated representation, different travel times at different times of day can be modelled with a parametricised function. Adapted from Schulz 2005, p.20, Fig. 3.2.
duce uncertainty in a particular simulation, only over repeated deterministic simulations.

Like many DTI networks, most DTD models obey the timetable without consideration of how accurate the timetable actually is. Although little work has been done, there is evidence that journey planners\(^6\) return considerably different results depending on whether the schedule or location-at-time AVL data are used to return shortest paths, at least in terms of the ranked order of route suggestions (Firmani et al., 2013, Allulli et al., 2014, and Delling et al., 2014 all consider Rome’s General Transit Feed (GTF) against a municipal journey planner using location-at-time AVL data).

### 2.4.3 Waiting time

In contrast to DTI network representations, waiting time in DTD models is based entirely on scheduled events rather than an estimate typically based on frequency (O’Sullivan et al., 2000; Liu, Bunker & Ferreira, 2010; Li, Lam, Wong & Sumalee, 2010). DTD representations accept such enforced waiting penalties as the primary limitation of the DTD model type (e.g. Lei & Church, 2010). This waiting penalty is neither a random passenger arrival assumption nor a model where passengers have incentives to reduce waiting time. Only one example of a DTD model was found that overrides this property to apply waiting time penalties more akin to DTI modelling (Martin, Jordan & Roderick, 2008).

Where DTD models have the capability to introducing a more realistic waiting time penalty is in the capacity for departure time choice (George, 2008). Delaying journey departure time can be rational if a passenger prefers to substitute time spent waiting at a bus stop for time spent waiting (or doing something else) at the journey’s origin (Small, 1982; Ettema & Timmermans, 2006; George, 2008; Nurul Habib, Day & Miller, 2009; Li et al., 2010; Tseng, Rietveld & Verhoef, 2012). However given deterministic vehicle arrival times, an algorithm may reduce waiting time to nothing when minimising overall disutility, rather than considering the real proclivity to allow a ‘safety margin’ in the presence of uncertainty (Knight, 1974; Noland & Small, 1995; Fosgerau & Engelson, 2011). More often, de-

---

\(^6\)Journey planners are essentially DTD networks capable of returning \(k\) shortest paths.
parture time choice is enabled so that an algorithm can minimise overall journey time, or so that a better trip destination arrival time can be achieved (Tseng & Verhoef, 2008; Tseng et al., 2012).

Ideally, DTD models should account for the likelihood and magnitude of early or late arrival, and the consequent disutility, when determining an optimal departure time (Knight, 1974). Such scheduling delays can be considered with the use of DTD models constructed with AVL data (Small, 1982; Carrion & Levinson, 2012). A general utility function is demonstrated in Equation 2.2 (Small, 1982; Carrion & Levinson, 2012).

\[
U(t_d; \text{PAT}) = \gamma_1 T + \gamma_2 \text{SDE} + \gamma_3 \text{SDL} + \gamma_4 \text{DL} \tag{2.2}
\]

Where \( t_d \) is the departure time; \( \text{PAT} \) is the given preferred arrival time; \( T \) the travel time (contingent on \( t_d \)); SDE and SDL are the degree of lateness and earliness given \( t_d, T, \) and \( \text{PAT};^7 \) DL (often omitted) is a binary term indicating early or late arrival relative to \( \text{PAT} \); and the \( \gamma \) coefficients are preference parameters that can be estimated for a population. The \( \gamma \) coefficients are negative by definition (Fosgerau & Karlstrom, 2010; Carrion & Levinson, 2012). The utility function is generally considered with discrete choice sub-modelling to find the optimal departure time, and is very computationally expensive as it requires enumeration of alternatives (a shortest path for many possible departure times).

An alternative, post-processing method of incorporating reliability is to consider mean lateness. Equation 2.3 compares scheduled journey time (given the best departure time determined from scheduled arrivals: SchedT) and the mean lateness at the destination (\( L^+ \)) that accrues in reality from making this decision (Carrion & Levinson, 2012). Early arrivals are typically ignored.

\[
U = \gamma_1 \text{SchedT} + \gamma_2 L^+ \tag{2.3}
\]

The value of \( \gamma_2 \) to be applied represents the cost of delay minutes, given by \( L^+ \). This term can be individual-specific, and may also vary with the time, trip purpose, and the need to make a transfer (Vincent, 2008). The mean-lateness technique does not provide a method to influence route choice in DTD models; it merely describes an outcome of the optimal scheduled route.

\[^7\text{SDE} = \max(0, \text{PAT} - (T + t_d))\]

\[^8\text{SDL} = \max(0, (T + t_d) - \text{PAT})\]
CHAPTER 2. LITERATURE REVIEW

2.5 Stochastic and time-dependent models

Stochastic and time-dependent (STD) network models define PT network edges with a cost modelled as a random variable drawn from a distribution of travel times, where the distribution itself varies with respect to time. An STD PT network can be pictured as representing travel time differently during peak and off-peak periods, which may also become more or less variable. Due to the difficulty of representing such networks as topological networks, non-graph representations are also possible (Dibbelt, Pajor, Strasser & Wagner, 2013).

In an STD network, for a given departure time, there can feasibly be a very large number of paths that have a positive probability of being the least cost path (Miller-Hooks & Mahmassani, 1998; Chang, Nozick & Turnquist, 2005; Tseng et al., 2012). Considering also that there is a departure time decision or on-time arrival target, it is clear that the task of identifying the single path (including departure time) that is most likely to be the least cost path in an STD representation is not at all a trivial task (Hall, 1986; Kaufman & Smith, 1993; Wellman, Ford & Larson, 1995; Horn, 2000; Chang et al., 2005; Schulz, 2005).

It has been shown that in an STD network, the appropriate method to determine the optimal path in the presence of random events is to use an adaptive decision rule (Hall, 1986; Wellman et al., 1995; Hickman & Bernstein, 1997; Pretolani, 2000; Opasanon & Miller-Hooks, 2006; Nie & Wu, 2009). A decision rule (or routing policy, or dynamic program) is distinct from a path in that it is the set of criteria that yields the next edge to take at each intersection based on the arrival time of the person at the intersection, and the conditions of the network (e.g. whether they have missed a service) (Hall, 1986; Pretolani, 2000; Nie & Wu, 2009). Pretolani (2000, Figs. 1–2, pp.319–320) presents a very intuitive example of an STD network and optimal policy.

Examples of STD models in the literature are largely theoretical; no software implementations currently exist in spatial network analysis tools. The illustrative implementations that exist typically require that the distributions hold no spatial or temporal dependence (Fu & Rilett, 1998; Miller-Hooks & Mahmassani, 2003; Nie & Wu, 2009; Dibbelt et al., 2013). For these reasons, the STD model type has not been considered for analysis as
2.6 Travel time reliability

Reliability of PT service is affected by different aspects of the transit environment. These include traffic congestion, weather, infrastructure repair, accidents, industrial land uses, traffic signals, pedestrian crossings, poor on-time departure at the depot, long boarding times, and poor scheduling (knock-on delays), among other causes (Sterman & Schofer, 1976; Abkowitz & Engelstein, 1983; Talley & Becker, 1987; Strathman & Hopper, 1993; Mazloumi, Currie & Rose, 2008; Chen et al., 2009; Li et al., 2010; Mazloumi et al., 2010; Mazloumi, Rose, Currie & Sarvi, 2011; Yetiskul & Senbil, 2012; Babaie, Schmöcker & Shariat-Mohaymany, 2014). Although the explanation of PT variability has attracted much research attention, less attention has been paid to including estimates of variability and unreliability in the construction of spatial PT accessibility models.

Vincent (2008) argues that there are three distinct meanings for the term ‘reliability’ in PT:

- **Punctuality**: vehicle adherence to a route schedule at the stop-level.

- **Variability**: similar to punctuality, except a service consistently behind schedule by the same magnitude is showing poor punctuality but no variability. Variability can occur with respect to stop arrival time, and travel time.

- **Cancellations**: trips that do not occur at all, or are cancelled after partial completion.

Although variability and unreliability are somewhat synonymous, some variability is expected by travellers and is thus not unpredictable (Yin, Lam & Miller, 2004; Vincent, 2008; Sweet & Chen, 2011; Carrion & Levinson, 2012; Bone, Wallis, O’Fallon & Nicholson, 2013; Coulombel & de Palma, 2014). Passengers may make an allowance for usual day-to-day variation, or occasional incident-related variation, but not severe, exceptional variation that cannot possibly be anticipated (e.g. network-wide disruption caused by a moderate earthquake) (Bone et al., 2013). Variability is also
heterogeneously valued by individual users, and for trips with different purposes (catching a flight versus meeting friends) (Bordagaray, dell’Olio, Ibeas & Cecín, 2014).

Passengers compensate for variability by changing mode, departure time, route, destination, reporting that they will be late but making no other adjustment, or cancelling the trip altogether (Abdel-Aty, Kitamura & Jovanis, 1995; Bates, Polak, Jones & Cook, 2001; Liu, Recker & Chen, 2004; Li, Hensher & Rose, 2010; Babaei et al., 2014). Models that consider reliability should ideally allow a mechanism for modelled passengers to react to unreliability on at least one of these bases. Departure time adjustment is common when there are few alternatives to a missed service (Bhat, 1998a; Bates et al., 2001; Li et al., 2010). However most studies on the value of reliability, and traveller adaptations to it, focus on driving. The findings of these studies cannot be adapted to the transit case because scheduled transit services, unlike driving, do not allow for a truly continuous departure time choice (Bhat, 1998b; Mazloumi et al., 2010; Benezech & Coulombel, 2013).

2.7 Summary

Following the typology established in Table 2.1, representations of PT and walking networks can be classified into four primary categories, albeit with significant scope for nuance within a model in any category. These categories are DTI, STI, DTD, and STD graphs.

DTI networks abstract away from schedules and so typically require cost functions that are aggregates of schedules. In principle, this actually provides significant scope for how cost functions can be defined to account for behavioural weights and multipliers for the effect of reliability. Yet waiting time has mostly been determined as some function of scheduled service frequency, or simply as a constant. Popular choices include taking half of the headway (perhaps with a cap), or a static value of between five and ten minutes. These assumptions severely misrepresent real choices that are made if passengers are seen to plan their arrival to a service, and if a service is (or is not) punctual.

An additional consideration of modelling PT with DTI network is the
complex topology required to do so. Common lines representing the travel between any pair of stops by several different routes must be included in the model to capture the benefits accruing to waiting time when passengers are indifferent amongst a set of overlapping routes. Such edges are not often considered; not even in the New Zealand accessibility analysis methodology (Abley & Halden, 2013).

STI models follow the same topological construction as a DTI network. Because edge costs or weights are represented as random variables drawn from probability distributions, STI models produce an estimate of overall travel time that is itself a probability distribution. There is little guidance on how these distributions should be formed in the presence of AVL data; and as timetables are generally considered perfectly deterministic, this is not a common representation for PT accessibility models despite the opportunity to move beyond point estimates of travel time.

When determining a shortest path in an STI simulation, an assumption must be made about how much information passengers have about the distributions and the realisations of the edge costs. Often, the STI modelling assumption is that the algorithm discovers each edge’s value upon arrival at the a node, and has recourse so that if this cost is very large, alternative paths may be explored. Simpler possibilities are allowing full specification of each edge’s simulated cost for each simulation (‘expected shortest path’); and never having full information (and simply choosing the ‘shortest expected path’). These are easier to implement because the standard Dijkstra’s algorithm can be used to identify the shortest path in either case.

A DTD network representation is essentially an itinerary rendered topologically. This allows for the realistic modelling of transfers, but it is considerably more difficult to consider passenger’s real incentive to allow a safety margin when deciding upon the departure time for their trip. Some models simply enforce a particular origin departure time, but more complex models implement a discrete choice sub-model that may account for expected variability of travel time.

It is exceedingly difficult to reconcile a time-dependent PT network model with stochastic edge costs in an STD representation. A very large number of possible paths could have a non-zero probability of being part
of a shortest path between an origin and a destination, for every possible departure time. Models of this form are not considered within this thesis, although in principle they remain a possible representation.

Within any PT model, the consideration of waiting time and its intersection with passenger arrival behaviour and service punctuality or headway variability deserves special attention. For time-independent models, the assumption of uniformly random passenger arrivals can be improved with information about real passenger decision-making. Alternatively, assuming rational passengers informed about a particular route’s proclivity to be early or late, an optimal arrival time can be determined, one that minimises expected waiting time. This applies particularly for infrequent routes.

AVL data provides a significant advancement in our understanding of the performance of PT networks at a very high level of detail: the same level of detail to which a timetable is published. Using theory of how passengers are expected to react to variability and punctuality, it is now possible to substitute scheduled data in PT accessibility models with AVL data, in order to add additional realism to our estimation of accessibility.
Chapter 3

Methodology

Four types of models can be used to represent public transport (PT) and so estimate travel time and accessibility. These models are:

1. Deterministic and time-invariant (DTI)
2. Stochastic and time-invariant (STI)
3. Deterministic and time-dependent (DTD)
4. Stochastic and time-dependent (STD)

This thesis does not consider STD graphs, as the complexity of the representation prevents shortest path queries from being implemented, or indeed, from being meaningful.

For each of the three remaining models, this chapter details the implementation of several variations. These variations capture different mechanisms of including waiting time (including transfers), drawn from the literature.

The research question is to determine whether substituting schedule information by automatic vehicle location (AVL) observations in PT models significantly affects accessibility model outcomes. To this end, each variation of each model is subject to two treatments: estimated costs formed from the General Transit Feed Specification (GTFS) data (scheduled treatment), or from the real-time information (RTI) time-at-location AVL data (observed treatment).

This chapter begins with a description of the data used to answer the research question. Section 3.2 briefly covers the software used to construct
the models and run the comparative analyses. The section presents a table summarising the models, model variations, and treatments, and finally gives detail on particularly challenging aspects of implementation. Sections 3.4–3.6 consider the three models, their variations and treatments in turn. Section 3.7 covers the methods of statistical analysis employed. Section 3.8 concludes the chapter.

3.1 Data

Two main datasets were used to model scheduled and observed PT arrival, departure and travel times: the Wellington region GTFS, and the time-at-location AVL data recorded by the Greater Wellington Regional Council (GWRC) RTI system. These datasets are described in Table 3.1.

The study had access to Snapper PT smartcard transaction data. However, this data was ultimately deemed unsuitable for answering the research question. Snapper does not record punctuality, record arrival or departure times at stops with no passenger activity, and is limited to three PT operators in the region (two bus and one cable car).
<table>
<thead>
<tr>
<th>Summary</th>
<th>Source</th>
<th>Date range</th>
<th>Limitations</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>PT schedule and geographic information, published in an open, standardised format.</td>
<td>Metlink</td>
<td>Apr. 2014 – Jul. 2014</td>
<td>• May not adequately describe PT in early 2014. &lt;br&gt;• Does not include stop dwell time. &lt;br&gt;• Does not include fare information. &lt;br&gt;• Bus schedules are recorded to a higher level of precision than trains.</td>
<td>• Uses scheduled stop times, not frequencies; headways are inferred. &lt;br&gt;• Lack of dwell times implies some real transfers are not shown.</td>
</tr>
<tr>
<td>Archived time-at-location AVL data for Wellington’s PT system, can be related to the GTFS. Records arrival and departure times, deviation from schedule at each stop and associated route data for every vehicle equipped with the technology.</td>
<td>GWRC</td>
<td>Oct. 2013 – Jul. 2014</td>
<td>• Occasional and undetectable device malfunctions: possible missing data. &lt;br&gt;• One second data precision may be overly precise. &lt;br&gt;• Cancellations cannot easily be identified. &lt;br&gt;• Has less complete route coverage than the GTFS. &lt;br&gt;• No information on patronage or vehicle capacity. &lt;br&gt;• Route names do not easily map to the GTFS.</td>
<td>• Arrival and departure events determined with geofencing around scheduled stops. &lt;br&gt;• Data extends beyond the stated date range, as the GWRC provided two datasets. Only dates within the overlapping extent are considered.</td>
</tr>
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Table 3.1: Data sources for building the PT system models for the scheduled and observed treatments.
 CHAPTER 3. METHODOLOGY

Pedestrian datasets were also required for the three models (Table 3.2). OpenStreetMaps (OSM) data was initially considered. After importing the Wellington OSM data to PostgreSQL with several automatic importers, the OSM data was deemed inferior to an alternative source of pedestrian network data developed by the New Zealand Institute for the Study of Competition and Regulation (NZISCR), as groundtruthing found that the OSM importers incorrectly identified pedestrian accessibility for several streets. The DTI and STI models used the NZISCR pedestrian network.

The DTD model used the OSM network, as OpenTripPlanner (OTP) (the software used to analyse the DTD network) worked most seamlessly with OSM in .osm.pbf format. That either pedestrian network may have errors is not important, as all comparisons of treatments are made on models with the same fundamental pedestrian network. Thus, differences in model results are limited to the source of PT data.

<table>
<thead>
<tr>
<th>Source</th>
<th>Currency</th>
<th>Limitations</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>NZISCR</td>
<td>2013 Closed data.</td>
<td>The author worked on improving this dataset when employed by the NZISCR.</td>
<td></td>
</tr>
<tr>
<td>OSM</td>
<td>2014 Changeable.</td>
<td>Some tags used to describe Wellington streets led to incorrect restrictions on pedestrian behaviour.</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2: Pedestrian datasets used to support the PT models.

In order to ensure that PT data was only drawn from ‘ordinary’ non-holiday weekdays, a supporting dataset was also required. This dataset was constructed with information from the Ministry of Education.\(^1\) The definition of a ‘school day’ is occasionally ambiguous: each year the Ministry of Education only dictates a window of dates between which schools may open and close for the year. The definition used in this thesis is liberal: any day a school legally could have been open is considered to be a school day, even though some schools may have been closed. Figure 3.1

\(^1\)http://www.minedu.govt.nz/theMinistry/EducationInNewZealand/SchoolTermsAndHolidays/SchoolTermsArchive.aspx
provides a visual example of why this distinction matters when considering PT, travel time, punctuality and variability.

Finally, Statistics New Zealand 2013 Census meshblocks and attribute dataset were used in analysis. Geometric centroids were constructed for each meshblock with the PostGIS ST_Centroid function. The centroid locations were not weighted, but they were subsequently associated with the location of the nearest node in the time-invariant topology: it is this final location that is used to represent trip origins for each meshblock.

### 3.1.1 Study area

Any city or region could have been used for this thesis; the only requirement is that there is a GTFS and that an equivalent time-at-location AVL dataset is available. The GWRC area has been chosen as a study area. The author lives in the region and attends university in Wellington city, the urban centre of the region. A GTFS exists for the region, published by Metlink, a division of the GWRC responsible for contracting public transport routes to private operators.

In addition to only considering AVL records of trips made in the morning peak (7–9am) on school-term weekdays in 2014, two of the models constructed in this thesis were limited to two contiguous cities within the Wellington region. Specifically, the DTI and STI models were limited to considering all PT routes that operate within or through Lower Hutt and Upper Hutt cities in the Hutt Valley. The models include services that begin and/or end beyond the extent of these two cities, such as the Wairarapa train service, and other train and bus services with their origins in the Hutt Valley. This restriction was made for the DTI and STI models because their complex topological representation (particularly common lines) meant that the size of the graph was unmanageable when the entire region was considered.

The spatial restriction was not required for the DTD. The DTD network used time-aggregation: the topology was largely handled by OTP rather than PostGIS as in the time-independent models.

---

Figure 3.1: In the case of this bus route in April 2014 (which consisted of two weeks’ of school term and two weeks’ of school holidays), buses better reflect the schedule during the school holidays. Note that the weekday schedule exhibits a small amount of variability because the schedule does not report the same travel time for all trips. Data drawn from GWRC RTI data.
3.2 Implementation

The three forms of PT models considered in this thesis have been implemented with open-source tools. These tools include PostgreSQL 9.3, PostGIS 2.1.0, pgRouting 2.0, OpenTripPlanner 0.11.0, GDAL, SciPy, and R. Scripts have been written in SQL, Python, R and Bash to conduct different data management tasks, accessing APIs, and statistical analyses.\(^3\)

**PostgreSQL** is an open-source, enterprise-grade object-relational database management system. It is suitable for constructing network datasets and processing AVL data because it tolerates very large tables (up to 32 TB) and rows (up to 1.6 TB), and supports the construction of large, efficient indexes. It is readily extensible with Procedural Language extensions to PostgreSQL (PL/pgSQL); developed extensions include PostGIS (for geographic object support) and pgRouting (for network topology and routing algorithms with PostGIS geometries). In contrast to SQLite (which was initially considered), it also has support for the SQL:2008 datatypes DATE, TIMESTAMP, and INTERVAL, the latter of which is particularly useful for storing in-vehicle time (IVT). The **ARRAY** datatype combined with support for large rows was useful when aggregating hundreds of AVL punctuality records per stop into an array from which an empirical cumulative distribution function (ECDF) of schedule punctuality could be constructed.

**pgRouting**\(^4\) enables geospatial routing functionality within the PostgreSQL/PostGIS spatial database system. Leveraging PostgreSQL and PostGIS’ efficient attribute and geometric indexing, it allows for high performance shortest path queries. Edge cost values can be dynamic; literally determined by functions at the moment a route is queried, without pre-computation. This makes it particularly suitable for querying the origin-destination pairs repeatedly with different cost functions.

Shortest path algorithms are fundamental to accessibility models with vector features, and enable the consideration of more than Euclidean dis-

---

\(^3\)Experiments with Graphserver (http://graphserver.github.io/graphserver/) and NetworkX (https://networkx.github.io/) gave disappointing results, both in terms of performance and in the size of graph they could support. They are also far less compatible with PostgreSQL, so would have made data management considerably more difficult.

\(^4\)http://pgrouting.org/.
tances. They are used to identify the path that minimises the cost (i.e. travel time) between any two points in a network. Therefore, they are generally used to find the path that a rational, informed traveller would seek to take when travelling between those two locations, and the associated (dis)utility of that trip, assuming that the network topology and cost structure adequately reflects reality. For instance, the shortest path and associated shortest travel time can be found for each location across a city to a destination feature or features. The result expressed as expected travel time defines a spatially-varying measure of accessibility to that amenity for each input trip origin.

The master branch of pgRouting currently supports ten different shortest path algorithms; and others have been implemented as proofs of concept. Different algorithms are used for different tasks, such as considering single or multiple destinations. Different algorithms often use different inputs that are relevant to their intended task, examples include Dijkstra’s algorithm, and the A* algorithm. A* preferentially considers network edges that are in the desired cardinal direction of travel before considering other edges, which frequently (but not necessarily) leads to a faster solution.

As discussed in Section 2, algorithms for time-dependent models are necessarily more complicated. While the pgRouting project has a time-dependent shortest path algorithm available for time-aggregated graphs,\(^5\) at the time of writing it has not been developed since 2011, is only available for an older version of PostgreSQL/PostGIS, and places a hard limit on the size of the network to which it can be applied. In addition, an equivalent but time-expanded representation failed due to the size of the graph.\(^6\)

In order to model a PT network with a DTD representation, after pgRouting options had been exhausted, the thesis considered OTP\(^7\) and in particular OTP Analyst.\(^8\) OTP is aimed at providing a similar service to Google Maps: efficiently providing customised trip itineraries. Both Google Maps

\(^5\)For example, the time-dependent shortest path algorithm

\(^6\)The model period was two hours at a temporal resolution of five seconds. On the complete PT network, it took fifteen minutes to solve a single shortest path query across Wellington using Dijkstra’s algorithm, and thousands were required.

\(^7\)http://www.opentripplanner.org/

\(^8\)http://www.opentripplanner.org/analyst/
and OTP accept transit data in the GTFS format, which was originally developed for use by Google Maps. OTP is distinct from Google Maps in that it is non-proprietary, open source, run locally, places no limit on the number of queries that can be issued, and allows the customisation of inputs to routing, such as passenger preferences.

**OTP Analyst** is a related set of tools that allow queries on a PT network beyond the itineraries that OTP produces. OTP Analyst exposes a REST API, and has resources for the determination of point-to-point routing, isochrones, and other queries related to the determination of PT accessibility. OTP Analyst also allows batch queries.

### 3.3 Model variations and treatments

Given the broad identification of three feasible models of PT networks (DTI, STI, and DTD), this thesis seeks to determine whether models formed on the basis of a schedule gives comparable results to instead using AVL data (the scheduled and observed treatments). However, there are several possible variations of each of the three models, and it is plausible that the consequence of this data substitution may differ with the model variations. To this end, five different variations on the DTI model are considered in this thesis, as well as two variations of STI model. Although only one form of DTD model (time-aggregated) is considered, two analysis techniques were conducted. Table 3.3 summarises the models, their variations, and the treatments applied in each case. Detail can be found in Section 3.4 (DTI), Section 3.5 (STI), and Section 3.6 (DTD).

---

9OTP Analyst v0.11.0 was used in this thesis: [http://docs.opentripplanner.org/apidoc/0.11.0/rest.html](http://docs.opentripplanner.org/apidoc/0.11.0/rest.html)
### Table 3.3: Summary of the PT model representations, their variations, and treatments.

For all **DTI** models, IVT is modelled by the average (scheduled or observed) travel time. For all **STI** models, IVT is a random variable $X \sim N(\bar{x}, s)$ (scheduled or observed travel time).

<table>
<thead>
<tr>
<th>Model</th>
<th>Variation</th>
<th>Treatment &amp; Detail</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DTI</strong></td>
<td>Half-headway, no transfer (HHNT)</td>
<td><strong>Scheduled treatment</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Waiting: 0.5$H_S$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Transfer: N/A</td>
</tr>
<tr>
<td></td>
<td>Half-headway and transfer (HHT)</td>
<td><strong>Observed treatment</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Waiting: 0.5$H_O$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Transfer: Average observed</td>
</tr>
<tr>
<td></td>
<td>Passenger choice and transfer (PCT)</td>
<td><strong>Path</strong></td>
</tr>
<tr>
<td></td>
<td>Static waiting and transfer (SWT)</td>
<td>HHNT: scheduled treatment, Equation 3.6 (p.53)</td>
</tr>
<tr>
<td></td>
<td>Headway function and transfer (HFT)</td>
<td>HHNT: observed treatment, Equation 3.6 (p.53)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>ESP: Shortest per simulation, Equation 3.6 (p.53)</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Waiting</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>HHNT: scheduled treatment, See Section 3.5.1 (p.53)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>HHNT: observed treatment, See Section 3.5.1 (p.53)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>ESP: Shortest per simulation, See Section 3.5.1 (p.53)</td>
</tr>
<tr>
<td><strong>STI</strong></td>
<td>SEP-S</td>
<td><strong>Isochrone</strong></td>
</tr>
<tr>
<td></td>
<td>SEP-O</td>
<td>See Table 3.5 (p.58) for model parameters</td>
</tr>
<tr>
<td></td>
<td>ESP</td>
<td>See Table 3.6 (p.61) for model parameters</td>
</tr>
<tr>
<td><strong>DTD</strong></td>
<td>Isochrone</td>
<td><strong>Trip</strong></td>
</tr>
<tr>
<td></td>
<td>Trip</td>
<td>See Table 3.5 (p.58) for model parameters</td>
</tr>
<tr>
<td></td>
<td></td>
<td>See Table 3.6 (p.61) for model parameters</td>
</tr>
</tbody>
</table>

$H_S$ and $H_O$ respectively refer to the scheduled and observed headways.
3.4 DETERMINISTIC AND TIME-INVARIANT MODEL

The five DTI model variations differ in terms of how waiting time and transfers are determined. The STI model variations have different assumptions about information revelation: shortest expected path (SEP), or expected shortest path (ESP). The DTD model only differs in terms of analysis task considered for comparison (an isochrone analysis, or single origin-destination trip).

The illustrative accessibility analyses conducted with these models mostly take the form of estimating travel time from each meshblock centroid to a point in downtown Wellington. This is repeated for both the scheduled and observed treatments, forming a paired observation. These paired observations enter the statistical analysis, which determines if the model as a whole predicts statistically similar estimates of travel time.

For the time-independent models, this point is the intersection of Lambton Quay and Molesworth Street in the Wellington central business district (CBD). This point is chosen to represent travel to the CBD from the Hutt Valley; a point further to the south would have biased routes in favour of the bus in most cases, as the trains terminate near the northern edge of the CBD, while most buses continue south.

The destination chosen for the DTD model trip analysis is different to the time-independent models: the intersection of Manners Mall and Cuba Street is selected. This was chosen simply as a representative point in central Wellington. This intersection is inappropriate for the time-independent models because they do not include the full range of possible PT routes that the DTD model does.

In addition, the DTD model also considers an isochrone travel time analysis: a count of how many jobs are estimated to exist within 30 minutes’ of walking and PT from the meshblock, under each treatment.

3.4 Deterministic and time-invariant model

The DTI model of a PT network represents travel times as constant, time-independent variables. Therefore this model aggregates the travel times of services over a time interval. This section discusses the variations on this model form selected to be representative of the range of examples found in the literature (see Section 2.3). Each variation is constructed twice: once for
each treatment. A treatment refers to whether scheduled travel times from the GTFS are used for cost parameters, or observed travel times from the RTI time-at-location AVL data. How time penalties have been calculated for each specific variation and treatment is the focus of this section. See Table 3.3 for a summary of variations and treatments of this model.

Topology The DTI model requires a considerably complex topological representation in order to include common lines, which represent the benefit of overlapping PT routes. This topology is reported in Appendix B. The size of the complete regional network was so large with this topological representation, that only PT routes operating in or through Lower Hutt and Upper Hutt cities were ultimately included. This spatial limitation is due to computational considerations: the size of the required network for the entire Wellington region would have extended into the millions of edges and made travel time analysis very computationally demanding, restricting the number of model forms that could be compared, with the only benefit an expanded geographical extent.

Headway Waiting time functions discussed in the remainder of this section mostly refer to service ‘headway’. In a time-independent context, the headway of a service is the expected time interval between the arrival of two ‘equivalent’ services. Because determining the correct value of a headway depends on both the origin stop and the destination stop, calculating the headway is not as trivial as it first seems. The interested reader is referred to Appendix C for detail on how headways have been calculated in this study.

3.4.1 Half-headway, no transfer (HHNT) variation

The HHNT variation of DTI model represents waiting time as being equal to half of the headway, and transfer edges have not been included. The waiting time assumption therefore represents the traditional assumption of uniformly random passenger arrival. This is a very common representation (see O’Sullivan et al., 2000; Asensio, 2002; Hess, 2005; Salon, 2006, 2009; Cheng & Agrawal, 2010; Joyce & Dunn, 2010; Hadas & Ranjitkar, 2012; Tribby & Zandbergen, 2012; Salonen & Toivonen, 2013). As in all
DTI models, IVT is represented as the average of scheduled or observed inter-stop travel times, as appropriate. It is expected that this model will produce relatively long travel times compared to the other DTI models because it ignores transfers, and disallows passenger arrival time choice even for very infrequent routes.

With respect to how close predictions of scheduled and observed travel times are, the difference may be go either way for any location. The source of any difference is limited to how well the real vehicle travel times and headways match the schedule, as there are no other mechanisms for differences to emerge.

Scheduled treatment

The scheduled treatment follows the above specification. The schedule is used to determine all information required to construct the aggregate cost functions. The required information is the average travel time between stops, and the average headway for all relevant services (for arrivals of one route on non-common edges, across all lines on a common line).

Observed treatment

The observed treatment simply substitutes the observed average travel time, and the observed average headway, for the equivalent values for each edge in the scheduled treatment.

3.4.2 Half-headway and transfer (HHT) variation

The HHT DTI variation is nearly identical to the HHNT variation, but the HHT includes transfer edges. This inclusion can be advantageous to a modelled passenger. For example, consider two different hourly services with one intersecting stop present on both routes. At this intersecting stop, the services could be scheduled to arrive within five minutes of each other. The appropriate waiting time when transferring from the first-arriving to the second-arriving route is five minutes, not half of the headway, because neither the passenger nor the vehicle have arrived at random.\(^{10}\) Further,

\(^{10}\)Note that in this study, when transfers are aggregated, only trips arriving at stops within 80m of each other (including the same stop), more than three minutes apart, and
when transferring from the later-arriving route, the waiting time is 55 minutes (assuming the first service does not dwell, which it might).

By definition, the shortest overall travel time for each origin-destination pair in this model cannot be greater than that found in the HHNT variation for the same pair, and is probably considerably smaller if a transfer is included in the optimal path.

The topology ensures that a transfer edge cannot be accessed without first waiting for and travelling on a component route. However, scheduled transfers can actually imply that the appropriate waiting time when moving from one route to another should be greater than one half of the headway. No topological mechanism can be included to stop a passenger intending to transfer instead ‘choosing’ the edge in the graph that represents arriving at random for a the transfer route. Ultimately, this reflects a limitation of the time-independent representation. In contrast, a time-dependent graph structure always applies the correct transfer penalty, without need for complicated topology to represent ‘movement’ within a stop.

Hadas & Ranjitkar (2012) explicitly acknowledge that modelling transfers explicitly leads to lower travel times, but did not include this possibility in their model. Salonen & Toivonen (2013) included scheduled transfers in their ‘intermediate’ DTI model.

Including transfer edges considerably increases the size of the graph. In turn, the computational time increases: queries where transfer edges are included take approximately four times longer than without these edges. Over thousands of queries, this inclusion translates to hours of additional processing time, and can rule out certain types of analyses, such as a complete origin-destination matrix with a large number of zones. This may explain why many models ignore explicit transfers, even if their inclusion can increase realism.

Scheduled treatment

The scheduled treatment is identical to that used in the HHNT DTI model variation, with the sole difference being that transfer edges are included. no more than 20 minutes apart, are considered to be transfers. All other services can be accessed via ordinary waiting edges.
These are represented as the average of scheduled transfer times.

**Observed treatment**

Similarly, for the observed treatment, transfers are captured as the average of observed transfer times. Note that some transfers that are not possible in the GTFS due to hard scheduling are occasionally possible in reality. These are captured by the DTI and are thus included as a possibility in route choice.

### 3.4.3 Passenger choice and transfer (PCT) variation

The PCT variation of DTI model allows waiting time costs to vary according to whether services are considered ‘frequent’ or ‘infrequent’. When a service is deemed frequent, passengers are assumed to arrive at random, and this is reflected in expected waiting time. When services are infrequent, a mechanism for passenger arrival time choice is enabled. The observed treatment is slightly more complex than the scheduled treatment, reflecting the richer data (i.e. probability of punctual vehicle arrival).

In this study, a service is defined as being ‘frequent’ when it arrives every ten minutes, or more frequently. All other services are deemed ‘infrequent’. Although the exact demarcation of this frequent/infrequent threshold is somewhat arbitrary, there is a general consensus that it is below ten minutes of headway where random passenger arrivals begin to dominate planned arrivals (Jolliffe & Hutchinson, 1975; Zhao, Dessouky & Bukkapatnam, 2006).

Transfer time is included in the same manner as the HHT variation. IVT is again the average of all relevant records’ IVT.

**Scheduled treatment**

For the scheduled treatment, the waiting time function for frequent services is determined with a function with wide support in the literature as an improvement on the simple half-headway waiting time function when headways are irregular (Welding, 1957; Holroyd & Scraggs, 1966; Osuna & Newell, 1972; Jolliffe & Hutchinson, 1975; Bowman & Turnquist, 1981;
Benezech & Coulombel, 2013). Note that this does not necessarily mean services are unpunctual: irregular headways can be scheduled.

With random passenger arrivals, the average passenger will be waiting for a period of time equal to half of the service headway as well as half of the ratio between the variance of the headway and average headway:

\[
E(\tilde{T}_{l,j}) = \frac{E(\tilde{H}_{l,j})}{2} \times \left(1 + \text{CoV}_{\tilde{H}_{l,j}}\right)
\]

(3.1)

\(E(\tilde{T}_{l,j})\) is the expected passenger waiting time at stop \(j\) on line \(l\).

\(E(\tilde{H}_{l,j})\) is the expected headway for vehicles on line \(l\) at stop \(j\).

\(\text{CoV}_{\tilde{H}_{l,j}}\) is the coefficient of variation (CoV) of headways on line \(l\) at stop \(j\), determined with Equation 3.2.

\(\sigma_{\tilde{H}_{l,j}}\) is the standard deviation of headways on line \(l\) at stop \(j\). (This is typically only meaningful at stops with overlapping routes, and hence irregularly-spaced headways—even when only considering the schedule.)

\[
\text{CoV}_{\tilde{H}_{l,j}} = \left(\frac{\sigma_{\tilde{H}_{l,j}}}{E(\tilde{H}_{l,j})}\right)^2
\]

(3.2)

For infrequent routes, Equation 3.1 probably over-estimates passenger waiting time by preventing modelled passengers from choosing to arrive close to the scheduled arrival of their service, when they have a strong incentive to do so. The simplest way to overcome this limitation is to cap the waiting time. 7.5 minutes has been chosen as a cap on Equation 3.1 to be consistent with the New Zealand accessibility analysis methodology (Abley & Halden, 2013).

**Observed treatment**

In the observed treatment, the cost function for frequent routes requires that the observed headway and observed standard deviation of headway are substituted for the equivalents drawn from the schedule. Note that some routes identified as frequent according to the schedule may become
‘infrequent’ by definition, and vice versa, in addition to just the parameters of the equation changing.

For infrequent routes, a more complex model of passenger choice is used here: an implementation of Bowman & Turnquist’s (1981) method of determining expected waiting time in the context of uncertain vehicle arrival times. Underlying Bowman & Turnquist’s (1981) technique is the principle that as services arrive with greater regularity, a rational passenger informed of this tendency for punctual arrival would choose to plan their arrival to be coincident with the typical vehicle arrival time. As AVL data allows us to measure punctuality directly, this technique makes excellent use of this high-resolution information.

The method divides the headway into small, discrete intervals, then computes a passenger’s utility for arriving within each interval. This utility is determined by considering the expected waiting time a passenger would experience when arriving at that point in time based on the probability of their intended service having already arrived (or not), and the average headway. These utilities are then used to inform a discrete-choice logit model that computes the probability that a passenger will choose to arrive within the given interval, given their risk preferences. These probabilities are then used to represent the distribution of arrival times across the population of passengers (Bowman & Turnquist, 1981).

\[
E[W(t)] = [1 - P(t)] \times W(t) + P(t) \times W'(t) \tag{3.3}
\]

\(E[W(t)]\) is the expected waiting time for a passenger arriving at time \(t\). \(t\) is specified relative to the schedule: \(t = 0\) is the scheduled arrival time of the service, and \(t\) can be positive or negative, with a maximum absolute value of \(H\) (the headway).

\(P(t)\) is the probability that the service would have arrived before the passenger, if the passenger arrives at \(t\). Thus, \(1 - P(t)\) is the probability that the service arrives after the passenger. \(P(t)\) is drawn from the

\(^{11}\) An alternative method that could be used is described in van Oort (2011). However van Oort’s (2011) requires the use of two arbitrary parameters to demarcate an interval within which all passengers are assumed to choose to arrive at random: it simply constrains the bounds of the random arrival time decision.
cumulative distribution function of bus arrival times relative to the schedule.

$W(t)$ is the expected waiting time if the passenger arrives before the vehicle.

$W'(t)$ is the expected waiting time if the vehicle arrives before the passenger and the service is therefore missed. $W'(t)$ is equal to $H$ in addition to $-t$ if the passenger arrived before the scheduled vehicle arrival; otherwise the positive value of $t$ is subtracted from $H$.

The expected waiting time in Equation 3.3 depends on the headway, and the adherence of the intended vehicle to schedule (Bowman & Turnquist, 1981). When services are reasonably punctual, longer waiting times decrease at first with respect to $t$. As the scheduled arrival time ($t = 0$) approaches, the risk and probability of a missed (i.e. early) service increases, so expected waiting time increases (Bowman & Turnquist, 1981).

An ECDF was constructed in Python using an array of all relevant vehicle arrivals recorded in the RTI. A probability density function (PDF) is then constructed from the ECDF by querying its value for every second in the interval bound by $-H_O$ and $+H_O$ (where $H_O$ is the average observed headway of relevant services, and the central value, $t = 0$ is the scheduled arrival of the service). From these, the probability of a vehicle having come, or being yet to come, can be determined for any value of $t$ (any arrival time).

The arrival time that a passenger is most likely to select depends on the utility function employed within this model, including passenger attitudes to risk and the likelihood of early and late arrivals. With a large number of discrete intervals (such as each second from $-H$ to $+H$), the logit model is approximated by Equation 3.4 (Bowman & Turnquist, 1981):

$$f(t) = \frac{-e^{U(t)}}{\int_{-H}^{0} e^{U(t)} dt}$$ (3.4)

The utility function (Equation 3.4) is related to waiting time. Bowman & Turnquist (1981) use the following model to make this relationship explicit:

$$U(t) = a \times E[W(t)]^b$$ (3.5)
The parameters $a$ and $b$ employed by Bowman & Turnquist (1981) are calibrated from observations of arriving passengers during a morning peak period. Generally, $a$ is less than zero, given that more time spent waiting is onerous and will lead to diminished utility. The exponent parameter, $b$, represents how the disutility of waiting increases with time, since it is probably not a linear function, and is expected to be greater than 1. Bowman & Turnquist’s (1981) empirical maximum likelihood estimation of the waiting time observations lead them to choose a value of $a = -1$. However, a value of $b = 0.55$ was chosen for the exponent. That the maximum likelihood estimation used to determine $b$ suggested 0.55 indicates a possible mis-specification in the utility function (Equation 3.4): it is possible that passengers consider more than the expected waiting time when selecting an arrival time, and these other considerations are not included in the function. The given values of $a$ and $b$ parameters nevertheless matched observed arrival patterns well (Bowman & Turnquist, 1981).

The advantage of Bowman & Turnquist’s (1981) method is clear from Figures 3.2 and 3.3. A reliable service allows optimal passenger arrival closer to the schedule, and with a smaller expected waiting time at the optimal moment, relative to a more unreliable service (note the left y-axes in Figures 3.2–3.3).

The power on the passenger’s utility function used in the observed DTI model (Equation 3.5) is 1, indicating the assumption of a risk-neutral passenger who simply seeks to minimise their expected waiting time, given they are fully informed of the probability distribution of service arrivals relative to the schedule. The assumption of risk neutrality could be relaxed, if adequate information about population preferences are available to justify a different value.

In reality, no passenger is perfectly informed of this distribution, and different passengers will have different attitudes to risk, probably depending on the activity they are travelling to. However, with repeated travel a passenger can certainly form an expectation of service arrival. The perfect information assumption is still less egregious than assuming all passengers arrive at random without any information or belief surrounding service punctuality.

A particular limitation of this method is that it assumes that the arrival
Figure 3.2: The arrival pattern of a real bus route at a particular stop (red), and expected passenger waiting time in relation to it (blue). The arrival data (red) is constructed from the RTI data: this is an actual demonstration of the calculation of waiting time for an edge in the PCT DTI model variation. The service graphed here is reasonably dependable, with almost no services ever observed to have arrived before the schedule. The optimal moment to arrive for a passenger who is fully-informed of the probability distribution can be found where the blue line reaches its minimum value. Based on the method of Jolliffe & Hutchinson (1975).
3.4. *DETERMINISTIC AND TIME-ININVARIANT MODEL*

Figure 3.3: Following Figure 3.2, this graph demonstrates equivalent information, but for a much more unreliable route arriving at a particular stop. A relatively larger proportion of observed vehicle arrivals have occurred before they were scheduled to arrive (red line). Consequently, there is a greater expected waiting time at any moment than for the stop/service shown in Figure 3.2, although much of the difference is due to a larger headway for this service, giving a greater penalty for a missed service. Based on the method of Jolliffe & Hutchinson (1975).
distributions of subsequent services to not overlap. The penalty used for a missed service is the average headway. In reality arrival distributions overlap. Therefore, this model is best applied when average headway is such that services are deemed to be infrequent and overlap is only minor. When the arrival distributions of vehicles begin to considerably overlap (or even overtake), the determination of expected waiting time becomes much more complex (Bowman & Turnquist, 1981).

Once the most likely passenger arrival time was determined for each waiting edge in the network, both the arrival time and the waiting time\textsuperscript{12} are written to disk for use in the shortest path algorithm. This method requires a significant amount of precomputation, so care must be taken to ensure that the appropriate headways are determined, and that the correct vehicle arrival observations enter the array.

3.4.4 Static waiting and transfer (SWT) variation

The SWT variation is a straightforward DTI model. A global waiting time penalty of ten minutes applies when boarding a service, and this also applies to transfers. Constant waiting time cost penalties have been used by Liao et al. (2011) (7.5 minutes), and Mavoa et al. (2012) (10 minutes) in New Zealand. It is also possible to vary the penalty according to mode, but this possibility is not explored.\textsuperscript{13}

Treatments The only difference between the observed and scheduled treatments is whether the schedule- and observation-based measures of average IVT are used.

3.4.5 Headway function and transfer (HFT) variation

The final DTI variation, HFT, assumes average scheduled or observed IVT and transfer time (as in the HHT and PCT variations). However the wait-

\textsuperscript{12}That is to say, the likely consequence of the chosen arrival time.

\textsuperscript{13}For example, Peipins et al. (2011, p.677) used a penalty of 16 minutes for buses, and 6 minutes for trains. These figures were based on average headways determined from across the entire schedule, with the mode-specific average headway then divided by two.
3.5. STOCHASTIC AND TIME-INVARIANT MODEL

The time function used is specified by the UK Association of Train Operating Companies’ Passenger Demand Forecasting Handbook, and cited in Vincent (2008). The function was found to hold a very close relationship to Vincent’s (2008) adjusted mean waiting time for several hundred Auckland and Wellington transit commuters.

Treatments For any waiting edge, the cost function is $0.72 H^{0.75}$. The observed treatment simply requires the substitution of the scheduled IVT, headway and transfer interval with the equivalent aggregates from appropriate RTI observations.

3.5 Stochastic and time-invariant model

The STI model of a PT network represents travel times as random variables, with a single distribution (i.e. it does not change with respect to time). All relevant travel times, whether from the schedule, or all observations from AVL data, are combined into this function.

Two variations which primarily differ in terms of how the shortest path is determined within an STI model are considered in this thesis: the shortest expected path (SEP) and the expected shortest path (ESP). They differ primarily in terms of how the shortest path is determined. A third alternative identified in the literature (the shortest path with recourse) cannot be represented in pgRouting, because pgRouting requires that the costs are specified once before the model is run, rather than being drawn multiple times throughout the routing procedure as is required for the shortest path with recourse.

Note that an important limitation of the STI models is that all costs are independent random variables, when in practice travel times should be temporally, spatially and topologically dependent (Fu & Rilett, 1998; Miller-Hooks & Mahmassani, 2003; Nie & Wu, 2009). To illustrate why this approach is problematic, two consecutive edges of the same transit route can readily have an unusually high cost (travels very slowly) followed immediately by an unusually low cost (travels very quickly). Moreover,
no relationship then exists between these random cost draws and the expected waiting time: the stochastic waiting time for this service could have been equivalent to the service arriving very early when in fact it was moving much slower than the schedule suggests.

The remainder of this section details how the waiting time and IVT random edge cost distributions were constructed within each variation, and for the scheduled and observed treatments in each case.

3.5.1 **Shortest expected path variation**

For the **shortest expected path (SEP)**, the shortest path is the optimal path found through the use of the DTI mode, HHNT variation. 100 simulations of edge costs are then made for all edges in this pre-identified shortest path. On the basis of these 100 simulations, the overall travel time is then its own random variable (Nie & Wu, 2009).

Because the DTI model is capable of returning two paths (the optimal one from each treatment), there is actually four possible outcomes of the SEP variation of STI model (Table 3.4). The ‘treatment’ of the SEP model refers to the cost distributions in each case.

<table>
<thead>
<tr>
<th>Shortest Path</th>
<th>Cost Distributions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Scheduled DTI</strong></td>
<td>Scheduled</td>
</tr>
<tr>
<td>STI-SS</td>
<td>STI-SO</td>
</tr>
<tr>
<td>STI-OS</td>
<td>STI-OO</td>
</tr>
</tbody>
</table>

Table 3.4: Because the DTI model can return two different shortest paths according to the treatment, and the STI model can permute the costs of each edge in these paths with two different distributions according to the treatment, there is a total of four travel time estimates that can be returned under the STI SEP variation.

**Scheduled cost distribution**

For the waiting edge(s) in this optimal path, 100 values of waiting time are required. In the scheduled case, there is little guidance on how waiting time should become stochastic. In this study, the typical assumption
of random passenger arrival and constant headways that leads to a half-headway penalty in the deterministic case is adapted to the stochastic case by using a uniform probability distribution: the waiting time is equally likely to take any value between 0 and $H_S$ (Equation 3.6).

Equation 3.6 also shows the determination for infrequent routes. In order to reflect passenger choice which can occasionally go awry, a Bernoulli distribution is used to determine a waiting time penalty. Passengers wait for five minutes with $p = 0.95$. The remainder of the time, the passenger effectively ‘misses’ the bus as it has already arrived; the penalty is then equal to the five minutes as well as the average headway determined from the schedule ($H_S$).

\[
E(W) = P_X(x) = \begin{cases} 
  p = 0.05: x = 300 + H_S & \text{if } H_S > 600 \\
  1 - p = 0.95: x = 300 & \text{if } H_S \leq 600 
\end{cases}
\]

(3.6)

These probabilities are used to add a small amount of stochasticity into the waiting time function for the scheduled case. It is not clear what value $p$ should take in the Bernoulli trial, given the assumption of punctual vehicles but somewhat imperfect humans, and the lack of any information except the schedule. Consequently, the values were chosen relatively arbitrarily, although the value of $p$ is identical between treatments.

Normal distributions with the mean and standard deviation of scheduled IVT were used to represent the distribution of IVT, for all transit edges in both the scheduled and observed cases. A different distribution could have been used; yet in an analysis of AVL public transportation data, Mazloumi et al. (2010) found that peak-hour travel times followed normal distributions. Log-normal distributions provided a better fit for off-peak services (Mazloumi et al., 2010).

**Observed cost distribution**

In the observed treatment, waiting time for infrequent services (less often than ten minutes) is determined on the basis of the optimal arrival time from DTI model variant 3. To introduce stochasticity alongside this value, a vehicle arrival time is then selected at random from the complete array
of vehicle arrivals recorded in the RTI at that stop for the appropriate services. The consequence of choosing the optimal time is therefore evaluated 100 times to represent waiting time.

By definition, the passenger-selected arrival time represents the best possible choice based on the entire record of service arrivals. However, it is possible that on many occasions buses either come too early, or arrive late. This ‘consequence’ can therefore imply that waiting times may be typically small, but sometimes large, depending on how variable vehicle arrivals really are around their scheduled arrival times.

For frequent services (at least every ten minutes), the waiting time is determined as a random variable drawn from a uniform distribution, as in the scheduled treatment, but using the observed headway ($H_O$):

$$X \sim U(0, H_O) \quad \text{if } H_O \leq 600$$

3.5.2 Expected shortest path variation

The second variation of STI model, the expected shortest path (ESP), also performs 100 simulations, but in each simulation the shortest path is re-determined on the basis of the realised random variables across the entire network. The ESP imagines that an individual knows all of the edge costs in the network before beginning their trip, and therefore will always choose the shortest path for that particular realisation of the stochastic network (Polychronopoulos & Tsitsiklis, 1996; Miller-Hooks & Mahmassani, 1998).

This is essentially 100 simulations of a DTI network, with the potential for each transit edge’s cost to be different in each simulation—again leading to a random variable representing overall travel time (Frank, 1969; Hassin & Zemel, 1985; Hollander & Liu, 2008).

The edge cost probability distributions are defined identically to the
3.6 Deterministic and time-dependent model

The DTD model is unlike the time-independent models. Time no longer needs to be aggregated; individual trips can be retained in the model, so that bus trips on the same route made at 7am and 8am can have different IVTs. Transfer time can be captured directly, based on the arrival of the two concerned services. These add substantial realism, but the relative disadvantage of the DTD model is that it is harder to represent an incentive to ‘arrive to wait’ for the first service a passenger seeks to catch. An algorithm that does not consider this incentive is therefore free to simply delay the trip departure time in order to reduce waiting time to nothing, which is unrealistic. In addition, the concept of a single shortest travel time no longer exists; rather, the shortest possible path and trip time inexorably depends on the target departure or arrival time of the trip.

As discussed earlier, the DTD model is made with OTP. OTP is capable of reading OSM pedestrian data and transit data in the GTFS format. OTP Analyst is used to conduct two different forms of analysis on this network: a measure of accessibility to jobs using isochrones, and a shortest path for a single origin-destination pair as in the DTI and STI models.

Scheduled treatment

The scheduled treatment of the DTD model is formed from the Wellington GTFS, with a small number of routes excluded from consideration when analyses are run. The excluded transit agencies are: East by West Ferry; Madge Coachlines Ltd.; Tranz Scenic; Tranzit Coachlines Wairarapa; Wellington Cable Car Ltd.; and Kapiti Coach Tours Ltd. Services run by these agencies (which constitute a small number of Wellington’s PT network, but can be locally influential) are excluded from the scheduled treatment.
treatment because they do not form part of the RTI system. Had these operators’ routes been included in the analysis, differences in modelled outcome between treatments would be expected to differ not only due to lack of schedule adherence, but also simple missing data in the observed treatment. This exclusion can only increase modelled travel times, so any outputs should be considered overestimates; however, bias in the comparison of treatments is removed through this action.

**Observed treatment**

For the observed treatment, a parser was written that is capable of reading the RTI data from PostgreSQL and exporting it according to the General Transit Feed Specification, in flat .txt files, compressed into a .zip file. Essentially, this is a fake ‘timetable’ that specifies everything that is recorded by the RTI, reformatted in the General Transit Feed Specification. This fake timetable is fed into OTP as though it is a real GTFS from a transit agency. Travel time queries can then be issued with reference to the AVL data instead of the schedule.

A GTFS file generally represents a normal week of PT activities with particular calendar-based exceptions noted, consequently the AVL-based ‘fake’ GTFS feeds could only be made representing about one week at a time, as OTP could not hold more information than this in memory. Therefore, 25 GTFS feeds were made to model all 102 non-holiday working days in 2014 up to (and including) 4 July 2014.

### 3.6.1 Isochrone analysis

One method of measuring accessibility to an amenity is to measure how much of that particular amenity is accessible to a particular population. For example, how many jobs are accessible to each meshblock, where ‘accessible’ is defined as being within 30 minutes’ travel time of the meshblock via PT and walking. This example isochrone analysis is conducted for the DTD model. The OTP Analyst REST resource ‘LIsochrone’ is capable of returning such isochrones as (multi-)polygon GeoJSON objects. This form of analysis requires a variety of parameters that set the terms of the analysis, such as the departure time. These parameters are listed in Table
3.6. DETERMINISTIC AND TIME-DEPENDENT MODEL

3.5.

In the scheduled case, only one representative date is required for analysis, as all ordinary workdays follow the same schedule. However in the observed case, what is within 30 minutes’ travel varies on a daily basis. This is of course the entire point: the shape and extent of all 30 minute travel time isochrones are found with OTP analyst, and recorded to disk. These are then converted from vector to raster with PostGIS and GDAL.\(^{15}\)

As rasters, these isochrones are each represented as a binary surface: 1 (reachable) and 0 (unreachable) (see Neutens et al., 2008).

For each meshblock, a composite surface is then constructed with a straightforward matrix manipulation:

\[
\sum_{N=1}^{N} n
\]

Where \(n\) is either 0 or 1 in each case, and \(1\ldots N\) are the daily isochrones for a particular meshblock. The output is a raster dataset indicating the probability of a particular cell having been determined to be within 30 minutes’ travel time from the meshblock centroid, given all the other parameters. The method of combining these daily raster isochrone surfaces is based on the method of reliable space-time prisms proposed by Chen et al. (2013) and also seen in Neutens et al. (2008). Figure 3.4 illustrates a real example of this analysis to further clarify the concept.

The shape and size of the accessible area can then be found by querying the composite layer: which parts of the city can a meshblock reach with a particular probability of arriving within 30 minutes? Two surfaces are considered in the observed treatment: the isochrone at \(p = 0.33\) and \(p = 0.95\). The \(p = 0.95\) isochrone represents the space that a traveller could have reached on time 19 mornings in 20, which is not unreasonable for a commute trip. The \(p = 0.33\) isochrone is more lenient (places that can be reached being late no more often than 13 days in every 20, approximately), and is by thereby at least the same size but probably larger than the \(p = 0.95\) isochrone.

\(^{15}\)The spatial resolution is 25 metres; the scheduled isochrones are also converted from vector to raster, and back to vector, to eliminate differences in shape and size emerging from this data conversion process, which is not required for the scheduled treatment.
### CHAPTER 3. METHODOLOGY

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fromPlace</td>
<td>Meshblock centroid, as latitude-longitude, for each meshblock.</td>
<td>Represents a trip origin.</td>
</tr>
<tr>
<td>mode</td>
<td>TRANSIT, WALK</td>
<td>The set of modes a user may use.</td>
</tr>
<tr>
<td>time</td>
<td>07:15:00</td>
<td>The ‘arrive by’ or ‘departure’ time for the trip.</td>
</tr>
<tr>
<td>arriveBy</td>
<td>FALSE</td>
<td>When false, the time parameter represents a ‘departure’ time time for the trip. Can be made flexible with clampInitialWait.</td>
</tr>
<tr>
<td>clampInitialWait</td>
<td>600</td>
<td>The maximum amount of time (seconds) by which a trip start time can be delayed to decrease overall travel time.</td>
</tr>
<tr>
<td>walkSpeed</td>
<td>3 mi/h (≈ 4.83 km/h)</td>
<td>The global walking speed.</td>
</tr>
<tr>
<td>precisionMeters</td>
<td>100</td>
<td>The precision of the isochrone extent (input to a concave hull algorithm).</td>
</tr>
<tr>
<td>optimize</td>
<td>QUICK</td>
<td>The shortest path algorithm returns the route that requires the least amount of time.</td>
</tr>
<tr>
<td>numItineraries</td>
<td>1</td>
<td>Returns only the best route.</td>
</tr>
<tr>
<td>bannedAgencies</td>
<td>EBYW, MADG, TRAN, T2WA, KCTL</td>
<td>A comma separated list of banned agencies, using the notation of the GTFS. These agencies are excluded as they do not exist in the RTI.</td>
</tr>
<tr>
<td>maxTransfers</td>
<td>1</td>
<td>How many transfers a route can use. 1 selected to prevent spurious transfers in the absence of a behavioural weight.</td>
</tr>
<tr>
<td>minTransferTime</td>
<td>-1</td>
<td>The minimum time (seconds) a transfer can be. -1 represents no restriction.</td>
</tr>
</tbody>
</table>

Table 3.5: Parameters and their values for the OTP Analyst LIsochrone REST resource, which is used to find a 30-minute travel time isochrone for each meshblock. Additional parameters exist, and their defaults were applied. The full documentation can be found here: [http://docs.opentripplanner.org/apidoc/0.11.0/resource_LIsochrone.html](http://docs.opentripplanner.org/apidoc/0.11.0/resource_LIsochrone.html).
Figure 3.4: One week of 30 minute travel time isochrones combined into an example surface, for an example meshblock in Petone, Lower Hutt. “Timely access” is achieved when an RTI isochrone indicated a particular raster cell was accessible given the time constraint. The red line indicates the extent of the 30 minute isochrone in the scheduled case, which can be compared to the extent of the RTI-derived extent for any probability of timely accessibility. In the full analysis, RTI isochrones from 102 days are considered, not just five consecutive days, but this example is drawn from a subset of the real data.
These two surfaces, and the single scheduled-treatment isochrone are then intersected with the meshblock dataset. All intersecting meshblocks are identified, and the number of jobs\textsuperscript{16} contained within are summed in each case. These final summations constitute the values to be tested for significant differences between treatments at the meshblock level.

### 3.6.2 Trip planner analysis

A more straightforward form of analysis than the isochrone methodology is to compute the scheduled shortest travel time from each meshblock to a particular destination, and then compute the average of the observed-treatment shortest paths’ travel times in order to form an equivalent pairwise comparison. This function (OTP Planner REST resource) also requires numerous parameters to be set. These are set out in Table 3.6. These parameters are the same between treatments.

### 3.7 Methods of statistical comparison

The research question seeks to determine whether substituting the schedule in a PT accessibility model with AVL observations significantly affects the model outcome. This problem lends itself naturally to a paired, repeated measures test design. For any definition of model, two outcomes are measured for each meshblock: the travel time (or number of accessible jobs) under the scheduled treatment, and under the observed treatment. Statistical tests on paired observations are then used to determine if the differences are significantly different from zero. If the differences are greater than zero over the population of meshblocks, then the inclusion of AVL data has significantly affected the outcome.

A repeated measures test design eliminates all sources of within-subject error, because a meshblock does not change between model simulations.

\textsuperscript{16}Specifically, the count of jobs used is the total number of people who stated that they work in a particular meshblock in the 2013 Census. This value is used as a denominator for the industry-specific counts of working people under the Australian and New Zealand Standard Industrial Classification (ANZSIC) 2006 divisions. This comes from the Statistics New Zealand Census Individual Table, Part 3a.
3.7. METHODS OF STATISTICAL COMPARISON

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fromPlace</td>
<td>Meshblock centroid, as latitude-longitude, for each meshblock.</td>
<td>Represents a trip origin.</td>
</tr>
<tr>
<td>toPlace</td>
<td>(-41.291244,174.776870)</td>
<td>Trip destination: the intersection of Manners Mall and Cuba Street in Wellington City.</td>
</tr>
<tr>
<td>mode</td>
<td>TRANSIT,WALK</td>
<td>The set of modes a user may use.</td>
</tr>
<tr>
<td>time</td>
<td>08:00:00</td>
<td>The ‘arrive by’ or ‘departure’ time for the trip.</td>
</tr>
<tr>
<td>arriveBy</td>
<td>TRUE</td>
<td>When true, the time parameter represents a target ‘arrive by’ time for the trip.</td>
</tr>
<tr>
<td>walkSpeed</td>
<td>3 mi/h (≈ 4.83 km/h)</td>
<td>The global walking speed.</td>
</tr>
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<td>optimize</td>
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<td>minTransferTime</td>
<td>-1</td>
<td>The minimum time (seconds) a transfer can be. -1 represents no restriction.</td>
</tr>
</tbody>
</table>

Table 3.6: Parameters and their values for the OTP Analyst Planner REST resource, which is used to find the travel time of one trip. Additional parameters exist, and their defaults were applied. The full documentation can be found here: http://docs.opentripplanner.org/apidoc/0.11.0/resource_Planner.html.
Tests of repeated measures can then find smaller differences to be statistically significant than tests with different subjects. Interpretation may be important to determine if any statistically significant difference is actually large enough to cause consternation about the on-going suitability of a schedule to describe accessibility.

Both parametric and non-parametric tests exist for repeated measures tests of differences. Non-parametric tests are used when the differences cannot be considered normally distributed. The distribution of the outcomes for the meshblocks does not need to be normal in order to use a parametric test (which has more power), only the paired differences.

A repeated measures analysis of variance (rANOVA) test is used to compare the differences in accessibility measures for multiple model variations at once. This tests whether at least one model result significantly differs from any of the others.

\[ H_0 : \mu_1 = \mu_2 = \mu_3 = \ldots = \mu_n \]  

The alternative hypothesis is that the related population means are not all equal: that at least one mean is different to any other mean.\(^{17}\) To reject the null hypothesis of the rANOVA test, the differences of one model definition need to vary from any one of the other results (e.g. across all ten DTI specifications), so the null hypothesis is often rejected. If the null hypothesis is rejected, post-hoc tests can be used to determine where model differences are apparent, and where they are not.

A correction is often required for multiple pairwise comparisons to counteract the problem of multiplicity given that multiple comparisons are being made (multiple model variations and treatments). The Holm-Bonferroni method was used to adjust the rejection region of each comparison to account for the increased probability of type I errors when making the multiple comparisons with rANOVA (Holm, 1975).\(^{18}\)

A non-parametric alternative test to the rANOVA is the Friedman rank sum test.\(^{19}\) This considers \(n\) blocks, which in this case are the meshblocks,
and \( k \) columns (or groups), which are the two model treatments in this case. It requires a single observation for each meshblock in each treatment. The test ranks each meshblock together, then considers the values of ranks within each treatment. The null hypothesis is that, controlling for the effect of a meshblock, the rank of the meshblock’s score is the same in each of the treatments. In other words, it determines whether the ordered list of meshblock scores is consistent across treatments.

The decision concerning whether to use the parametric or non-parametric test is determined by whether or not the distribution of the differences is normally distributed. This can be determined by plotting the distribution as a histogram, and comparing it visually against a normal distribution with the same mean and standard deviation. Quantile-quantile plots, box-plots and other methods also give similar information. More formal tests of normality are also possible but were not required in this study.

Following a rejection of the null hypothesis in either the rANOVA or Friedman’s test, post-hoc pairwise tests can be used to determine precisely which pairs of models have significant differences between them. The post-hoc pairwise comparison in the parametric case is simply paired \( t \)-tests. The repeated measures \( t \)-test considers the null hypothesis that the average differences due to treatment are zero.

For the non-parametric case, the Wilcoxon signed-rank test is used to consider whether the mean ranks differ. The null hypothesis is that the population distributions of the accessibility model outcomes are identical. The sign test can be used as an alternative to the Wilcoxon signed-rank test. A post hoc test for the Friedman test is also possible.\(^{20}\) The null hypothesis is that the mean rank differences across two treatments for a paired sample are zero.

A linear regression model can also be fit to test the agreement between accessibility model predictions between treatments of the same model, given that data is paired. Given a linear model \( y = \alpha + \beta x + \epsilon \), where

---

\(^{20}\)This is only available in R as a user-contributed script: [http://www.r-statistics.com/2010/02/post-hoc-analysis-for-friedmans-test-r-code/](http://www.r-statistics.com/2010/02/post-hoc-analysis-for-friedmans-test-r-code/). No Python implementation of the test exists in major libraries (SciPy, StatsModels, etc.) at the time of writing.
a perfect model agreement is \( y = x \), the root-mean square error (RMSE) can be used to gauge how well (or poorly) the results of two models agree. Usually such a method requires a *reference method* known to be a good predictor of the phenomenon, and a *test method*, with any error then linked to the test method.

Alternatively, in the absence of a known reference method, a *comparative method* can be used. A comparative method does not make a claim to be correct, but it is still useful to compare the results of comparative and test methods to determine the RMSE of the prediction. This error is more difficult to interpret, as the true difference from reality is not known for either model; but if the differences are large, then at least one of the two models is not a good predictor. The RMSE is determined according to Equation 3.10.\(^{21}\) In terms relevant to this thesis, a set of accessibility model outputs at the meshblock level for the scheduled case constitute the comparative method, and the parallel model results for the observed treatment represent the test method. Once regressed, if the RMSE is zero, then the models accord perfectly;\(^{22}\) if RMSE is large, at least one model is a poor predictor of accessibility.

\[
\text{RMSE} = \sqrt{\frac{\sum_{t=1}^{n}(x_{1,t} - x_{2,t})^2}{n}} \tag{3.10}
\]

Where \( x_{1,t} \) and \( x_{2,t} \) represent two different estimates of the same phenomenon for the same subject, neither of which is known to be closer to reality than the other.

### 3.8 Summary

In order to ascertain whether data on the observed performance of a PT model leads to significantly different accessibility model outcomes relative to models formed only on the basis of the schedule, this thesis considers a variety of model definitions for which this answer may differ.

Five distinct variations of DTI model have been identified, as well as two models of STI model, and two forms of analysis for a DTD model.

---

\(^{21}\)The standard notation, \( \sqrt{\frac{\sum_{t=1}^{n}(y_t - \hat{y_t})^2}{n}} \), is used when there is a reference method.

\(^{22}\)Although they may still both poorly represent real behaviour and vehicle operation.
For each variation, it has been demonstrated how AVL and GTFS data is included to form the parallel model treatments.

Pairwise, repeated-measures methods of statistical analysis are used to determine whether differences in accessibility model outcomes between the scheduled and observed treatments are significant for any particular model variation. Using a repeated measures design eliminates within-subject variation, meaning the null hypothesis of a lack of difference is more readily rejected. For this reason, interpretation is required for any result deemed to be statistically significant: is the difference large in practical terms?
Chapter 4

Results

This chapter describes the results of the three public transport (PT) models, for each of their variations, between the scheduled and observed treatments. It explores circumstances that may explain the results that are external to the data treatment used.

4.1 Deterministic, time-independent model

For the deterministic and time-invariant (DTI) model, this thesis compares the two treatments (scheduled from the General Transit Feed Specification (GTFS); observed from the real-time information (RTI)) for five different model variations, representing a range of definitions of PT costs found in the literature. Refer to Table 3.3 (p.38) for a summary of the cost functions of each model variation and treatment.

Expected travel times under each treatment and model type across all Hutt Valley meshblocks are shown in Figure 4.1. Whether the use of observed data predicts longer or shorter travel times than the schedule differs according to the particular model variation. The null hypothesis of all the means being equal is rejected ($p < 0.001$, both rANOVA and Friedman rank sum tests). Post-hoc $t$-tests (Figure 4.1) indicate that the most substantial differences are found for the half-headway, no transfer (HHNT), half-headway and transfer (HHT), and passenger choice and transfer (PCT) variations, of between 3.4–5.7 minutes on average across Lower Hutt. The static waiting and transfer (SWT) and headway function and transfer (HFT) models, in contrast, have small differences, of 1.6 and
0.6 minutes. Table 4.1 can be used as a reference for estimated differences across models and between treatments.

Values of root-mean square error (RMSE) for predictions of a regression model (based on scheduled versus observed treatments) indicate the same pattern to the differences noted in Table 4.1. In particular, the RMSE for the HHNT and PCT model variations are 4.45 and 3.26 minutes, respectively (Figure 4.5).

Figure 4.1: Pairwise comparisons of the distributions of PT travel times for meshblocks. The scheduled treatment (GTFS, left) is compared to the observed treatment (RTI, right) for each DTI model variation.

The HHNT and HHT model variations predict larger RTI-derived travel times, indicating that real average headways are longer than scheduled. The direction of the difference is the opposite for the PCT variation, which applies different waiting time functions according to whether a given service is frequent or infrequent. Headways are generally longer in the observed treatment and more routes are therefore considered infrequent and subject to passenger arrival time choice. If infrequent services offering service to the CBD are also punctual, more infrequent services enable considerably smaller waiting times than under the scheduled case.

The SWT variation has a close match between observed and scheduled
Table 4.1: Mean difference in travel time between all pairs of DTI model variations. In each cell, the mean difference is first given with the significance of the p-value of the $t$-test indicated. The second line is the 95% confidence interval for the mean difference. The bottom two lines in each cell repeat the information, with the log of the estimated travel time for each model. Light purple indicates a mean difference of five to ten minutes; dark purple of more than ten minutes. Red text indicates the comparison being made is between the scheduled and observed treatments of the same model variation, which are the most important comparisons. Asterisks denote the level of significance according to usual conventions (e.g. *** corresponds to $p \leq 0.001$).
treatments. The SWT uses a ten minute waiting penalty for all services, in both treatments, so differences are limited to differences in in-vehicle time (IVT). Because all waiting penalties are equal, infrequent but direct express services are more likely to be advocated as part of the shortest path for many meshblocks than other model definitions. The HFT variation has an even greater degree of similarity between observed and scheduled treatments.

The estimated travel times for all pairs of models are positively correlated at the 99% level, which is a useful check that the pattern of relatively higher and lower travel time estimates is consistent across the geographic area between treatments. It does not, however, provide evidence that the estimates of travel time are the same under each treatment. Figure 4.2 presents a full matrix of correlation coefficients. The correlation between the same models (different treatments) is considered, as are comparisons between different model variations. Between treatments of the same model variation, correlation coefficients are only as low as 0.94 for the HHNT, PCT and SWT variations. The highest correlation between treatments is 0.98 for the HFT variation.

A non-parametric alternative post-hoc test (Friedman) that compares differences in ranks of scores according to each group has almost the same result, with one major difference. The only comparison of treatments to not return a significant difference is the HFT model variation. Indeed, the p-value for the HFT comparison of treatments is 0.403, indicating that the null hypothesis cannot be rejected, if the distribution of the differences is indeed non-normal. Given that in the paired t-tests this model has the smallest mean difference in magnitude of any of the pairs (-38 seconds), that the non-parametric test finds a non-significant result in this case is not surprising.

Because the two models give very different p-values, it is important to determine whether the distribution of the difference between the estimated travel times from the scheduled treatment of the HFT variation, and its observed treatment, is normally distributed, to decide which result holds credence. Figure 4.4 plots this histogram; and Figure 4.3 plots a comparative case (PCT model variation), where the distribution of the difference is much more obviously non-normal. The distribution of the treat-
4.1. DETERMINISTIC, TIME-INDEPENDENT MODEL

Figure 4.2: Correlation matrix between the ten different model estimates of travel times. Pearson correlation coefficients are reported in the upper panel. The lower panel plots estimated travel times, pairwise. The axes are stated in seconds. All relationships exhibit statistically significant correlation at the 99\% level. (S) and (O) refer to the scheduled or observed treatment for each model variation.
ment differences in for the HFT variation is generally normal, but with a small number of outliers that are very unusual if the data is truly normally distributed. In the centre of the distribution, there are two peaks, although this pattern is not discernible with different histogram class breaks.

Figure 4.3: The distribution of the differences in estimated travel times between treatments, for the DTI model HFT variation. The blue line traces a normal distribution with the same mean and standard deviation of the differences (it is not centred on zero). Values on the x-axis are stated in minutes. A score of zero indicates parity between the estimates.

Due to the remaining ambiguity in the normality of the distribution, and the different levels of statistical significance, another non-parametric test is considered. The Wilcoxon-signed rank test returned a median difference in scores of -40.5 seconds (scheduled treatment tends to predict shorter trips than the observed), with a 95% confidence interval of -46 to -34 seconds. The null hypothesis that the true difference is equal to 0 can be rejected ($p < 0.001$) for the HFT variation. It is, however, clear that the HFT variation does not have a considerable distance between absolute estimates of travel time with respect to the treatments.
4.1. DETERMINISTIC, TIME-INDEPENDENT MODEL

Histogram of the differences in travel time: DTI model PCT variation

Figure 4.4: The distribution of the differences in estimated travel times between treatments, for the DTI model PCT variation. The blue line traces a normal distribution with the same mean and standard deviation of the differences (it is not centred on zero). Values on the x-axis are stated in minutes. A score of zero indicates parity between the estimates.
Figure 4.5: Linear regression models for the scheduled and observed treatments of the five DTI model variations. The blue line indicates model parity; the red line is the linear model. The RMSE and $r^2$ are also given.
4.2 Stochastic, time-independent model

Recall that for the stochastic and time-invariant (STI) model, three variations of the model are considered, each with two treatments. A sharp distinction can be drawn between the expected shortest path (ESP) variation, and the two shortest expected path (SEP) variations. The ESP variation performs a shortest path analysis 100 times with different simulated costs. The SEP variations instead take as given the shortest path from either the DTI model HHNT scheduled treatment, or observed treatment, and then simulates 100 random travel times on the path. These 100 random costs may be drawn from a distribution estimated from the schedule, or from distributions constructed with automatic vehicle location (AVL) data—this constitutes the STI model treatment.

Table 4.2 presents the differences due to treatments in each of the three variations. Results very strongly indicate that the treatment produces significantly different estimates of travel time, and of the variation of travel time. The scheduled treatment universally predicts longer trips, and exhibits less variation in travel time.

<table>
<thead>
<tr>
<th>STI variation</th>
<th>Difference due to treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Median travel time</td>
</tr>
<tr>
<td>ESP</td>
<td>10.94***</td>
</tr>
<tr>
<td>SEP-O</td>
<td>4.37***</td>
</tr>
<tr>
<td>SEP-S</td>
<td>3.44***</td>
</tr>
</tbody>
</table>

Table 4.2: Summary of results for the STI model variations. See Table 3.3 (p.38) for a summary of the model variations. After 100 simulations of each model under each treatment, the median travel time for a meshblock is noted and the difference in this measure across treatments within the same model is recorded. This table presents the median of those median differences, for all included meshblocks. A positive median value indicates a larger value for the scheduled treatment. Minutes are used for units. For the median differences, the reported test result is the significance of the Wilcoxon non-parametric test.
4.2.1 Expected shortest path variation

Stated in absolute rather than relative terms, the mean of the mean travel times for all meshblocks in the observed treatment of the ESP model variation is 31 minutes; for the scheduled treatment, the value is 42 minutes. The consequent difference of around 11 minutes is larger than any of the differences found between treatments of the DTI model. The value of the median difference is within one minute of the overall mean. Figure 4.6 maps the mean travel times and their differences according to treatment.

There is not a single meshblock for which the schedule treatment predicts a shorter travel time than the observed treatment. Therefore the result of any of the statistical tests is a foregone conclusion. Using a non-parametric Wilcoxon test finds a median difference between treatments of 10.94 minutes ($p < 0.001$).

As each meshblock also holds a standard deviation of the 100 simulated travel times, the difference in the standard deviations can also be compared between models. This value is a key difference in output between the STI and DTI models. The mean standard deviation of travel time in the scheduled treatment is 1.3 minutes, whereas it is significantly higher under the observed treatment, at 3.5 minutes. Unlike the travel time, some meshblocks in the scheduled treatment actually exhibits more variance than under the observed treatment. This is a surprising result, although it is limited in spatial extent (Figure 4.7).

A histogram of the difference in standard deviation clearly shows two outliers that have a greater standard deviation of travel time in the scheduled treatment, of more than five minutes. These are also clear in the north of Map C, Figure 4.7. Non-normality is nevertheless clear; so a non-

\footnote{Variation in IVT arises in limited instances in the scheduled case due to stated variations in scheduled travel times, particularly for common lines with routes that can take different roads between the same pairs of stops.}

\footnote{Inspection reveals that these two meshblocks (1914300 and 1914400) are quite isolated from surrounding meshblocks, and their best path into Wellington always relies upon the Wairarapa train line, boarding the train at Maymorn Station. The Wairarapa train has a very high headway; and due to the model definition, the passengers boarding at Maymorn are considered to miss the train 5% of the time; with no other model option but to wait for the next train to arrive from the Wairarapa. At stations south of Maymorn, the electrified Upper Hutt train line begins, allowing shorter headways and...}
4.2. STOCHASTIC, TIME-INDEPENDENT MODEL

Figure 4.6: Maps of the mean travel time for the STI model (ESP variation), after 100 iterations. Travel time is recorded from the centroid of each meshblock to the intersection of Molesworth Street and Lambton Quay in central Wellington (not shown; off map approximately 10 kilometres to the southwest).
CHAPTER 4. RESULTS

Figure 4.7: Maps of the the variation in travel time for the STI model (ESP variation, DTI scheduled treatment path), after 100 iterations. The CoV is the standard deviation of the travel time estimates divided by the mean of those same observations, for each meshblock.
parametric Wilcoxon test for paired samples is used. This indicates that the median difference between the scheduled treatment-predicted standard deviation of travel time and that from the observed treatment is greater than two minutes; variation in travel times from the observed model is larger.

### 4.2.2 Shortest expected path variations

The SEP variation should predict travel times that are more similar between treatments than the ESP variation, since both treatments in the SEP are determined on the basis of the exact same path through the network, but with different edge cost distributions. Unlike the ESP model which has route choice, a passenger cannot avoid a missed service, so waiting times are perhaps more realistic in both SEP treatments.

As expected, the SEP variations predict a smaller difference in travel time than the ESP variation. The median differences of 3.44 and 4.37 minutes (scheduled treatment longer) are in the same direction but smaller than the treatment difference found for the DTI model PCT variation (5.7 minutes). However the distributions of the differences for the SEP variations both have heavy tails and are not considered normally distributed. Spatially, it is clear that the most considerable differences occur in the meshblocks furthest from Wellington City.

Unlike the ESP variation, the differences in travel time due to treatments in the SEP case are both positive and negative. Nevertheless, the tendency is still quite clearly for the scheduled travel time to be longer. This tendency can be quantified with the slope of a linear regression model, and the use of RMSE (Figures 4.8–4.9). RMSE values are 3.33 (scheduled DTI path), and 4.25 minutes (observed DTI path).

**Path: DTI scheduled treatment**

A Wilcoxon non-parametric test for paired samples indicates that the scheduled-treatment predicts significantly longer travel times than the observed treatment alternatives when trains are missed. In the observed treatment, while considerable variation exists in both waiting and travel time at these meshblocks, the optimal waiting time function allows early enough arrivals to avoid missed services with greater than 95% success.
Figure 4.8: Linear regression model of the STI model SEP-S variation under the scheduled (y-axis) and the observed treatments (x-axis). The blue line indicates parity between estimates, where points lie if the scheduled and observed treatments predict the same value. The red line indicates a fitted linear regression model, to which the red function, $r^2$, and RMSE apply. Units are minutes.

The travel time between the scheduled and observed treatments differs by a median of 3.4 minutes ($p < 0.001$). Figure 4.10 maps the absolute and relative estimates of travel time across treatments.

In terms of the difference in estimated standard deviation of travel time, a median difference of -1.8 minutes was found (observed treatment shows greater variability, $p < 0.001$, Wilcoxon).

**Path: DTI observed treatment**

The observed treatment predicts travel times that are shorter by 4.4 minutes at the median (Wilcoxon non-parametric for paired samples, $p < 0.001$). Figure 4.11 maps the absolute and relative estimates of travel time across treatments. Refer to Section 3.5 (p. 51) for an explanation of how this model definition differs from the previous model’s.

The observed treatment was also more variable than the scheduled treatment, with a median difference in the median standard deviation of 1.2 minutes (Wilcoxon non-parametric for paired samples, $p < 0.001$).
4.2. STOCHASTIC, TIME-INDEPENDENT MODEL

Figure 4.9: Linear regression model of the STI model SEP-O variation under the scheduled (y-axis) and the observed treatments (x-axis). The blue line indicates parity between estimates, where points lie if the scheduled and observed treatments predict the same value. The red line indicates a fitted linear regression model, to which the red function, $r^2$, and RMSE apply. Units are minutes.
Figure 4.10: Maps of the mean travel time for the STI model (SEP variation, DTI HHNT variation scheduled treatment path), after 100 iterations. Travel time is recorded from the centroid of each meshblock to the intersection of Molesworth Street and Lambton Quay in central Wellington (not shown; off map approximately 10 kilometres to the southwest).
4.2. STOCHASTIC, TIME-INDEPENDENT MODEL

Figure 4.11: Maps of the mean travel time for the STI model (SEP variation, DTI HHNT variation *observed treatment path*), after 100 iterations. Travel time is recorded from the centroid of each meshblock to the intersection of Molesworth Street and Lambton Quay in central Wellington (not shown; off map approximately 10 kilometres to the southwest).
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4.3 Deterministic, time-dependent model

For the deterministic and time-dependent (DTD) model, the observed-treatment, 30 minute PT isochrones predict significantly fewer accessible jobs than the scheduled treatment, with both isochrones ($p = 0.33$, which includes some places that are on average arrived at later than 30 minutes, and at $p = 0.95$, which demarcates all spaces that can be reached within 30 minutes while only being late once in every 20 days). Figure 4.18 demonstrates the disparity as a linear regression model.

The isochrone method is limited by the fact that very large differences only manifest for meshblocks in an approximate 30 minute transit time ‘halo’ beyond downtown Wellington (Figures 4.12–4.13). For all other meshblocks, the disparity is much smaller, even if the size and shape of the isochrones are very different. Instead, the method that compares the median travel times to the Wellington CBD between treatments is preferred.

Whether the entire Greater Wellington Regional Council (GWRC) area is considered, or just a subset of the region (Wellington, Lower Hutt, and Upper Hutt cities), the difference in travel time due the treatments is also significant at the 99% level. The median difference between the RTI average travel time and the GTFS estimate, restricted to the Wellington and Hutt Valley meshblocks, is 4.7 minutes (Table 4.3).

<table>
<thead>
<tr>
<th>DTD variation</th>
<th>Difference due to treatment</th>
<th>Median</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isochrone (0.33)</td>
<td>162 jobs***</td>
<td>14126 jobs</td>
<td></td>
</tr>
<tr>
<td>Isochrone (0.95)</td>
<td>5955 jobs***</td>
<td>20516 jobs</td>
<td></td>
</tr>
<tr>
<td>CBD commute</td>
<td>-4.7 minutes***</td>
<td>7.45 minutes</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.3: Summary of results for the DTD model variations. The median difference is stated in terms of the scheduled outcome parameter subtracted by the observed treatment’s outcome. The test in all three cases is the Wilcoxon non-parametric test for paired samples.
4.3. DETERMINISTIC, TIME-DEPENDENT MODEL

Figure 4.12: Map of the differences in the number of accessible jobs, with meshblocks with a difference less than $-1.5 \times \text{IQR}$ (inter-quartile range) symbolised in blue. White polygons represent meshblocks with less extreme differences, and grey meshblocks have no data (OTP could not place the meshblock centroid on a pedestrian edge within tolerance). No meshblocks have a difference value greater than $1.5 \times \text{IQR}$. Difference computed between the 30 minute $p = 0.95$ RTI isochrone and the 30 minute GTFS isochrone.
Figure 4.13: Map of the differences in the number of accessible jobs, with meshblocks with a difference less than \(-1.5 \times \text{IQR}\) symbolised in blue, and those with a difference greater than \(1.5 \times \text{IQR}\) as orange polygons. White polygons represent meshblocks with less extreme differences, and grey meshblocks have no data (OTP could not place the meshblock centroid on a pedestrian edge within tolerance). Difference computed between the 30 minute \(p = 0.33\) RTI isochrone and the 30 minute GTFS isochrone.
4.3. DETERMINISTIC, TIME-DEPENDENT MODEL

4.3.1 Limitations

Departure time is not constrained to any particular time for the analysis measuring travel time to the CBD. In practice, the algorithm does not capture waiting for the first service, which may be the only service required for a shortest path. This applies equally in the scheduled and observed treatments, but implies that although in reality a bus arrival is uncertain, the algorithm does not acknowledge that. If an allowance is made for this property of the algorithm, we would expect the observed treatment to predict even longer travel times due to passenger ‘safety margins’ and occasional missed services. Yet despite this, the observed treatment still predicts significantly longer average travel times than the scheduled treatment: a median difference of 10.4 minutes. These differences are not normally distributed with some extreme values. All of the meshblocks exhibiting a difference greater than $1.5 \times \text{IQR}$ are in the Kāpiti Coast, Wairarapa, and the northern part of Porirua.

While there is a high level of between-day variation in the travel times returned from the observed treatment, as demonstrated in Figure 4.14, the standard deviations of the differences for the Kāpiti Coast, Wairarapa, and the northern part of Porirua find that these sub-regions exhibit extremely high levels of variation in travel time, with standard deviations between 30 to 77 minutes in the most extreme cases (see Figure 4.15). Although the standard deviations for most meshblocks appear to be a fair reflection of the real variation in travel time, that these extreme values are so high and so geographically concentrated suggests that there is an unforeseen data issue. However, legitimately cancelled services would also be expected to cause such an effect, as the scheduled treatment cannot simulate real

\[3\] It is most likely that this stems from missing observations in the AVL on particular dates, due to hardware malfunction. That is, some services did run but their AVL information was not captured. This effect is suspected because a similar pattern does not emerge in the time-independent models, where all available AVL data points were aggregated, rather than accessed individually. Additionally, it is important to note that the highest standard deviations are found north of the Porirua inlet. Here, there is only the Kāpiti train line, and no alternative bus services. The Kāpiti train already exhibits high variation, and combined with missing observations, modelled standard deviations of travel time above 60 minutes become marginally plausible, though are still considered unrealistic.
Because meshblocks with suspiciously high standard deviations may bias the result, the travel time analysis is run again, restricted to meshblocks from Wellington, Lower Hutt, and Upper Hutt cities. In Figure 4.15, it is clear that these sub-regions exhibit much more subdued variation and generally have more similar estimates of travel time across treatments than the outlying areas. Therefore this subset is likely to support the null hypothesis of there being no difference in the estimates of travel time across treatments.

After subsetting, the difference in travel time is reduced to a median difference of 4.7 minutes—but this difference is still significantly different from zero. Figures 4.16 and 4.17 maps the difference between the two treatments. It demonstrates that areas where the schedule is optimistic, pessimistic, or neither, are clustered. This is reasonable when nearby meshblocks will rely on the same PT routes.
Figure 4.14: Comparison of the 10th and 90th percentile of travel times returned from the OTP Planner REST resource, across different days, at the meshblock level. As noted in the text, the arrival time is constrained to be by 8am, and only routes that exist in both the GTFS and the RTI are included (e.g. no harbour ferry). Travel time units are minutes.
Figure 4.15: Map of the standard deviation of the daily travel times to the marked destination (arriving before 8am, for 2014 school-term and working weekdays), observed treatment. Grey meshblocks have no data (OTP could not find a valid walk and transit route). Note the possibly erroneously high standard deviation of travel time for the inset area north of Porirua (Camborne, Plimmerton and Pukerua Bay).
4.3. DETERMINISTIC, TIME-DEPENDENT MODEL

Figure 4.16: A contrast between the GTFS-derived expected travel time to the marked destination, and the median travel time from the RTI estimates. Note that the bus and train routes have only been presented separately in order to distinguish them; both modes are present in each model.
Figure 4.17: The difference in travel time (minutes) between the median RTI estimate and the GTFS estimate (travel time to marked destination in DTD model).
4.3. **DETERMINISTIC, TIME-DEPENDENT MODEL**

(a) Note that the left figure is a subset of the right figure.

Figure 4.18: Linear regression models for the scheduled and observed treatments of the trip- and isochrone-based DTD analyses. The blue line indicates model parity; the red line is a linear best fit. The RMSE and $r^2$ are also given. When comparing these to the DTI and STI equivalents, note that the DTD figures also include results for over 1 000 meshblocks in Wellington City, in addition to the Hutt Valley.
Chapter 5

Discussion

Empirically, it has been determined that for each model comparison considered in this thesis, the use of real-time information (RTI) automatic vehicle location (AVL) data in place of the General Transit Feed Specification (GTFS) produces a difference in estimated travel time (or number of accessible jobs) that is statistically significant at the 99 percent level. Therefore, the models based on the public transport (PT) schedule do not produce similar accessibility model results to equivalent models with travel time estimates made from RTI data (time-at-location AVL). However, the difference that AVL data makes to estimates of travel time varies in magnitude and direction according to the various model forms that are considered, and the difficulty of incorporating AVL data in each model variation also varies. A practitioner will need to balance the expected difference in accessibility model outcome that AVL data makes, the intended accuracy of their application, and the costs of including AVL data, when deciding whether the schedule is adequate for a particular model.

Literature on the issue of unreliability of travel time in accessibility modelling is limited in several respects. Firstly, the literature has mostly only been applied to the driving case, which is not subject to many of the same issues as PT, such as including a mid-trip ‘safety margin’, departure time choice constrained by a schedule, route cancellations, and overlapping routes (e.g. Choosumrong et al., 2012; Chen et al., 2013; Mattingly & Morrissey, 2014). The consideration of unreliability in route choice and accessibility modelling is still at an exploratory level, even for driving (Brennand, 2010). Both of Wellington’s existing strategic PT planning mod-
els, the Wellington Transport Strategy Model (WTSM) and the Wellington Public Transport Model (WPTM), use cost functions that do not allow for the inclusion of reliability (Brennand, 2010). While a body of theoretical literature exists considering how PT unreliability should affect estimates of different aspects of PT travel, this theory has not been fully exploitable until the recent adoption of complete, high-resolution AVL technology (Mazloumi et al., 2010; Li-Jun et al., 2011; Nassir et al., 2011; Yetiskul & Senbil, 2012).

This study demonstrates the application of the theoretical literature on reliability of PT in conjunction with AVL data for a wide variety of PT accessibility models within the Wellington region, particularly the Hutt Valley. The results extend the limited research into the potential disparity between accessibility model estimates based on the schedule and those based on real PT performance (e.g. Firmani et al., 2013; Allulli et al., 2014; Delling et al., 2014). Compared to the existing insights, which consider journey planners and do not attempt to quantify expected travel time error, this research considers deterministic and time-invariant (DTI) and stochastic and time-invariant (STI) network models in addition to a time-dependent one. Practitioners designing a PT system model that is similar to any of the representations considered as part of this thesis should note that the use of the schedule is not a reliable proxy for the observed information over a city. However, in the absence of a complete AVL record, using the schedule to construct a model may still be necessary. In such cases, this study contributes an estimate of the magnitude of travel time error that can be expected for different definitions of PT accessibility model.

This research corroborates and extends Chen et al.’s (2013) results, which introduced the ‘reliable space-time prism’ approach. Chen et al. find that for the driving case, more than half of the edges in their study area have highly variable travel times; they argue that the use of deterministic edge costs to model travel and activity decision-making is simply erroneous when road networks are congested. This thesis finds that for morning

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1The SATURN platform used in the regional WTSM has a hard-coded generalised cost function that does not include reliability; the scheduled in-vehicle time of rail services in WPTM is taken at face value, with no consideration for the schedule adherence of trains, which has ranged from 70–98% (arrival within 5 minutes of schedule) in 2014, depending on the line (Jones & Ford, 2012; Tranz Metro, 2014)
weekday commutes to the city centre, the schedule predicts commutes that are several minutes faster or slower than those found using AVL data, depending on the model definition. The largest travel time disparity is greater than ten minutes. Similarly, for a 30 minute Wellington commute, a median of nearly 6000 fewer jobs can be reached according to the AVL (with on-time arrival of $p = 0.95$), relative to the estimate developed from the scheduled model.

5.1 DTI models with constant waiting time

For Lower and Upper Hutt, using average observed in-vehicle time (IVT) with a constant waiting time penalty of ten minutes (including transfers) leads to travel times that are typically shorter than the schedule, when travelling to Wellington City. Because the model applies the same waiting penalty in both treatments, this difference accrues entirely due to the expected IVT of services recorded from AVL data relative to their scheduled IVT. A different waiting time penalty, could be used (e.g. Lester & Walton, 2009; Liao et al., 2011; Mavoa et al., 2012) and it would have the same result in terms of the effect of treatments. This difference is closer to zero than all but one of the DTI models.

5.2 DTI models with waiting time as a function of headway

Three DTI models use a waiting time penalty that is a proportion of the headway, similar to many models found in the accessibility literature (e.g. O’Sullivan et al., 2000; Hess, 2005; Hadas & Ranjitkar, 2012; Tribby & Zandbergen, 2012; Abley & Halden, 2013; AECOM, 2013; Salonen & Toivonen, 2013). The differences between the three model variations consist of the precise waiting time function, and whether transfers are considered separately; waiting time is in either case a function of the (scheduled or observed) frequency. Other waiting time variations are not considered in this thesis, in particular the application of behavioural weights (Abley & Halden, 2013; AECOM, 2013).
The two models with a linear half-headway waiting time penalty lead to much larger average differences (3.4 to 5.4 minutes). DTI models without constant or punctuality-informed estimates of arrival time choice lead to shorter overall travel times under the schedule. This result suggests that real headways for shortest paths are likely to be much longer than the scheduled headways on the shortest scheduled paths, for the morning peak.

A waiting time function which returns smaller values whose growth with respect to headway becomes smaller as the headway increases (DTI HFT model) demonstrates the smallest difference between treatments. This suggests that such a definition of waiting time can be used to reduce disparity between scheduled and observed model outcomes. As already noted, observed headways tend to be longer than scheduled headways, but this model definition offsets this effect. That is not to say that this model is ‘better’ than the others: such an inference is not valid, as no model has undergone calibration with real PT route and waiting time choices. Instead, it indicates that with the headway function and transfer (HFT) model definition, this study provides evidence that the disparity between schedule-based and AVL-based shortest paths is minimised, relative to the other model definitions that have been considered.

5.3 DTI models with passenger arrival time choice

The passenger choice and transfer (PCT) DTI model variation uses the most complicated waiting time function, with Bowman & Turnquist’s (1981) method determining the expected waiting time of passengers using infrequent services. A random arrival assumption is still applied to waiting for frequent services (arriving every 10 minutes or more frequently), adjusted by the coefficient of variation in vehicle arrivals. The scheduled treatment, in the absence of an equivalent to observed vehicle arrival times, uses a cap on waiting time of 7.5 minutes, as in the New Zealand accessibility analysis methodology (Abley & Halden, 2013). This model comparison therefore represents a particularly important comparison between treatments. On the one hand, it considers a peer-reviewed method of determining transit accessibility on the sole basis of limited information from a
5.4 STI MODELS: expected shortest path

A major advantage of having access to AVL data is that probability distributions of travel time can be constructed that allow repeated simulation and the production of range estimates of travel time (Hollander & Liu, 2008). The STI models produced in this thesis are not true stochastic models in that the shortest path algorithm doesn’t directly access probability distributions. Support for such a representation does not exist in existing software implementations of spatial network datasets. Instead, probability distributions are constructed for the stochastic edges, and a distinct
DTI network (and shortest path solution) constructed for each of 100 simulations, under both shortest expected path and expected shortest path model variations.

Introducing limited forms of variability into the STI model based on the scheduled costs, the GTFS-based models produce longer estimates of travel time than the RTI-derived estimates in all three STI model variations. Indeed, the single largest difference between treatments for any of the nine weekday commute experiments was found for the expected shortest path (ESP) STI network variation, which was just short of eleven minutes. This pattern of a pessimistic GTFS travel time is universal for all meshblocks under this variation: not a single meshblock exhibits an average AVL-derived travel time that is shorter than the schedule equivalent.

This comprehensive result is particularly notable: the magnitude of this difference is likely to be due to the definitions of waiting time between treatments, which are somewhat different due to limitations imposed by the scheduled data, and the additional opportunities afforded by the AVL data. For the scheduled case, a uniformly random arrival assumption is used. This stands in contrast to an estimate of passenger choice in the observed case, with stochasticity introduced by simulating random bus arrivals drawn from the real observational record, in conjunction with the optimal decision. Although we have seen in the results of the PCT DTI model that the passenger choice assumption leads to faster travel times overall with AVL data, this result is compounded in the stochastic case.

Some of the treatment difference arises from the definition of IVT: in each treatment, IVT is considered to be a normally distributed variable, with the mean and standard deviation from scheduled or observed travel time records as appropriate. As edge costs in the observed treatment are generally more variable, the distribution is wider for each edge. As the ESP model re-simulates route choice in each iteration, the algorithm tends to select unusually fast transit edges in each simulation. Such unusually fast edges are not as common in the scheduled treatment, with more constrained travel time distributions. This does not suggest that either the observed or scheduled treatment is in some way unfair. Both models follow a standard definition and their results reflect the different input data available for their construction.
5.5 STI models: shortest expected path

The shortest expected path models have a median difference in treatments in the same direction as the expected shortest path, but the magnitude is less: 3.4 minutes in the case where the optimal path is pre-determined from the scheduled treatment of the DTI half-headway, no transfer (HHNT) variation; and 4.4 minutes from the observed treatment of that DTI model.

Regression models of the differences in travel time predicted by the scheduled and observed models indicate that although there is a bias of several minutes in favour of faster GTFS trips, the correlation between the results is strong. Mismatch increases as travel times became longer. On the basis of this, there is some evidence that for an STI shortest expected path (SEP) model, a simple transformation of results obtained from a model based on the GTFS may be appropriate, to compensate for an absence of AVL data. However, this result may not hold out of sample.

Results for the comparison of variation in travel time for the STI models reveal a surprising result: the magnitude of the differences in median standard deviation of travel time are quite small and are bidirectional when the relationship is expected to be comprehensively in favour of less variability from GTFS estimates. Although, as expected, the RTI is more variable overall, with a median difference of 1.17–1.84 minutes, the amount of variability in the GTFS travel time estimates is still surprisingly large. In particular, there are a small number of influential meshblocks that exhibit extremely large variability in travel time under the scheduled treatments.

The surprisingly high route variability in the scheduled treatment reflects the presence of common lines in shortest paths (the major source of variable travel times in the scheduled treatment). The variability of the outcome is not an adequate reflection of travel time variation caused by traffic congestion or anything else, merely the different routes one may choose when travelling between the same pair of stops, and their different travel times recorded in the schedule (particularly express and non-express train services).

In the STI model, like the others, preferences are not considered, beyond the single goal of minimising expected travel time. Preferences could also be included as stochastic parameters, potentially from meshblock-
level demographic data (Hollander & Liu, 2008). This method would potentially shrink the estimated difference for this model comparison: as people are probably more likely to be risk averse than risk seeking, they would tend to trade speed for dependability in the observed treatment, diminishing the relative pessimism of travel time under the scheduled treatment.

5.6 DTD models

Two accessibility analyses are conducted with the deterministic and time-dependent (DTD) PT model to test the effect of treatments: a comparison of travel time into the Wellington CBD (8AM target arrival time), and a count of the number of jobs within a 30 minute commute (leaving between 7:15–7:25AM) for 102 working-weekday mornings in 2014. Both analyses indicate that the schedule is considerably optimistic about travel time. This effect is in the opposite direction to the result found in the STI models, and most of the DTI models, including the model with a constant waiting time penalty. This is important to note, because the DTD model is unable to model an incentive to ‘arrive to wait’, but does correctly capture the time penalty of transferring.

The conservative isochrone (30 minutes’ travel time with $p = 0.95$) indicated that the scheduled estimate of accessibility is uniformly optimistic for Wellington meshblocks, while the larger $p = 0.33$ isochrone revealed the existence of a small number of places where more jobs could be reached within 30 minutes’ travel than predicted by the schedule—which is expected given that the $p = 0.33$ isochrone, by definition, includes areas that are more often than not unreachable within 30 minutes’ travel time. That the $p = 0.33$ variant still demonstrated that the GTFS was optimistic in measuring accessibility reveals just how optimistic it is.

5.7 Limitations

A significant limitation of the DTD methodology in this thesis is the extent to which hardware failures of the AVL system lead to absent observations: buses and trains that did run, but were not recorded in the AVL,
cannot enter the observed treatment. Using the GTFS, this problem is avoided. The average rate of AVL hardware failure is approximately 3% of services not being captured on any one day (Nathan White, 2014, pers. comm.). Conversely, AVL data captures the real effect of service cancellations through exactly the same mechanism, while the GTFS alone cannot. The direction of the bias is therefore difficult to discern and may cancel overall, according to the distribution of cancellations and hardware failures. This problem is very difficult to address: there is no reliable method to find missing records of the database.

Care must also be taken to ensure that the AVL is a complete record of the transit system. To facilitate the fair comparison of treatments in this methodology, routes in the GTFS are deliberately excluded from consideration because they are not part of Wellington’s RTI system. If the emphasis of this study were instead to accurately measure employment accessibility rather than measuring disagreement in pairwise estimates of accessibility, the GTFS may have been a superior option to the AVL purely due to the completeness of the GTFS. Alternatively, AVL data could be used to supplement GTFS data, with scheduled timetables as a fallback for services absent from any AVL record, or simulations made on the basis of the characteristics of similar routes.

Whether the results differ for access to particular industries has not been considered, so these results are considered provisional. Access to industrial and other similar jobs occurring predominantly outside of the central business district (CBD) may or may not differ considerably between scheduled and observed treatments. The pattern of bus and train routes leading to non-CBD employment hubs is considerably different and generally inferior. Yet the routes to these non-CBD districts may not be subject to the same degree of traffic congestion, so the use of the GTFS may in fact be adequate.
Chapter 6

Conclusions

The General Transit Feed Specification (GTFS) is increasingly being published by public transport agencies, and consumed by application developers and transit riders. It is also becoming an incredibly useful tool for public transport planning and various forms of accessibility analysis. However, non-real-time GTFS can only provide information about what should happen. It provides no information about how reliably the timetable is replicated in practice or how passengers respond to reliability through their selection of origin departure time, stop arrival time, and route selection.

A variety of representations of public transport systems exist, and this thesis explains their structure, assumptions, limitations and provides detail on how three models can be formed from the GTFS, using the Wellington region as case study. The model construction is then repeated using automatic vehicle location (AVL) data (Wellington Regional Council’s real-time information (RTI) system). This allows comparisons to be made between models that vary as little as possible between the GTFS representation and the AVL representation. A summary of model results is given in Table 6.1.

6.1 Key findings

- Existing accessibility models that include public transport (PT) overwhelmingly assume that the schedule is an adequate reflection of reality. As so many models take the schedule as a given, few models have adequate data on the true performance of a PT system to be...
<table>
<thead>
<tr>
<th>Model</th>
<th>Variation</th>
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<th>Test</th>
<th>Sig.</th>
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<td>HHNT</td>
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<td>No transfers; average IVT; waiting time $1/2H$</td>
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<td>Average IVT and transfers; waiting time $1/2H$</td>
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<td>STI</td>
<td>ESP</td>
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<td>Edge costs are the same as in STI-ESP, without route choice (optimal path from DTI HHNT variation, observed treatment)</td>
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<td>+5,955 jobs</td>
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<td>For non-holiday weekdays in 2014, departing 7:15–7:25 AM</td>
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<td>+162 jobs</td>
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Table 6.1: Summary of model results. Differences are stated as GTFS (scheduled) treatment less the RTI AVL (observed) treatment. n refers to the number of meshblocks for which a treatment difference was estimated (for computational and data-integrity reasons, not all meshblocks could be considered in the time-independent model forms).
6.1. KEY FINDINGS

able to move beyond simple assumptions of passenger arrival and waiting times.

- AVL data provides an opportunity to substitute schedule information with a high-resolution and (almost) complete representation of real system performance. Section 3 discusses a methodology to do so.

- There is only some existing evidence to suggest that journey planner results made from AVL significantly differ from those made solely on the basis of the schedule. By extension, accessibility models are subject to the same problem. This research extends the literature on this point, and intentionally assesses a variety of accessibility models to ensure the results have wide applicability.

- For every model constructed in this study, the use of AVL data as a substitute for GTFS data lead to differences in estimated travel time (or number of accessible jobs) that are significant with 99 percent confidence. Travel towards the Wellington City central business district (CBD) is considered, for working mornings during early 2014.

- For deterministic and time-invariant (DTI) models with constant waiting times, the GTFS is found to be pessimistic, by 1.6 minutes.

- For DTI models with waiting time penalties that are functions of the scheduled or real frequency of service, the GTFS is found to be relatively optimistic, predicting travel times that are, at the median, 38 seconds to 5.35 minutes different from the observed model.

- Comparing a capped, random passenger arrival waiting time penalty within a schedule-based model against a model made with RTI data and using theory about passenger arrival time choice to guide estimates of waiting time lead to the biggest disparity between treatments for the DTI models, of 5.7 minutes. The observed model predicts faster trips, indicating that a waiting time penalty that assumes random passenger arrivals may over-estimate waiting time.

- For all stochastic and time-invariant models, however defined, the GTFS data predicts pessimistic travel time estimates, relative to ob-
served treatment models using probability distributions formed from
the RTI. This applies strongly to the stochastic and time-invariant
(STI) model that re-calculates shortest paths (expected shortest path
(ESP)).

- Deterministic and time-dependent (DTD) model results indicate that
in a time-dependent representation the GTFS is relatively optimistic
about travel time in the weekday morning peak. In particular, this
model has the poorest root-mean square error (RMSE), representing
widespread disagreement in travel time and estimated employment
accessibility between the treatments across all meshblocks.

6.2 Implications

Using AVL data in PT accessibility modelling leads to outputs that are sig-
nificantly different to outputs of models constructed only with the sched-
ule. As such, this thesis recommends that AVL data be used in preference
to the schedule if AVL data is present and it is possible to include this data.
If third-party software must be used to read standardised data (such as the
GTFS) into an accessibility model, it is noted that converting AVL to match
this text-file-based standard is reasonably straightforward using most rel-
lational database management systems, provided the required fields can
be populated (route, trip, stop, etc.).

In cases where AVL data is not collected or made available, this thesis
contributes an estimate of the likely magnitude AVL data can make to es-
timation of travel time and accessibility. Although the exact magnitude
of difference will vary between geographical and temporal contexts and
scales, this thesis nevertheless provides a first-pass estimate for models
under a wide variety of definitions (whether time-dependent, time-independent
and deterministic or stochastic).

Comparing across model variations, rather than the scheduled and ob-
served data treatments, models using a waiting time penalty equal to half
of the headway lead to longer travel times than models which made differ-
ent assumptions (passenger choice, static penalties, and non-linear func-
tions of frequency). The magnitude of inter-model difference is frequently
larger than the difference due to the treatments. Theoretically and in practice, the half-headway assumption overestimates waiting time. Although the passenger choice model is only practicable with AVL data stored in a (very large) array, the model does make use of the theoretical literature on how people make this decision in the face of uncertainty. Because PT headways are not constant even when only using the schedule, at the very least, a model that accounts for the variation in headways should improve estimation.

6.3 Future research

Further work needs to be performed to determine how the inclusion of behavioural weights on travel time might widen model predictions, or narrow them. For instance, if waiting times are longer according to AVL data, a relatively large multiplier of time would exacerbate the differences found in this research.

In all cases, neither treatment has undergone calibration by comparing the modelled shortest paths with routes passengers really choose (with their own preferences and imperfect information). Consequently, while pairwise disagreement can be captured, no judgement can be passed about the true accuracy of any model in describing the true movement of passengers. This is particularly acute for using RMSE to measure disagreement. Although all models demonstrated RMSE values of at least one minute, this result could emerge even if both treatments do a poor job at describing accessibility. Comparing AVL model results against a calibrated model could assess its absolute accuracy, rather than its accuracy against a scheduled model.

Attention in this thesis has focused on disparity in raw estimates of travel time, with only one of the models being applied to a pure accessibility analysis to all workplaces as listed in the 2013 New Zealand Census. The spatial pattern of employment may mean that the results of this analysis, which focuses on trips to a single high-employment city centre, may not be applicable in non-mono-centric cities or for access to industries and other amenities that are not as centrally located. For the same reason, consideration of whether the differences can be also be anticipated in off-peak
and weekend periods is also warranted. This thesis outlines methodologies to construct the models that can be used to consider accessibility in other cities.
Bibliography


BIBLIOGRAPHY

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Appendices
Appendix A

Modelled public transport routes

Table A.1: The bus and train routes included in both the GTFS and RTI. A bold common name for the route indicates whether the routes operates in or through Lower Hutt or Upper Hutt: these routes are the only ones considered by the DTI and STI models. The DTD model considers all of the routes in this table. Note that for the RTI, direction is not expressed by a single field but is rather inferred from the order of stops visited. Additional routes exist in the GTFS that are not included in this table. In particular, these include all of the Wairarapa bus routes.

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<th>Common route name</th>
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Appendix B

Time-independent model topology

With the exception of particular sub-models ignoring transfer edges, all deterministic and time-invariant (DTI) and stochastic and time-invariant (STI) model forms in this thesis use the same network topology. Only the cost functions vary to give rise to different estimates of travel time.

In the simplest case, a topological walking network can be constructed as a graph of walking edges connected by nodes (intersections) where turns are permissible. To model a public transit network that interfaces with the pedestrian network, special nodes that are spatially and topologically coincident with particular pedestrian edges can be added: these represent features like bus stops or train stations.

pgRouting only allows edge costs (i.e. travel time) to be stored and enter the shortest path calculation on edges and not nodes. Consequently, a simple topological construction cannot correctly model important properties of using public transport, including waiting and transferring. These aspects require their own edges. A single public transport stop then becomes a complex arrangement of walking edges, edges linking the pedestrian edges to the transit stops, edges from transit stops to each of the outgoing vehicle services, and edges allowing direct transfers between transit vehicles. Visually, this becomes unmanageable when many different routes use the same stop, and travel to a variety of different downstream stops.
B.1 The common lines problem

Accurate accounting of in-vehicle time (IVT), waiting and transfer time becomes more complicated once the presence of multiple routes serving the same stop pairs is acknowledged. For illustration, if you are at a transit hub intending to travel down a transit corridor served by many of the routes that visit the hub, your expected time-independent waiting and transit time should be determined from all services that you are indifferent between. Ten different routes may serve ten different ultimate destinations, but all make use of the transit corridor. If you are travelling to one of the ultimate destinations, you are only concerned with one of these ten; if, however you are simply travelling down the corridor, you can pick from any one of these ten routes.

This general issue is known as the ‘common lines’ problem, and represents a very important edge case for time-independent models which can struggle to correctly model this situation without an unwieldy topology (Yin et al., 2004). The usual approach to developing estimates of waiting time and IVT that are robust to this issue is to consider both an edge’s origin and destination nodes and a subset of ‘attractive’ arriving services (Chriqui & Robillard, 1975; Marguier & Ceder, 1984; De Cea & Fernández, 1993; Lam et al., 1999; Sumalee et al., 2011; Szeto et al., 2013).

The routes that are deemed attractive must at a minimum be equivalent in terms of the origin and destination stops that they serve. Therefore, both an express and a non-express service may be deemed part of the attractive set. Topologically, this means that public transport (PT) routes can no longer be modelled as a small number of route-specific sequential edges. Instead, all possible connections between any pair of stops served by more than one route need to be included in the topology.

Topologically, Chriqui & Robillard (1975) therefore modelled the common lines problem not as one where there are multiple edges between the stops A and B, but rather where only a single edge is stored, that contains composite properties of all attractive lines. Figures B.1–B.3 demonstrate how such a topology may be formed for a schematic network. Note that this is a very simplified image, only showing the effect of considering common lines on the number and arrangement of transit edges. In addition to these, waiting time edges must be constructed for each transit edge, as
well as transfer edges. The real situation can become messy, and introduce non-trivial computational strain.

Figure B.1: A hypothetical network graph, \( G_0 = (N, L) \). Where all nodes \( N \) and non-common lines \( L \) are represented. Adapted from Yin et al. (2004), itself based on De Cea & Fernández (1993).

Figure B.2: If direct stop connections are also shown, additional edges are required to represent travel from A to Y, and X to B. Adapted from Yin et al. (2004), itself based on De Cea & Fernández (1993).

Figure B.3: The network shown with only the common lines. Edge \( S4 \) represents common line travel starting at X, and ending at Y. For such a trip, a passenger will be indifferent among all arriving red and purple services, as they overlap for this section. However, with A as an origin and travelling to Y, a passenger can either choose to take common edge \( S3 \) and transfer to \( S4 \), or simply take \( S2 \), which requires no transfer. Unlike non-common lines, common lines only represent direct travel between two stops. For a long overlapping sections of stops, the number of required edges to adequately model the situation expands considerably. Adapted from Yin et al. (2004), itself based on De Cea & Fernández (1993).

Methods that attempt to define the subset of all possible routes that a passenger would consider ‘attractive’, do this based on speed, directness,
frequency, mode, or some other consideration (Chriqui & Robillard, 1975; Marguier & Ceder, 1984; De Cea & Fernández, 1993; Lam et al., 1999; Gentile, Nguyen & Pallottino, 2005; Szeto et al., 2013). It is also possible to consider that all arriving routes are equally appealing, and to aggregate all possible direct connections between any two routes into a common line with a travel time and frequency defined as the weighted average of the components (De Cea & Fernández, 1993). This latter approach is taken in the time-independent models of this thesis.

Whether all arriving routes or only a subset of them are deemed attractive, the waiting time function is in either case defined with respect to the ‘attractive’ set of routes. Each common line edge in the network can be associated with attributes such as mean travel times (actual or scheduled), variances and co-variances of the IVTs, as well as capacity or available seats (Szeto et al., 2013).

### B.1.1 Implementation

In this thesis, the network of common lines is first found by taking each route and expanding it from a series of sequential edges, to a representation where each stop has as many edges emanating from it as it has downstream stops. The number of additional edges this requires may be found for a single route according to the following formula:

\[
\frac{n^2}{2} - 0.5n
\]  

(B.1)

where \( n \) is the number of stops on the route. This is less than \( n^2 \) because additional edges are only required in the direction of travel.

After manipulating each route into a common lines representation, these edges need to be aggregated according to their origin and destination stops. It is at this point that conditional logic could also be considered (e.g. only including routes in the aggregation that meet a frequency threshold).

Actually aggregating all of the common lines with SQL is considerably computationally intensive, particularly for automatic vehicle location (AVL) data, which requires an aggregation across \( \frac{n^2}{2} - 0.5n \) additional edges for every route, every trip, for every day of observation. The joining and aggregation procedure, with bottlenecks identified and assisted with ap-
B.2. TRANSFERS

Transfer time is more difficult to determine than aggregate IVT, headway, and punctuality as the appropriate time penalty differs depending on whether the transfer movement is between common lines, non-common lines, or a mixture of the two (non-common to common; common to non-common). Each of the tens to hundreds of different possible transfers at a single stop can potentially have different scheduled and observed transfer times, according to the arrival patterns of the incoming and outgoing routes associated with each edge.

All possible transfer opportunities were determined from the schedule, under the definition of a transfer as being the arrival of a service from a different route (identified by the GTFS route_id) at the same stop or at any stop within 80m, where the scheduled difference in arrival time is greater than or equal to 20 seconds and less than or equal to 20 minutes. 20 minutes was used as the upper limit as that is the definition of a ‘transfer’ to get a fare concession when using Snapper between bus services.

Aggregations were made on the basis of a derived table of all transfer opportunities according to the first stop (A), the egress transfer stop (X1), the boarding transfer stop (X2), and the final egress stop (B). Stops X1 and X2 may or may not be the same stop.

In order to determine a time-independent estimate of transfer time
suitable for inclusion in the DTI network, aggregations were then made over groups of routes, A, X1, X2 and B stops for transfers between non-common edges only; and over A, X1, X2 and B stops for transfers between common edges. That is, for transfers between common lines, one need not consider what routes are being used, just the transfer stop location.

When considering a transfer from a common line to a non-common line (and vice versa), there is a corner case: by definition, the common line represents travel on the same route as the transferring non-common line. Therefore, care should be taken to ensure that the aggregate transfer time does not incorporate transfers from the route represented by the non-common line, to itself. The calculation of a transfer prevents such ‘self-transfers’ from being included in the cost estimate.
Appendix C

Time-independent headway

In addition to topology, the definition of a headway is also consistent between time-independent models that are considered in this study. This section of the appendix details why the idea of a headway is simple to define conceptually, but difficult to correctly implement in a PT model.

The standard definition of a headway is the interval of time between the arrival of two vehicles at the same stop, that are going to the same destination stop. This applies equally to both non-common and common lines, although it is necessary to ensure that if an attractive subset of a common lines is considered, that the average headway refers only to these.

In the specific context of waiting time, the headway is typically some portion of the ‘penalty’ associated with missing an intended service. Passengers who miss their service can also walk to their destination, walk and then wait at a different stop, or take a different service from the same stop (change route): in practice the penalty for a missed service can be less than the headway, which is merely the upper bound.

An important corner case is that two routes could be considered equivalent for the purposes of determining headway if they offer service from the same origin but to slightly disjoint destinations (near enough for a passenger to trivially walk between them, but not identical). This situation has not been considered in this thesis.
APPENDIX C. TIME-INDEPENDENT HEADWAY

C.1 Calculating headway from the GTFS

For the purposes of calculating headway definitively from a GTFS file, I follow the recommendation of the Transit Capacity and Quality of Supply Manual (TCQSM), 2nd Edition (Kittelson & Associates, KFH Group, Parsons Brinckerhoff Quade & Douglass & Hunter-Zaworski, 2003). A scheduled headway is thus the scheduled time between any vehicles travelling between any pair of stops, even if not via the same route between these stops.

Further, for each vehicle arrival for any stop pair, the next equivalent service must be a service that serves the same pair of stops and arrives at least three minutes after the first vehicle, at the origin stop. This follows the TCQSM guideline regarding the determination of service frequency at transfer points (Kittelson & Associates et al., 2003, p.3.29):

Some judgement must be applied to bus stops located near timed transfer centers. There is a considerable difference in service from a passenger’s perspective between a bus arriving every 10 minutes and three buses arriving in a row from a nearby transfer center every 30 minutes, even though both scenarios result in six buses per hour serving the stop. In general, buses on separate routes serving the same destination that arrive at a stop within 3 minutes of each other should be counted as one bus for the purposes of determining service frequency [level of service (LOS)].

Attention is also paid to ensuring that service additions and exceptions are considered (from calendar_dates) in addition to the usual schedule for a particular date. Following this definition, the headway of every scheduled service was obtained.

The maximum value an individual headway may take in this study is two hours. This is to avoid very large (perhaps peak-only) services from unduly influencing the determination of the average headway, while acknowledging that one or more component services has a very considerable penalty when missed.

These same definitions of headway calculation are applied to the RTI, too. However this is of course a much more considerable computational
task: every recorded arrival in the RTI has at least one associated headway. In the case of common lines, each observation may have several headways, depending on a passenger’s ultimate destination.

When calculating headways from the RTI for a particular time-window (e.g. 7–9am as in this study), it is not enough to just subset the AVL data to observations from this window. This is because such an action will prevent the determination of headway for a service that (for example) arrives at 8:55am, with the next relevant service due in 30 minutes.